Child Labor, Intergenerational Earnings Mobility and Economic Growth

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Submitted to the Department of Economics of Amherst College
in partial fulfillment of the requirements for the degree of
Bachelor of Arts with Honors

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May 5, 2006

Acknowledgements

I am grateful to Professor Sami Alpanda for thoughtful advice. I also would like to thank Professors Daniel Barbezat, Jessica Reyes, Steven Rivkin, and Geoffrey Woglom for useful comments. Professors Daniel Velleman and Katherine Tranbarger in the Department of Mathematics and Computer Science helped me in simulation programs. Katherine Mooney at the Writing Center provided invaluable editorial assistance. This research has been supported by the Student Research Award from the Dean of the Faculty's office. My education at Amherst College has been funded by the Doshisha Neesima Scholarship and the Rotary Foundation Ambassadorial Scholarship.

Abstract

This paper analyzes the effects of child labor on intergenerational earnings mobility and economic growth. The model is a two-period overlapping-generations model based on Maoz and Moav (1999). In their model, if an individual invests in education in the first period of his life, then he will become educated labor in his second period. Their model suggests that if an economy has a larger initial number of educated workers, then the economy has higher wage equality and higher intergenerational earnings mobility. I introduce to their model the choice to work in the first period of an individual's life. In comparison to their model, I find that a child-labor economy has lower mobility, a lower rate of growth, and a larger number of educated workers in the steady state. Increase in the number of uneducated workers by the number of child workers increases the wage of educated adult labor and decreases the wage of uneducated adult labor, with the result that more children of educated parents can easily invest in education, and fewer children of uneducated parents can afford education.

1 Introduction

In many parts of the world, children cannot afford education, and they work. This paper analyzes the effects of child labor on intergenerational earnings mobility and economic growth. The model is a two-period overlapping-generations model based on Maoz and Moav (1999). I introduce the choice to work in the first period of an individual's life. In comparison to their model, a child-labor economy has lower mobility, a lower rate of growth, and a larger number of educated workers in the steady state.

Economists have been trying to figure out the relationship between economic growth, income equality, and intergenerational earnings mobility empirically. Some suggest that a more developed country has higher equality and higher mobility. Ozdural (1993) finds that the United States has higher equality and higher mobility than Turkey does. Others suggest that growth does not change income equality. Li, Squire, and Zou (1998) find that income inequality is stable within countries using the Gini coefficient for 49 developed and developing countries between 1947-94. Some suggest that income equality is positively correlated with mobility. According to Atkinson, Rainwater, and Smeeding (1995), Sweden has highest income equality and the States has lowest equality among OECD countries. Gustafsson (1994), Bjorklund and Jantti (1997), and Osterberg (2000) find that Sweden has higher intergenerational mobility than the States. Gottschalk and Smeeding (1997) find that Germany has higher income equality than the States does. Couch and Dunn (1997) find that Germany has higher intergenerational earnings mobility than the States does. However, the amount of panel data measuring intergenerational mobility is very limited. Some developed countries like Japan and most developing countries do not have data comparable to the Panel Study of Income Dynamics or the National Longitudinal Survey, which are commonly used to estimate intergenerational mobility in the States. Solon (2002) explains that sample differences bias estimation results and that it is difficult to compare different countries' mobility using different surveys. Lee and Solon (2006) also point out that even in one nation different surveys give different estimation results due to large sampling errors. Thus economists do not have empirical consensus on the relationship between growth, equality, and mobility.

It is, however, important to speculate about this relationship. The Maoz-Moav model gives a theoretical explanation that a more developed economy has higher wage equality and higher intergenerational earnings mobility. Their production function consists of educated labor and uneducated labor. If an economy is more developed – in other words, if the number of educated workers in the economy is larger – then the wage of educated labor is higher, the wage of uneducated labor is lower, so the wage gap between educated labor and uneducated labor is smaller than it is in a less developed economy. A more developed economy also has higher mobility, because the smaller wage gap enables more children of uneducated parents to invest in education.

We see that these explanations do not change when we introduce child labor to their model. A more developed economy has higher wage equality and higher mobility. Child labor, however, in both less developed economies and more developed economies, results in children of educated parents investing in education more easily. It becomes much more difficult for children of uneducated parents to acquire education.

Economists pay attention to the roles of children's ability and their transfers from parents. Galor and Tsiddon (1997) assume that capital markets are perfect, and thus ability is more important than parental capital for children's mobility. In their model, high ability is needed to use technology until the economy has innovation and technology becomes more accessible to everyone. First, the wage gap between individuals who can use technology and those who cannot increases, and mobility based on ability increases. Once technology becomes more accessible, the wage gap and mobility decrease and become more persistent based on individuals' parental capital.

If we assume imperfect capital markets, then transfers from parents become more important for children's mobility, as Owen and Weil (1998) suggest. Since capital markets are imperfect, even if a child has high ability, if he receives a very small transfer from his parent, then it is hard for the child to acquire education. Their production function consists of capital, uneducated labor, and educated labor. Having the two state variables, capital and labor, leads to multiple steady-state equilibria; two economies starting with identical conditions except different initial wealth distributions can reach different rates of mobility, inequality, and per capita income.

The existence of multiple steady states makes it difficult to analyze the dynamics of growth path, so Maoz and Moav (1999) consider the one state variable, labor. Their production function consists of educated labor and uneducated labor. Imperfect capital markets, specifically no capital, is assumed. In a less developed economy, many children who have high ability and uneducated parents cannot afford education because of the small transfers from the parents. As an economy grows, the transfers increase, and it will be possible for some children of uneducated parents to acquire education.

I keep the no capital assumption of the Maoz- Moav model when I analyze a child-labor economy. In reality, a child from a low-income family often cannot borrow capital for his education even if he has high ability. Not many countries offer financial aid as the States does. I consider a transfer from a parent, namely, a parent's wage, as the dominating factor in his child's education decision.

Hassler and Mora (2000) model a transfer as information such as how to run a business. In their model, when the growth rate is low, the social environment is changing slowly. Information given by a parent is more important for children's future earnings than their actual ability, so mobility is low. When an economy grows rapidly, the social environment is changing quickly. Parental information is less valuable, and ability is more important, so mobility is high. Their interpretation of a transfer is interesting, but for simplicity, we stick with Maoz and Moav's view which treats a transfer strictly as a part of a parent's wage.

I explain the detail of the Maoz-Moav model in chapter 2, and then we discuss a child-labor economy in chapter 3. The last chapter is a conclusion.

2 Maoz-Moav Model

Maoz and Moav's (1999) overlapping-generations model suggests that a more developed economy has higher wage equality and intergenerational earnings mobility.

2.1 Description of the Model

We consider a two-period overlapping-generations model with no population growth and no capital. We have infinite discrete time, and, in each period, educated and uneducated workers whose numbers are determined endogenously produce a single homogeneous good that can be used for either consumption or for investment in human capital.

Technology

The production function in period t is $Y_t = AE_t^{1-\alpha}U_t^{\alpha}$ where Y_t is aggregate output, A is total factor productivity, E_t is the number of educated workers, and U_t is the number of uneducated workers. The number of people in each generation is normalized to one: $E_t + U_t = 1$. The wage for educated labor, w_t^e, and the wage for uneducated labor, w_t^u, are

$$w_t^e = \frac{\partial Y_t}{\partial E_t} = (1-\alpha)AE_t^{-\alpha}(1-E_t)^{\alpha},$$

$$w_t^u = \frac{\partial Y_t}{\partial U_t} = \alpha AE_t^{1-\alpha}(1-E_t)^{\alpha-1},$$

respectively.[1] These marginal products tell that as the number of educated workers increases, the wage of educated labor increases, and the wage of uneducated labor decreases, so the wage gap

[1] $w_t^u < w_t^e$ iff $E_t < 1-\alpha$.

between educated labor and uneducated labor becomes smaller.

Individuals

An individual lives for two periods and has one parent and one child. In the first period of his life, he does not work but receives a transfer from his parent and spends all of it on either consumption or a combination of consumption and investment in education. If he only consumes the transfer and does not invest in education, then he will become uneducated labor in his second period. If he invests in education in his first period, then he will become educated labor in his second period. He works in his second period and receives a wage based on his labor type. He consumes some of his wage and gives the rest to his child as a transfer.

We denote U^i as lifetime utility for an individual i born in period t, c_t^i as his consumption in t, c_{t+1}^i as his consumption in t+1, and x_{t+1}^i as his transfer to his child in t+1. His lifetime utility function is

$$U^i = \log c_t^i + \log c_{t+1}^i + \log x_{t+1}^i .$$

His budget constraints in periods t and t+1 are

$$c_t^i + \delta^i h_t^i = x_t^i$$

$$c_{t+1}^i + x_{t+1}^i = w_{t+1}^i$$

where $\delta^i = 1$ and $w_{t+1}^i = w_{t+1}^e$ if he invests in education; $\delta^i = 0$ and $w_{t+1}^i = w_{t+1}^u$ otherwise. h_t^i is his cost of education.

The cost of education is not the same for everyone, and it depends on an individual's ability. θ_t^i is an ability parameter and uniformly distributed over the interval $(\underline{\theta}, \overline{\theta})$, where $\underline{\theta} \geq 0$. The higher i's ability is, the lower θ_t^i is.

$$h_t^i = \theta_t^i (a + bY_t)$$

where $0 \le a$, $b \in [0,1]$. Thus h_t^i is uniformly distributed over the interval $(\underline{h}_t, \overline{h}_t)$, namely $(\underline{\theta}(a + bY_t), \overline{\theta}(a + bY_t))$.

