Math 111, Fall 2011: Midterm I Study Guide

The first midterm will take place on Friday 30 September at 9:00am in Seeley Mudd 206. It will be 50 minutes long. You will not be allowed to use notes, books or calculators of any kind. The exam will cover sections 1.1 through 2.2. (That is everything we covered in class up until Friday 23 September.)

Format of the exam

The exam will have five questions, each will be worth 10 points. You should do as many of the questions as fully as possible. You will get partial credit if you have the right idea for a question, even if you do not get it completely correct. You should also explain your answers fully, to an amount of detail appropriate to the question. The rule of thumb is that you should explain the key steps needed to solve the problem, but do not have to explain minor steps that you use along the way. In the solutions to the practice exam problems, I'll try to indicate what is expected.

Syllabus

Here is a summary of what you need to know and be able to do. Unless stated otherwise, you may use any fact or result from class or from the sections of the textbook we have covered. You may *not* use any results from later in the textbook. You should:

- functions (1.1)
  - understand the concept of a function whose inputs and outputs are real numbers
  - know how to describe functions in words, with a table of values, with a graph or with a formula
  - be able to translate between the different ways to represent a function (e.g. use a table or a formula to draw a graph, or read off a table of values from the graph)
  - know what it means for a function to be odd or even, and be able to tell if a function is odd or even using either the formula, or a graph of the function
  - know what is meant by the domain and range of a function and be able to find the domain and range of a function based either on the graph, a description in words, or a formula
- standard functions (1.2)
  - know what is meant by a linear, quadratic, polynomial or power function, and know examples of these
  - be able to draw the graph of a given linear or quadratic function, marking where the graph crosses the x- and y-axes

- be able to draw the graph of the power functions x^{-1}, x^{-2} and $x^{1/2}$
- know the definition of the sine, cosine and tangent functions
- know (or be able to work out) the values of the sine, cosine and tangent functions at all multiples of $\pi/6$ and $\pi/4$.
- be able to draw graphs of the sine, cosine and tangent functions, indicating where they cross the x-axis, where they have maximum and minimum values and where they have vertical asymptotes
- know what is meant by the absolute value function and be able to draw its graph

- new functions from old (1.3)
  - know how to draw the graph of $g(x)$ using the graph of $f(x)$ when $g(x)$ is of the form
    $$f(x) + c, f(x + c), cf(x), f(cx)$$
  - know how to find a formula for $g(x)$ when the graph of $g(x)$ is obtained from the graph of $f(x)$ by translating or stretching vertically or horizontally
  - be able to combine multiple transformations of the above types
  - know what is meant by adding, multiplying, dividing or composing two functions

- the limit of a function (1.5)
  - be able to find the limit of a function at a particular point using a graph of the function, including explaining when the limit is $\pm\infty$, or does not exist for some other reason
  - be able to find one-sided limits of a function at a particular point and know how the values of the one-sided limits are related to the value of the limit (if it exists)
  - be able to sketch a possible graph of a function with given limits
  - know what is meant by a vertical asymptote for a function and understand the relationship between vertical asymptotes and infinite limits
  - be able to tell if a (one-sided or two-sided) limit is $+\infty$, or $-\infty$, or neither, based on looking at x-values a small distance either side of the limit point
  - know that the limits of $f(x)$ and $g(x)$ at $x = a$ are equal if f and g are equal for x-values close to, but not equal to, a

- limit laws (1.6)
  - know the limit laws for adding, subtracting, multiplying and dividing functions and be able to use them correctly
  - understand when limit laws cannot be used to find the limit of a sum, difference, product or quotient of two functions

- precise definition of a limit (1.7)

- know and understand the precise definition of what it means to say that

$$\lim_{x \to a} f(x) = L$$

- be able to use the precise definition (that is, the ϵ/δ-definition) to prove that a limit of a constant or linear function is equal to a particular value (in this test, you will not need to deal with limits of other types of functions, and you will not need to show that a limit does not exist, or is not equal to a particular value)

- continuity (1.8)
  - know what it means for a function to be continuous at a given value
  - be able to tell from the graph of a function where that function is continuous and where it is not continuous
  - be able to use the fact