Feasibility Condition

In period t, consumption of children investing in education, their costs of education, consumption of children not investing in education, consumption of educated labor, and consumption of uneducated labor sum to the output in the economy.

Firms maximize profits (zero profit), individuals maximize utility, and the feasibility condition holds.

2.2 Solving the Model

We first maximize the t+1 part of the utility function, $\log c_{t+1}^i + \log x_{t+1}^i$, subject to the t+1 part of the budget constraints, $c_{t+1}^i + x_{t+1}^i = w_{t+1}^i$. The maximum utility in t+1 is

$2\log w_{t+1}^i - 2\log 2$ with $c_{t+1}^i = x_{t+1}^i = \frac{w_{t+1}^i}{2}$, which is true regardless of an individual i's choice in t. If i invests in education in t, his lifetime utility will be

$\log(x_t^i - h_t^i) + 2\log w_{t+1}^e - 2\log 2$, and if not, $\log x_t^i + 2\log w_{t+1}^u - 2\log 2$.

Thus he invests in education in period t

iff $\log(x_t^i - h_t^i) + 2\log w_{t+1}^e - 2\log 2 \ge \log x_t^i + 2\log w_{t+1}^u - 2\log 2$

iff $$h_t^i \le x_t^i \left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e} \right)^2 \right]$$

Applying the result $x_{t+1}^i = \frac{w_{t+1}^i}{2}$ to x_t^i, he invests in education

iff $$h_t^i \le \frac{w_t^i}{2} \left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e} \right)^2 \right]. \tag{1}$$

An individual i who is a child of an educated parent invests in education in t

iff $$h_t^i \le \frac{w_t^e}{2} \left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e} \right)^2 \right].$$

An individual i who is a child of an uneducated parent invests in education in t

iff $$h_t^i \le \frac{w_t^u}{2} \left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e} \right)^2 \right].$$

These conditions tell us that individual i's education decision in t depends on his parent's wage in t and the wage gap between uneducated labor and educated labor in t+1. For example, for two individuals whose education costs are the same, if one is a child of an educated parent and the other is a child of an uneducated parent, then the child of an educated parent can invest in education more easily than the other due to the different transfers they receive. If the wage of educated labor decreases and the wage of uneducated labor increases as the economy grows, then the situation for these two individuals will be different from the above case. There are two effects of this change. Although the child of an educated parent still has an advantage due to his parent's wage, since the wage gap will be smaller, the advantage will be smaller. Meanwhile, the gap of future wages will also be smaller, so both children's incentives for education investment will be smaller.

We denote $\hat{h}_t^e = \frac{w_t^e}{2}\left[1-\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right]$, $\hat{h}_t^u = \frac{w_t^u}{2}\left[1-\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right]$,

and we call them critical values.[2] A child of an educated parent invests in education if and only if his education cost is lower than or equal to the critical value, $\hat{h}_t^e$. Similarly, a child of an uneducated parent invests in education if and only if his education cost is lower than or equal to the critical value, $\hat{h}_t^u$. In period t, given the number of educated workers, all the children of uneducated parents have the same critical value, and all the children of educated parents share another critical value.

An individual's cost of education is based not on his parent's labor type but on his own ability and the level of output in the economy.

$$h_t^i = \theta_t^i \left(a + bY_t\right)$$

and h_t^i is uniformly distributed over the interval $\left(\underline{h}_t, \overline{h}_t\right)$. We denote the c.d.f. functions of $\hat{h}_t^e$ and $\hat{h}_t^u$ as $F_t\left(\hat{h}_t^e\right)$ and $F_t\left(\hat{h}_t^u\right)$, respectively.[3] $F_t\left(\hat{h}_t^e\right)$ means the proportion of children who have educated parents and invest in education in t, and $F_t\left(\hat{h}_t^u\right)$ means the proportion of children who have uneducated parents and invest in education in t. The equation for the dynamic behavior of the number of educated workers is

$$E_{t+1} = E_t F_t\left(\hat{h}_t^e\right) + \left(1 - E_t\right) F_t\left(\hat{h}_t^u\right).$$

Since E_t is the number of educated parents in t, $E_t F_t\left(\hat{h}_t^e\right)$ is the number of children who have educated parents and invest in education in t. Since $\left(1 - E_t\right)$ is the number of uneducated

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[2] We express $\hat{h}_t^e$ and $\hat{h}_t^u$ using the marginal products of educated labor and uneducated labor in Appendix for 2.2.

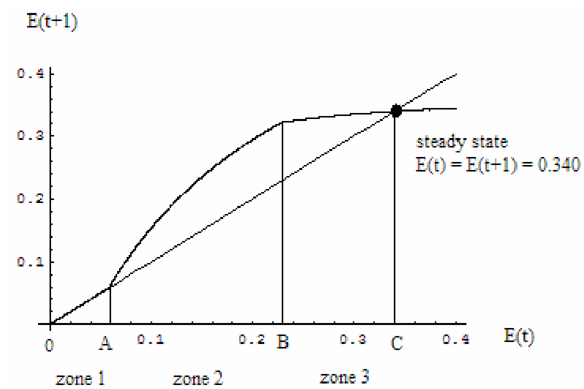
[3] The equations for $F_t\left(\hat{h}_t^e\right)$ and $F_t\left(\hat{h}_t^u\right)$ are in Appendix for 2.2.

parents in t, $(1-E_t)F_t\left(\hat{h}_t^u\right)$ is the number of children who have uneducated parents and invest in education in t.

2.3 Numerical Example

We let $A=1$, $\alpha=0.5$, $\overline{\theta}=5$, $\underline{\theta}=1$, and $a=b=0.05$.[4] The results are in the following graphs, where E_t, E_{t+1}, w_t^e, w_t^u, $\hat{h}_t^e$, $\hat{h}_t^u$, $\underline{h}_t$, and $\overline{h}_t$ are denoted by E(t), E(t+1), we(t), wu(t), h^e, h^u, h-low, and h-high, respectively.

Graph 2.1

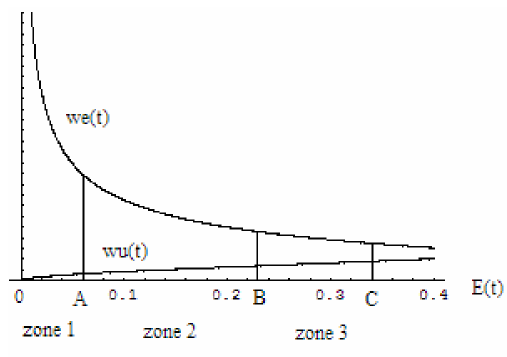


Graph 2.1 shows what the number of educated workers in period t+1, E_{t+1}, will be given the initial number of educated workers in t, E_t. If an economy has a very small initial number of educated workers (zone 1), then the dynamic behavior function stays on the 45 degree line. This means that the numbers of educated workers in the two periods are the same, and the economy does not grow; we call zone 1 a poverty trap. If an economy's initial number of educated workers is large enough (zone 2), then the behavior function is above the 45 degree

[4] The equations with these numerical values are in Appendix for 2.3.

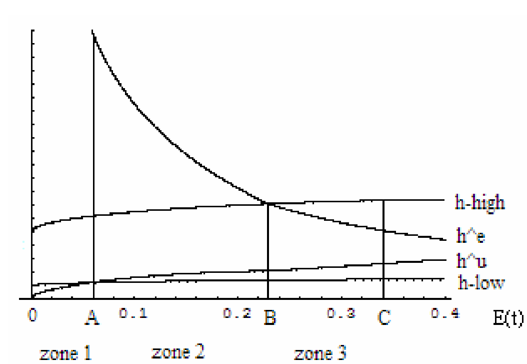
line, so the number of educated workers in t+1 is larger than that of t, which means the economy grows. If an economy's initial number of educated workers is very large (zone 3), then the economy grows but not as fast as an economy in zone 2. The function goes back to the 45 degree line at C, and the intersection of the function and the line is the steady state for the number of educated workers.

Graph 2.2



Graph 2.2 tells what the wage of educated labor, w_t^e, and the wage of uneducated labor, w_t^u, are given the number of educated workers in an economy. The former is higher than the latter, and the former decreases faster than the latter increases.[5]

Graph 2.3



[5] $E_t < 1-0.5$ guarantees $w_t^u < w_t^e$.

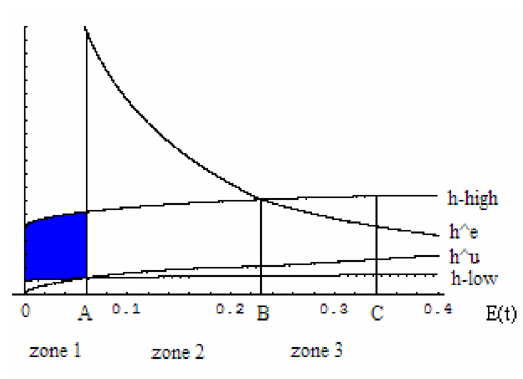
Graph 2.3 explains the relationship between $\hat{h}_t^e$, $\hat{h}_t^u$, $\underline{h}_t$, and $\overline{h}_t$. Children's costs of education are uniformly distributed between $\underline{h}_t$ (h-low) and $\overline{h}_t$ (h-high) depending on their abilities. An individual invests in education if and only if his cost of education is lower than or equal to the critical value, $\hat{h}_t^e$ or $\hat{h}_t^u$, depending on his parent's labor type. Table 2.1 and the following pictures show who invests in education in each zone.

Table 2.1

		children of educated patents investing in education	children of uneducated parents investing in education
zone 1	$\hat{h}_t^u < \underline{h}_t < \overline{h}_t < \hat{h}_t^e$	All	None
zone 2	$\underline{h}_t < \hat{h}_t^u < \overline{h}_t < \hat{h}_t^e$	All	Between $\underline{h}_t$ and $\hat{h}_t^u$
zone 3	$\underline{h}_t < \hat{h}_t^u < \hat{h}_t^e < \overline{h}_t$	Between $\underline{h}_t$ and $\hat{h}_t^e$	Between $\underline{h}_t$ and $\hat{h}_t^u$

In zone 1, we have no mobility. Children whose costs are in the following shaded area invest in education; all of them are the offspring of educated parents.

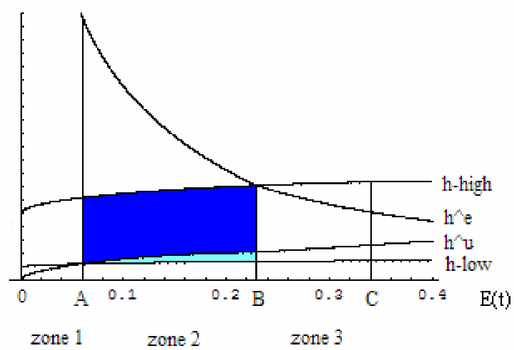
Graph 2.3 (zone 1)



Since the wage gap between educated labor and uneducated labor is very large, the gap in transfers is also very large. All the children of educated parents invest in education, which means that the child who has an educated parent and the lowest ability among all children can invest in education. No child of uneducated parents invests in education, which means that the child who has an uneducated parent and the highest ability cannot afford education. Thus the numbers of educated workers in t and t+1 are the same.

In zone 2, we have only upward mobility. Children of educated parents whose costs are in the darker (upper) area and the lighter (lower) area invest in education, and children of uneducated parents whose costs are in the lighter (lower) area invest in education.

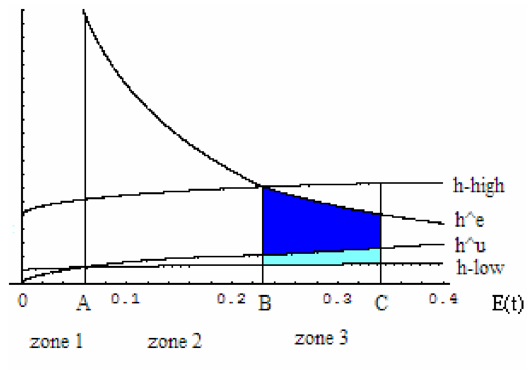
Graph 2.3 (zone 2)



The wage gap is small enough so that children who have uneducated parents and very high abilities can invest in education. All children of educated parents invest in education regardless of their abilities. Thus the number of educated workers in t+1 becomes larger than the number of educated workers in t.

In zone 3, we have both upward and downward mobility. Children of educated parents whose costs are in the darker (upper) area and the lighter (lower) area invest in education, and children of uneducated parents whose costs are in the lighter (lower) area invest in education.

Graph 2.3 (zone 3)



The wage gap is very small. Children who have uneducated parents and higher abilities invest in education. Children who have educated parents and very low abilities find that not investing in education in t and becoming uneducated in t+1 maximizes their lifetime utility, so they do not invest in education. Since the wage of educated labor decreases faster than the wage of uneducated labor increases, the number of children who have educated parents and decide not to invest in education is larger than the number of children who have uneducated parents and invest in education. Thus the number of educated workers in t+1 is larger than that in t, but this increase is not as large as in zone 2.

In conclusion, the economy has three types of mobility and corresponding growth given the initial number of educated workers in an economy. If the number is very small (zone 1), then the wage gap is very large, and the gap in transfers is very large. Only children of educated parents can invest in education, and no child of uneducated parents can afford education, so we have no mobility, and the economy does not grow. If an economy's initial number of educated workers is large enough (zone 2), then the wage gap is small enough so that children who have uneducated parents and very high abilities can invest in education. All children of educated parents can invest in education. We have only upward mobility, and the economy grows. If an economy's initial number of educated workers is very large (zone 3), then the wage gap is very small, so children who have educated parents and very low abilities decide not to invest in education. Children who have uneducated parents and higher abilities invest in education. We have both upward and downward mobility, so the economy grows but not as much as in zone 2. The economy eventually reaches the steady state for the number of educated workers. The number of educated workers indicates how much an economy is developed. As the number becomes larger, an economy becomes more developed. The Maoz-Moav model suggests that a more developed economy has higher wage equality and higher intergenerational earnings mobility.

3 Analysis of Child Labor

This chapter investigates the effects of child labor on intergenerational earnings mobility and economic growth. I first assume myopic expectation with regard to the wage differential between educated labor and uneducated labor in order to analyze a child-labor economy using wages in exactly two periods. Then I introduce the choice to work in the first period of an individual's life. We see that when an economy has child labor, children of educated parents can invest in education more easily, and it is much more difficult for children of uneducated parents to acquire education. A child-labor economy has lower mobility, a lower rate of growth, and a larger number of educated workers in the steady state.

3.1 Myopic Expectation

In the Maoz-Moav model, the future wage gap worked as an incentive for education investment, and rational expectation was assumed. In order to analyze child labor using wages in exactly two periods in section 3.2, we introduce myopic expectation. We do not consider child labor in this section but focus on how myopic expectation changes a non-child-labor economy. We assume that an individual does not know the future wage gap and uses the current wage gap as a prediction for the future wage gap.

An individual i invests in education if and only if his education cost is lower than or equal to his critical value. We recall equation 2,

$$h_t^i \le \frac{w_t^i}{2}\left[1-\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right], \qquad (1)$$

and replace w_{t+1}^u and w_{t+1}^e by w_t^u and w_t^e, respectively. We obtain

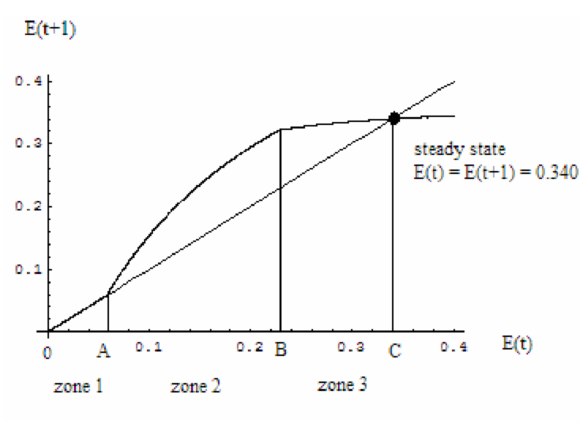
$$h_t^i \le \frac{w_t^i}{2}\left[1-\left(\frac{w_t^u}{w_t^e}\right)^2\right].$$

This replacement has two effects on i's incentive in its size and its rate of change. If there is no growth, then the wage gap does not change, and the individual's prediction is exactly right. If an economy grows, educated labor's wage increases, and uneducated labor's wage decreases, so the future wage gap is smaller than the current wage gap. The individual makes a decision based on the current wage gap, which is larger than the future wage gap, so he has a larger incentive to acquire education in comparison to the Maoz-Moav model. Since the size of the incentive is larger than before, its rate of change for the same amount of time is also larger than before.

In the following graphs, we see how these changes affect growth and mobility. We use the same numerical values as those from Maoz and Moav (1999). The graphs on the left side are from chapter 2, and the graphs on the right side are the results given myopic expectation.

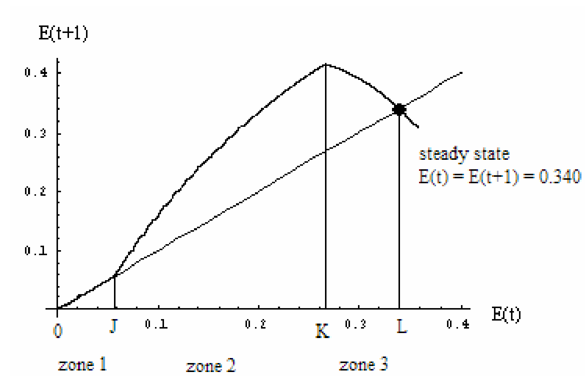
Rational Expectation

Graph 2.1



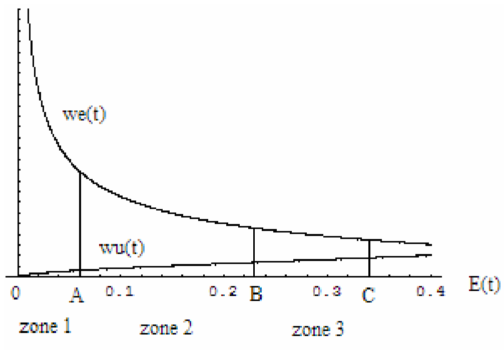
Myopic Expectation

Graph 3.1

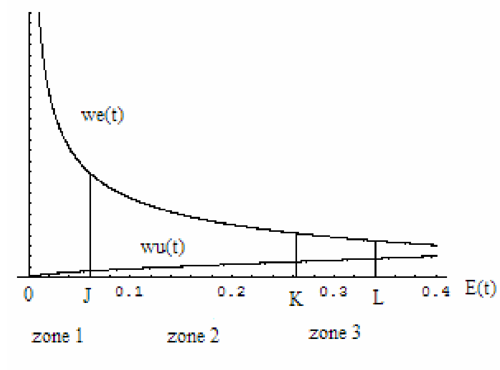


Two economies reach the same level of steady states but arrive there in different ways. Since economies do not grow in zone 1, there is no difference there. Myopic expectation gives a wider range of zone 2 and a higher growth rate in the zone. In zone 3, the growth rate decreases rapidly, which makes the range of the zone smaller.

Graph 2.2

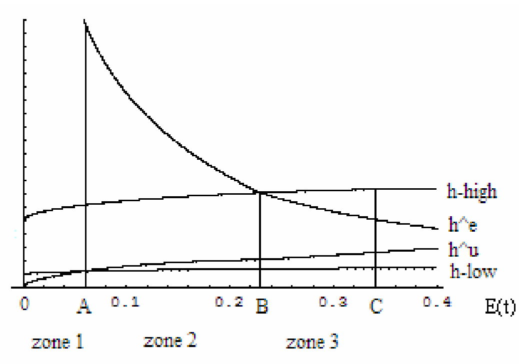


Graph 3.2

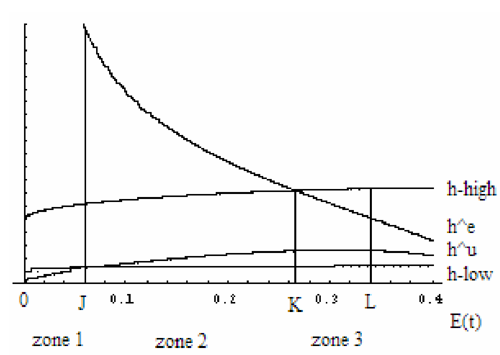


Graphs 2.2 and 3.2 are identical to each other, because we did not change the wages.

Graph 2.3

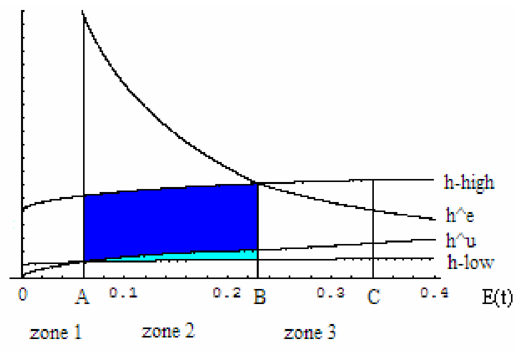


Graph 3.3

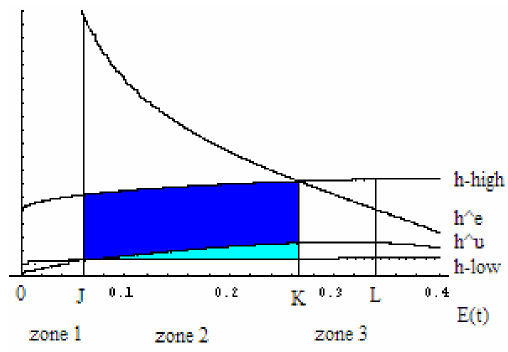


There is no change in intergenerational mobility in zone 1, since an economy does not grow there. In zone 2, more children invest in education than before. The following picture describes zone 2. Children of educated parents whose costs are in the darker (upper) areas and the lighter (lower) areas invest in education, and children of uneducated parents whose costs are in the lighter (lower) areas invest in education.

Graph 2.3 (zone 2)



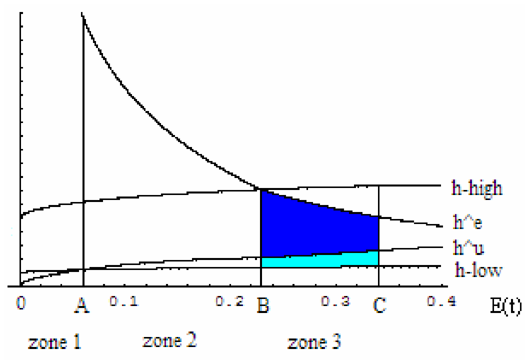
Graph 3.3 (zone 2)



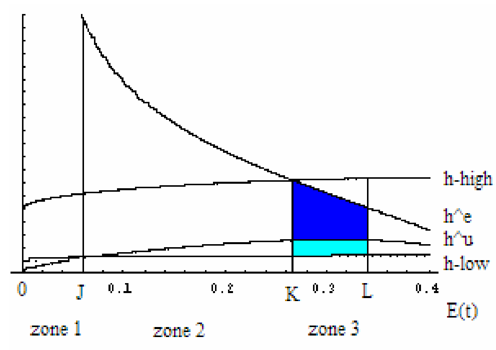
Comparing the two lighter (lower) areas, more children who have uneducated parents and higher abilities invest in education due to larger incentives. The wider rage of zone 2 in graph 3.3 also indicates that children who have educated parents and lower abilities invest in education in a more developed economy. In other words, the child who has an educated parent and the lowest ability decides not to invest in education at B in graph 2.3, but in graph 3.3, he keeps investing until K due to a larger incentive.

In zone 3, more children decide not to invest in education than before. Children of educated parents whose costs are in the darker (upper) areas and the lighter (lower) areas invest in education, and children of uneducated parents whose costs are in the lighter (lower) areas invest in education.

Graph 2.3 (zone 3)



Graph 3.3 (zone 3)



Since the rate of change in incentive is larger than before, once the wage gap becomes very small at K, more children decide not to invest in education. They maximize their lifetime utility by consuming more in their first period.

Our myopic expectation assumption leads to a larger size of incentive and its larger rate of change. The larger size results in more children investing in education in zone 2, so the economy has a wider zone 2 and a higher growth rate there. The larger rate of change results in more children becoming uneducated, so the growth rate decreases rapidly in zone 3, which makes the range of zone 3 smaller.

3.2 Child Labor

By accepting myopic expectation, we are able to analyze the effects of child labor on intergenerational earnings mobility and economic growth using wages in exactly two periods. I introduce the choice to work in the first period of an individual's life.

3.2.1 Description of the Model

Technology

The production function in period t is $Y_t = AE_t^{1-\alpha}U_t^{\alpha}$ as it was before, but the number of uneducated workers, U_t, is different due to child labor. The number of adult educated workers in t is E_t, and the number of adult uneducated workers in t is $1-E_t$. The number of children who invest in education in t and will become educated adult labor in t+1 is E_{t+1}, and the number of children who work in t and will become uneducated adult labor in t+1 is $1-E_{t+1}$. Thus we express total uneducated labor in t as $U_t = (1-E_t)+\varsigma(1-E_{t+1})$, where $0 \le \varsigma \le 1$ is the relative productivity of child labor. We have more uneducated workers than in the Maoz-Moav model. Our production function is

$$Y_t = AE_t^{1-\alpha}\left[(1-E_t)+\varsigma(1-E_{t+1})\right]^{\alpha}.$$

The economy is competitive, and factors are paid their marginal products. We denote the wages of educated adult labor, uneducated adult labor, and child labor as w_t^{ea}, w_t^{ua}, and w_t^{uc}, respectively.

$$w_t^{ea} = \frac{\partial Y_t}{\partial E_t} = (1-\alpha) A E_t^{-\alpha} \left[(1-E_t) + \varsigma (1-E_{t+1}) \right]^{\alpha}$$

$$w_t^{ua} = \frac{\partial Y_t}{\partial (1-E_t)} = \alpha A E_t^{1-\alpha} \left[(1-E_t) + \varsigma (1-E_{t+1}) \right]^{\alpha-1}$$

$$w_t^{uc} = \frac{\partial Y_t}{\partial (1-E_{t+1})} = \alpha \varsigma A E_t^{1-\alpha} \left[(1-E_t) + \varsigma (1-E_{t+1}) \right]^{\alpha-1}$$

Since there are more uneducated workers in this economy, the wage of uneducated adult labor is lower than what it was before, and the wage of educated labor is higher than what it was before.

Individuals

An individual has a choice to work in his first period. He receives a transfer from his parent in his first period, and he either invests in education or works. If he invests in education, then he spends all of the transfer on a combination of consumption and investment. If he works, then he spends all of the transfer and his child-labor wage as consumption. The lifetime utility function for an individual born in t is

$$U^i = \log c_t^i + \log c_{t+1}^i + \log x_{t+1}^i .$$

His budget constraints in periods t and t+1 are

$$c_t^i + \delta^i h_t^i = x_t^i + (1-\delta^i) w_t^{uc}$$

$$c_{t+1}^i + x_{t+1}^i = w_{t+1}^i$$

where $\delta^i = 1$ and $w_{t+1}^i = w_{t+1}^{ea}$ if he invests in education; $\delta^i = 0$ and $w_{t+1}^i = w_{t+1}^{ua}$ otherwise. h_t^i is the cost of education. I introduced the $(1-\delta^i) w_t^{uc}$ expression to the budget constraint in t, which increases individual i's opportunity cost to attend school. If he does not invest in education, then he can gain more utility from consumption in t than before, because he has his child-labor wage to spend on consumption.

Other assumptions remain the same. We examine how the new wages and the new opportunity cost affect the education decision of both children of educated parents and children of uneducated parents and thus intergenerational earnings mobility and economic growth.

3.2.2 Solving the Model

We know the maximum utility in t+1 is $2\log w_{t+1}^{i} - 2\log 2$ with $c_{t+1}^{i} = x_{t+1}^{i} = \dfrac{w_{t+1}^{i}}{2}$.

If individual i invests in education in t, his lifetime utility will be

$\log\left(x_t^i - h_t^i\right) + 2\log w_{t+1}^{ea} - 2\log 2$, and if not, $\log\left(x_t^i + w_t^{uc}\right) + 2\log w_{t+1}^{ua} - 2\log 2$.

Thus he invests in education in period t

iff $$\log\left(x_t^i - h_t^i\right) + 2\log w_{t+1}^{ea} - 2\log 2 \geq \log\left(x_t^i + w_t^{uc}\right) + 2\log w_{t+1}^{ua} - 2\log 2$$

iff $$h_t^i \leq x_t^i - \left(x_t^i + w_t^{uc}\right)\left(\frac{w_{t+1}^{ua}}{w_{t+1}^{ea}}\right)^2$$

By applying $x_{t+1}^{i} = \dfrac{w_{t+1}^{i}}{2}$ to x_t^i, he invests in education

iff $$h_t^i \leq \frac{w_t^i}{2} - \left(\frac{w_t^i}{2} + w_t^{uc}\right)\left(\frac{w_{t+1}^{ua}}{w_{t+1}^{ea}}\right)^2$$

Note that w_t^i is not the wage for child labor in t but the wage for adult labor – the generation born in t-1 – in t. $w_t^i = w_t^{ea}$ or w_t^{ua} depends on the labor type of i's parent. From the myopic expectation assumption,

$$h_t^i \le \frac{w_t^i}{2} - \left(\frac{w_t^i}{2} + w_t^{uc}\right)\left(\frac{w_t^{ua}}{w_t^{ea}}\right)^2 .$$ [6]

If i's parent is educated, then i invests in education in t

iff $$h_t^i \le \frac{w_t^{ea}}{2} - \left(\frac{w_t^{ea}}{2} + w_t^{uc}\right)\left(\frac{w_t^{ua}}{w_t^{ea}}\right)^2 .$$

If i's parent is uneducated, then i invests in education in t

iff $$h_t^i \le \frac{w_t^{ua}}{2} - \left(\frac{w_t^{ua}}{2} + w_t^{uc}\right)\left(\frac{w_t^{ua}}{w_t^{ea}}\right)^2 .$$

These conditions tell us that if the wages of educated adult labor and uneducated adult labor are the same as before, it becomes harder for any child to attend school because of the child-labor wage an individual receives by choosing work in his first period. In other words, both children of educated parents and children of uneducated parents have higher opportunity costs.

The wages of adult workers, however, are not the same as before. The wage of educated adult labor is higher than before, so children of educated parents receive larger transfers. We are interested in how the increases in transfers and the opportunity costs affect the decisions of children of educated parents. The wage of uneducated adult labor is lower than before, so children of uneducated parents receive smaller transfers than before. The decrease in transfers and increase in the opportunity costs make it much harder for children of uneducated parents to attend school.

We denote $\hat{h}_t^{ea} = \frac{w_t^{ea}}{2} - \left(\frac{w_t^{ea}}{2} + w_t^{uc}\right)\left(\frac{w_t^{ua}}{w_t^{ea}}\right)^2$, $\hat{h}_t^{ua} = \frac{w_t^{ua}}{2} - \left(\frac{w_t^{ua}}{2} + w_t^{uc}\right)\left(\frac{w_t^{ua}}{w_t^{ea}}\right)^2$,

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[6] Without myopic expectation, we will need E_{t+2} for w_{t+1}^{ua} and w_{t+1}^{ea}, and we will not be able to find the unique E_{t+1} corresponding to an initial E_t using the dynamic behavior function we have.

and we call them critical values.[7] A child of an educated parent invests in education if and only if his education cost is lower than or equal to the critical value, $\hat{h}_t^{ea}$. Similarly, a child of an uneducated parent invests in education if and only if his education cost is lower than or equal to the critical value, $\hat{h}_t^{ua}$.

An individual's cost of education is based not on his parent's labor type but on his own ability and the level of output in the economy as it was before.

$$h_t^i = \theta_t^i \left(a + bY_t\right)$$

and h_t^i is uniformly distributed over the interval $\left(\underline{h}_t, \overline{h}_t\right)$. We denote the c.d.f. functions of $\hat{h}_t^{ea}$ and $\hat{h}_t^{ua}$ as $F_t\left(\hat{h}_t^{ea}\right)$ and $F_t\left(\hat{h}_t^{ua}\right)$, respectively.[8] $F_t\left(\hat{h}_t^{ea}\right)$ means the proportion of children who have educated parents and invest in education in t, and $F_t\left(\hat{h}_t^{ua}\right)$ means the proportion of children who have uneducated parents and invest in education in t. The equation for the dynamic behavior of the number of educated adult workers is

$$E_{t+1} = E_t F_t\left(\hat{h}_t^{ea}\right) + \left(1 - E_t\right) F_t\left(\hat{h}_t^{ua}\right).$$

Since E_t is the number of educated parents in t, $E_t F_t\left(\hat{h}_t^{ea}\right)$ is the number of children who have educated parents and invest in education in t. Since $\left(1 - E_t\right)$ is the number of uneducated parents in t, $\left(1 - E_t\right) F_t\left(\hat{h}_t^{ua}\right)$ is the number of children who have uneducated parents and invest in education in t.

[7] We express $\hat{h}_t^{ea}$ and $\hat{h}_t^{ua}$ using the marginal products of educated adult labor and uneducated adult labor in Appendix for 3.2.2.

[8] The equations for $F_t\left(\hat{h}_t^{ea}\right)$ and $F_t\left(\hat{h}_t^{ua}\right)$ are in Appendix for 3.2.2.

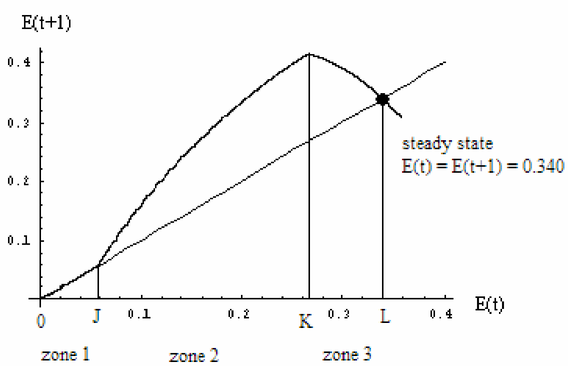
3.2.3 Numerical Example 1

We let $A=1$, $\alpha=0.5$, $\bar{\theta}=5$, $\underline{\theta}=1$, and $a=b=0.05$.[9] We first compare a non-child-labor economy with a child-labor economy with the relative productivity 0.5, which means that child labor's marginal product is one half of uneducated adult labor's marginal product. The graphs on the left side are for the non-child-labor economy, which are the same as the graphs we saw in section 3.1. The graphs on the right side are for the child-labor economy with the relative productivity 0.5. In the graphs E_t, E_{t+1}, w_t^{ea}, w_t^{ua}, w_t^{uc}, $\hat{h}_t^{ea}$, $\hat{h}_t^{ua}$, $\underline{h}_t$, and $\bar{h}_t$ are denoted by E(t), E(t+1), wea(t), wua(t), wuc(t), h^ea, h^ua, h-low, and h-high, respectively.

No Child Labor

$\varsigma=0$

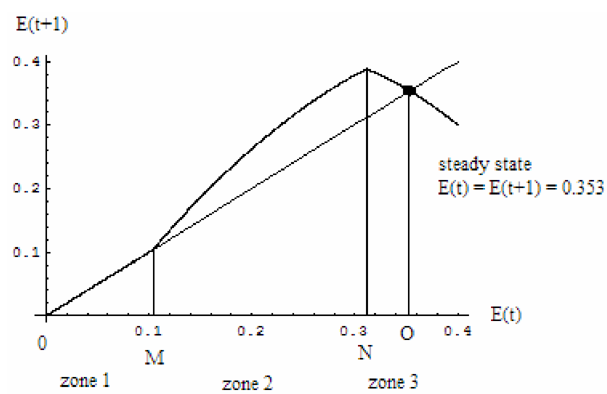
Graph 3.1



Child Labor

$\varsigma=0.5$

Graph 3.4



Graph 3.4 indicates that a child-labor economy has a longer poverty trap, a lower rate

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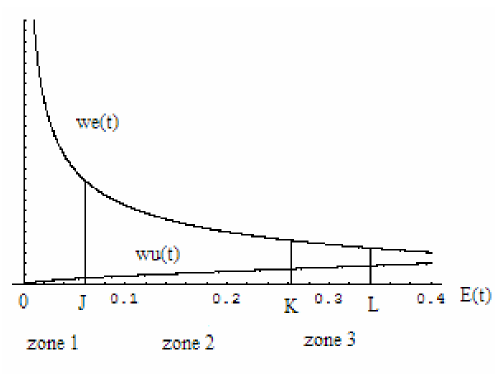
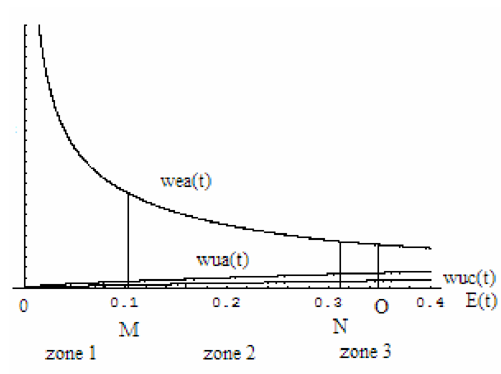
[9] The equations with these numerical values are in Appendix for 3.2.3.

of growth in zone 2, a smaller decrease in the growth rate in zone 3, and a higher level of steady state for the number of educated adult workers.

Graph 3.2

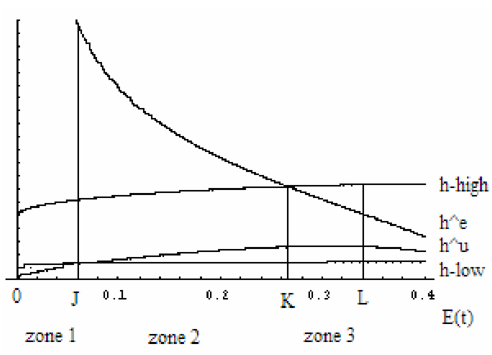


Graph 3.5

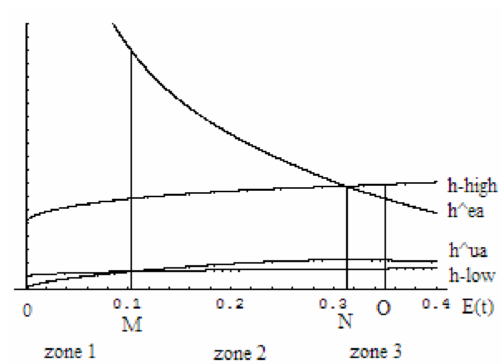


As we discussed, the increase in the number of uneducated workers increases the wage of educated adult labor and decreases the wage of uneducated adult labor.

Graph 3.3



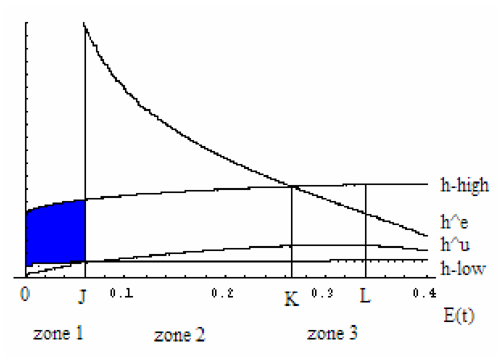
Graph 3.6



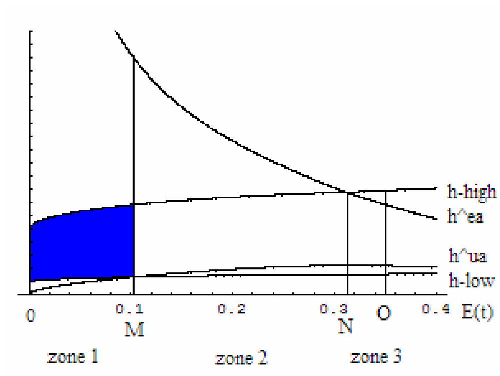
Graph 3.6 shows that introducing child labor does not change the basic relationship between the critical value of children of educated parents, the critical value of children of uneducated parents, the lowest cost of education, and the highest cost of education. Child labor, however, results in lower mobility.

We have a wider range of zone 1. Children whose costs are in the following shaded areas invest in education; all of them are the offspring of educated parents.

Graph 3.3 (zone 1)



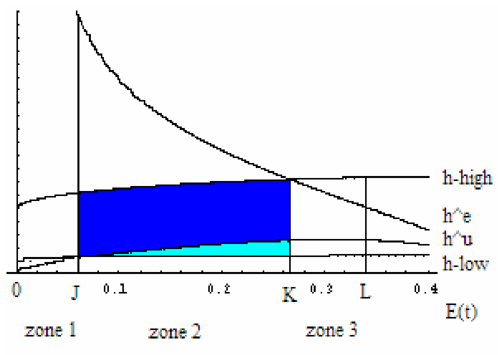
Graph 3.6 (zone 1)



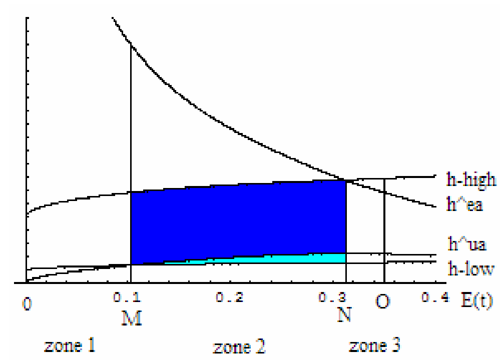
Since the smaller transfers and the higher opportunity costs make it much more difficult for children of uneducated parents to invest in education, the child who has an uneducated parent and the highest ability cannot acquire education until M. In the non-child-labor economy, he can afford education if the initial number of educated workers in the economy is larger than J, but in the child labor economy, the wage gap at J is not large enough for him to pay for his education.

In zone 2, although children of uneducated parents invest in education in both economies, the numbers are different. Children of educated parents whose costs are in the darker (upper) areas and the lighter (lower) areas invest in education, and children of uneducated parents whose costs are in the lighter (lower) areas invest in education.

Graph 3.3 (zone 2)



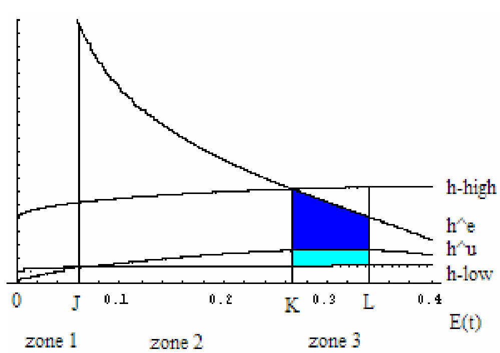
Graph 3.6 (zone 2)



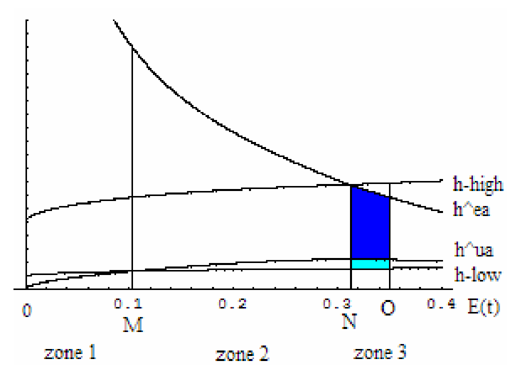
Comparing the two lighter (lower) areas, in the child-labor economy fewer children of uneducated parents invest in education. Again, this is because of the smaller transfers and the higher opportunity costs of schooling. Some children who have uneducated parents and higher abilities and were able to afford education in the non-child-labor economy cannot invest in education in the child-labor economy.

In zone 3, the number of children who decide not to invest in education is smaller in the child-labor economy. Children of educated parents whose costs are in the darker (upper) areas and the lighter (lower) areas invest in education, and children of uneducated parents whose costs are in the lighter (lower) areas invest in education.

Graph 3.3 (zone 3)



Graph 3.6 (zone 3)



Although children of educated parents have higher opportunity costs than before, the increase in the transfers from their parents overcomes the increase in the opportunity costs. It becomes much easier for children of educated parents to invest in education. Thus in the child-labor economy, fewer children who have educated parents and lower abilities decide not to invest in education in zone 3.

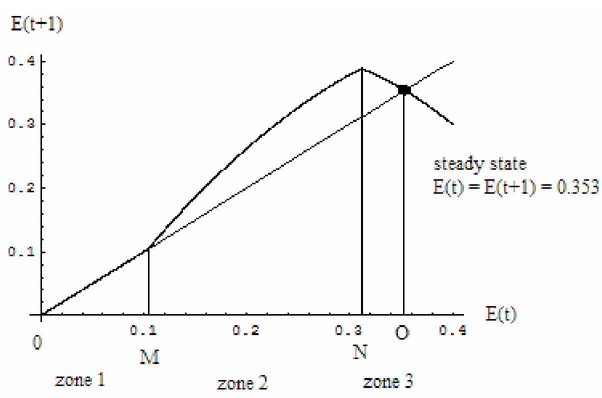
In conclusion, the child-labor economy has lower mobility, a longer poverty trap, a lower growth rate in zone 2, a smaller decrease in the growth rate in zone 3, and a higher level of steady state for the number of educated adult workers. Since the smaller transfers and the higher opportunity costs of education make it much more difficult for children of uneducated parents to invest in education, the economy needs a smaller wage gap for the children to attend school. That is to say, the economy has a longer poverty trap. Even if the economy’s initial number of educated adult workers is large enough, since fewer children of uneducated parents can afford education, the child-labor economy has lower mobility and a lower rate of growth. Although children of educated parents also have higher opportunity costs than before, the increase in their transfers is larger than the increase in the opportunity costs, so children of educated parents can invest in education more easily. Fewer of them decide not to invest in education in zone 3, so the economy has a smaller decrease in the growth rate and reaches a higher level of steady state.

3.2.4 Numerical Example 2

We next compare two child-labor economies; one has the relative productivity 0.5, and the other has the relative productivity 1. We see what intergenerational earnings mobility and economic growth will be if the relative productivity of child labor increases. The marginal product of educated adult labor increases, and the marginal product of uneducated adult labor decreases. As we discussed in section 3.2.3, the transfers of educated parents increase, transfers of uneducated parents decrease, and the opportunity cost of education increases. The graphs on the left are from section 3.2.3, and the graphs on the right are the results given $\varsigma = 1$.

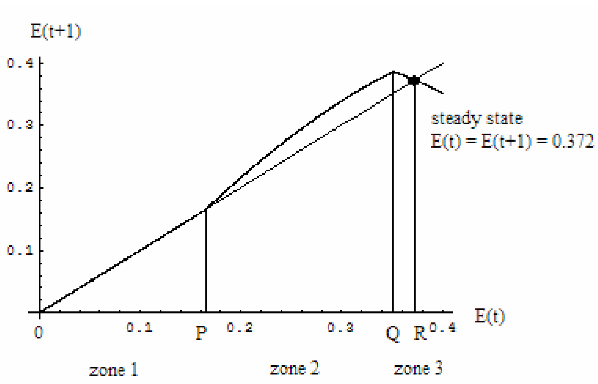
$\varsigma = 0.5$

Graph 3.4



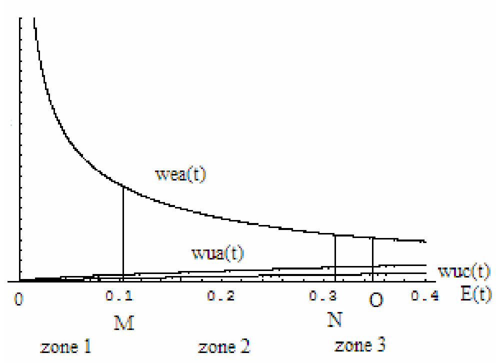
$\varsigma = 1$

Graph 3.7

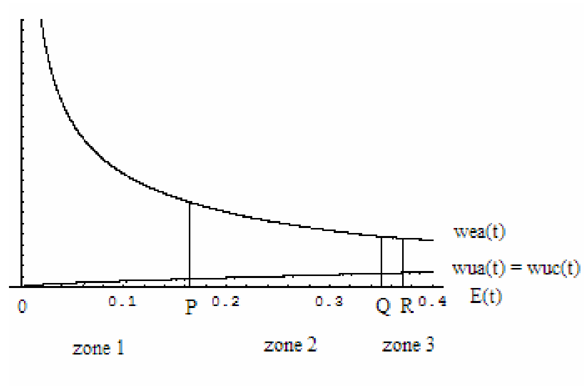


The higher productivity of child labor leads to a longer poverty trap, a lower rate of growth in zone 2, a smaller decrease in the rate in zone 3, and a higher level of steady state.

Graph 3.5

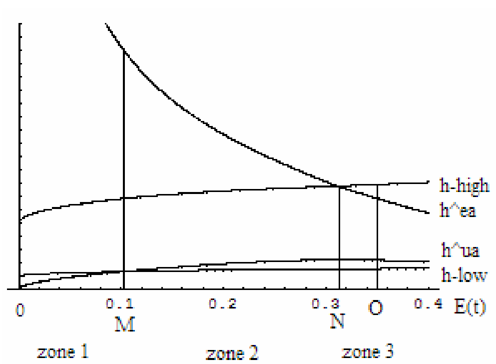


Graph 3.8



Graph 3.8 indicates that the higher productivity of child labor results in a higher wage of the educated adult labor and a lower wage of uneducated adult labor.

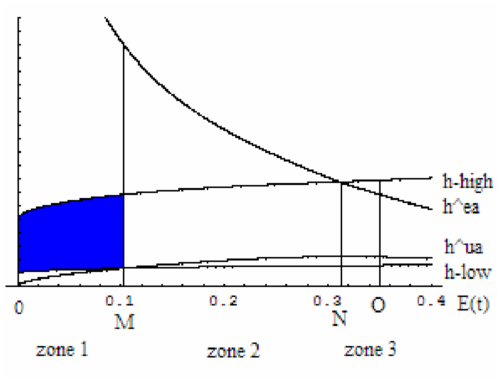
Graph 3.6



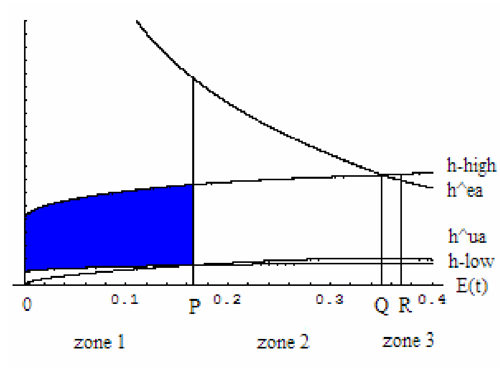
Graph 3.9

Since the wage of educated adult labor increases and the wage of uneducated adult labor decreases, the economy with the higher productivity has lower mobility. We have a wider range of zone 1. Children whose costs are in the following shaded areas invest in education; all of them are the offspring of educated parents.

Graph 3.6 (zone 1)



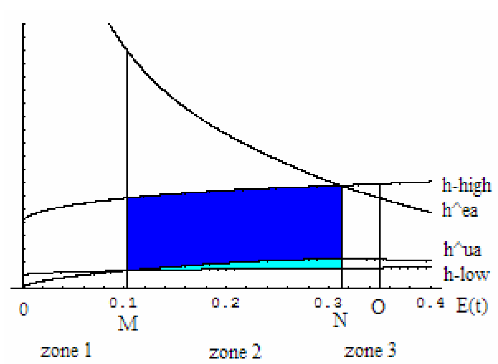
Graph 3.9 (zone 1)



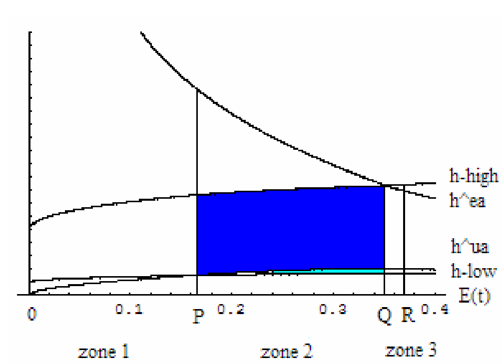
In graph 3.6, the child who has an uneducated parent and the highest ability can invest in education if the initial number of educated adult workers in an economy is larger than M, but he cannot afford education until P in graph 3.9 due to the larger gap in parents' wages.

In zone 2, fewer children invest in education. Children of educated parents whose costs are in the darker (upper) areas and the lighter (lower) areas invest in education, and children of uneducated parents whose costs are in the lighter (lower) areas invest in education.

Graph 3.6 (zone 2)



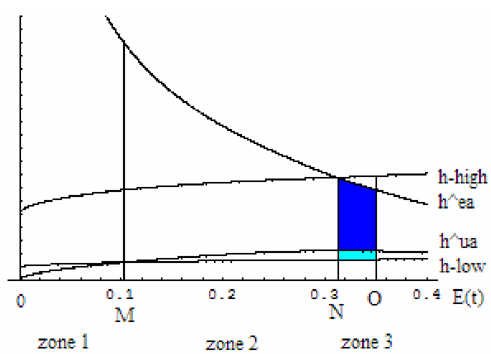
Graph 3.9 (zone 2)



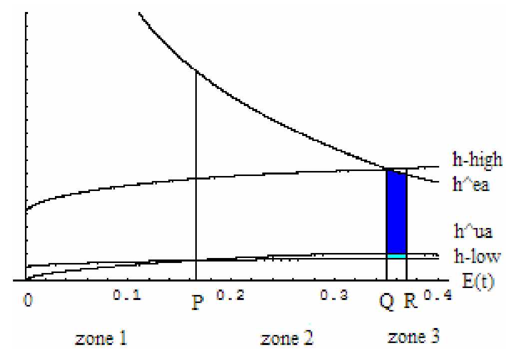
The smaller transfers and the higher opportunity costs make it much more difficult for children of uneducated parents to invest in education. More children who have uneducated parents and higher abilities cannot afford education.

In zone 3, fewer children decide not to invest in education. Children of educated parents whose costs are in the darker (upper) areas and the lighter (lower) areas invest in education, and children of uneducated parents whose costs are in the lighter (lower) areas invest in education.

Graph 3.6 (zone 3)



Graph 3.9 (zone 3)



Although children of educated parents have higher opportunity costs than before, the increase in their transfers is larger than the increase in the opportunity costs, so children of educated parents can invest in education more easily. Thus fewer children who have educated parents and lower abilities decide not to invest in education in graph 3.9.

In conclusion, the higher relative productivity of child labor leads to lower mobility, a longer poverty trap, a lower rate of growth in zone 2, a smaller decrease in the growth rate in zone 3, and a higher level of steady state for the number of educated adult workers. Increasing the relative productivity means increasing the marginal product of child labor, which increases the marginal product of educated adult labor and decreases the marginal product of uneducated labor. Both children of educated parents and children of uneducated parents have higher costs of education. Children of educated parents receive larger transfers than before, and this increase is larger than the increase in the opportunity costs, so children of educated parents can invest in education more easily. Children of uneducated parents receive smaller transfers than before, and this decrease and the increase in the opportunity costs make it much more difficult for children of uneducated parents to acquire education. Since fewer children of uneducated parents invest in education, the economy has a lower growth rate in zone 2. Since fewer children who have educated parents and lower abilities decide not to invest in education, the economy has a smaller decrease in the growth rate in zone 3 and reaches a higher level of steady state.

4 Conclusion

This paper analyzed the effects of child labor on intergenerational earnings mobility and economic growth. The model was a two-period overlapping-generations model based on Maoz and Moav (1999). I introduced the choice to work in the first period of an individual's life. When an economy has child labor, children of educated parents can invest in education more easily, and it is much more difficult for children of uneducated parents to acquire education. A child-labor economy has lower mobility, a longer poverty trap, a lower rate of growth, and a larger number of educated adult workers in the steady state.

For a less developed economy, the costs of having child labor are a higher possibility to be in a poverty trap and a lower growth rate. If the number of educated adult workers in the economy is very small, it is more likely that the economy stays in a poverty trap and does not grow. If the number is large enough, then the economy grows slowly.

Both empirical and theoretical research is left for the future. If more panel data become available, then we could compare more countries' intergenerational earnings mobility. We could add more periods to the model and calibrate the parameters from the data. It is also possible to assume that a child's ability is not independent of his parent's labor type, and we could change the distribution of abilities. Although we assumed that children either work or attend school, considering children who do both is an interesting topic. By introducing capital, we could analyze a child-labor economy where a child can borrow capital to pay for his education and save his child-labor wage for his consumption and transfer in his second period.

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Appendix for 2.2

We express $\hat{h}_t^e$ and $\hat{h}_t^u$ using the marginal products.

$$\hat{h}_t^e = \frac{(1-\alpha)AE_t^{-\alpha}(1-E_t)^{\alpha}}{2}\left[1-\left(\frac{\alpha}{1-\alpha}E_{t+1}(1-E_{t+1})^{-1}\right)^2\right]$$

$$\hat{h}_t^u = \frac{\alpha AE_t^{1-\alpha}(1-E_t)^{\alpha-1}}{2}\left[1-\left(\frac{\alpha}{1-\alpha}E_{t+1}(1-E_{t+1})^{-1}\right)^2\right]$$

The proportion of children who have educated parents and invest in education in t is

$$F_t\left(\hat{h}_t^e\right) = \frac{\hat{h}_t^e - \underline{\theta}(a+bY_t)}{\overline{\theta}(a+bY_t)-\underline{\theta}(a+bY_t)}$$

$$F_t\left(\hat{h}_t^e\right) = \frac{\frac{(1-\alpha)AE_t^{-\alpha}(1-E_t)^{\alpha}}{2}\left[1-\left(\frac{\alpha}{1-\alpha}E_{t+1}(1-E_{t+1})^{-1}\right)^2\right]-\underline{\theta}\left(a+bAE_t^{1-\alpha}(1-E_t)^{\alpha}\right)}{\left(\overline{\theta}-\underline{\theta}\right)\left(a+bAE_t^{1-\alpha}(1-E_t)^{\alpha}\right)}$$

The proportion of children who have uneducated parents and invest in education in t is

$$F_t\left(\hat{h}_t^u\right) = \frac{\hat{h}_t^u - \underline{\theta}(a+bY_t)}{\overline{\theta}(a+bY_t)-\underline{\theta}(a+bY_t)}$$

$$F_t\left(\hat{h}_t^u\right) = \frac{\frac{\alpha AE_t^{1-\alpha}(1-E_t)^{\alpha-1}}{2}\left[1-\left(\frac{\alpha}{1-\alpha}E_{t+1}(1-E_{t+1})^{-1}\right)^2\right]-\underline{\theta}\left(a+bAE_t^{1-\alpha}(1-E_t)^{\alpha}\right)}{\left(\overline{\theta}-\underline{\theta}\right)\left(a+bAE_t^{1-\alpha}(1-E_t)^{\alpha}\right)}$$

Appendix for 2.3

The equations with numerical values are

$$F_t\left(\hat{h}_t^e\right) = \frac{0.25(1-E_t)^{0.5} E_t^{-0.5}\left(1-E_{t+1}^2(1-E_{t+1})^{-2}\right) - \left(0.05+0.05E_t^{0.5}(1-E_t)^{0.5}\right)}{0.2+0.2E_t^{0.5}(1-E_t)^{0.5}}$$

$$F_t\left(\hat{h}_t^u\right) = \frac{0.25(1-E_t)^{-0.5} E_t^{0.5}\left(1-E_{t+1}^2(1-E_{t+1})^{-2}\right) - \left(0.05+0.05E_t^{0.5}(1-E_t)^{0.5}\right)}{0.2+0.2E_t^{0.5}(1-E_t)^{0.5}}$$

$$\hat{h}_t^e = 0.25(1-E_t)^{0.5} E_t^{-0.5}\left(1-E_{t+1}^2(1-E_{t+1})^{-2}\right)$$

$$\hat{h}_t^u = 0.25(1-E_t)^{-0.5} E_t^{0.5}\left(1-E_{t+1}^2(1-E_{t+1})^{-2}\right)$$

$$\underline{h}_t = 0.05+0.05E_t^{0.5}(1-E_t)^{0.5}$$

$$\overline{h}_t = 0.25+0.25E_t^{0.5}(1-E_t)^{0.5}$$

$$w_t^e = 0.5(1-E_t)^{0.5} E_t^{-0.5}$$

$$w_t^u = 0.5(1-E_t)^{-0.5} E_t^{0.5}$$

Since $F_t\left(\hat{h}_t^e\right)$ and $F_t\left(\hat{h}_t^u\right)$ are the c.d.f. functions, $0 \le F_t\left(\hat{h}_t^e\right) \le 1$ and $0 \le F_t\left(\hat{h}_t^u\right) \le 1$. For any $F_t\left(\hat{h}_t^e\right) < 0$ and $F_t\left(\hat{h}_t^u\right) < 0$ we assign 0, and for any $1 < F_t\left(\hat{h}_t^e\right)$ and $1 < F_t\left(\hat{h}_t^u\right)$ we assign 1.

Appendix for 3.2.2

We express $\hat{h}_t^{ea}$ and $\hat{h}_t^{ua}$ using the marginal products.

$$\hat{h}_t^{ea} = \frac{\dfrac{(1-\alpha)AE_t^{-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha}}{2} - \left(\dfrac{(1-\alpha)AE_t^{-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha}}{2} + \alpha\varsigma AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}\right)}{\left(\dfrac{\alpha}{1-\alpha}E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2}$$

$$\hat{h}_t^{ua} = \frac{\dfrac{\alpha AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}}{2} - \left(\dfrac{\alpha AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}}{2} + \alpha\varsigma AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}\right)}{\left(\dfrac{\alpha}{1-\alpha}E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2}$$

$$F_t\left(\hat{h}_t^{ea}\right) = \frac{\left[\dfrac{\dfrac{(1-\alpha)AE_t^{-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha}}{2} - \left(\dfrac{(1-\alpha)AE_t^{-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha}}{2} + \alpha\varsigma AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}\right)}{\left(\dfrac{\alpha}{1-\alpha}E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2}\right] - \underline{\theta}(a+bY_t)}{\overline{\theta}(a+bY_t) - \underline{\theta}(a+bY_t)}$$

$$F_t\left(\hat{h}_t^{ua}\right) = \frac{\left[\dfrac{\dfrac{\alpha AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}}{2} - \left(\dfrac{\alpha AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}}{2} + \alpha\varsigma AE_t^{1-\alpha}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{\alpha-1}\right)}{\left(\dfrac{\alpha}{1-\alpha}E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2}\right] - \underline{\theta}(a+bY_t)}{\overline{\theta}(a+bY_t) - \underline{\theta}(a+bY_t)}$$

Appendix for 3.2.3

The equations with numerical values are

$$F_t\left(\hat{h}_t^{ea}\right)=\frac{\left[\begin{array}{l}0.25E_t^{-0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}-\\ \left(0.25E_t^{-0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}+0.5\varsigma E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}\right)\\ \left(E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2\end{array}\right]-\left(0.05+0.05E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}\right)}{4\left(0.05+0.05E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}\right)}$$

$$F_t\left(\hat{h}_t^{ua}\right)=\frac{\left[\begin{array}{l}0.25E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}-\\ \left(0.25E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}+0.5\varsigma E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}\right)\\ \left(E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2\end{array}\right]-\left(0.05+0.05E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}\right)}{4\left(0.05+0.05E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}\right)}$$

$$\begin{array}{rl} & 0.25E_t^{-0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}-\\ \hat{h}_t^{ea}= & \left(0.25E_t^{-0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}+0.5\varsigma E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}\right)\\ & \left(E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2\end{array}$$

$$\begin{array}{rl} & 0.25E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}-\\ \hat{h}_t^{ua}= & \left(0.25E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}+0.5\varsigma E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}\right)\\ & \left(E_t\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-1}\right)^2\end{array}$$

$$\underline{h}_t=0.05+0.05E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}$$

$$\overline{h}_t=0.25+0.25E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}$$

$$w_t^{ea}=0.5E_t^{-0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{0.5}$$

$$w_t^{ua}=0.5E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}$$

$$w_t^{uc}=0.5\varsigma E_t^{0.5}\left((1-E_t)+\varsigma(1-E_{t+1})\right)^{-0.5}$$

Since $F_t\left(\hat{h}_t^{ea}\right)$ and $F_t\left(\hat{h}_t^{ua}\right)$ are the c.d.f. functions, $0\le F_t\left(\hat{h}_t^{ea}\right)\le 1$ and $0\le F_t\left(\hat{h}_t^{ua}\right)\le 1$. For any $F_t\left(\hat{h}_t^{ea}\right)<0$ and $F_t\left(\hat{h}_t^{ua}\right)<0$ we assign 0, and for any $1<F_t\left(\hat{h}_t^{ea}\right)$ and $1<F_t\left(\hat{h}_t^{ua}\right)$ we assign 1.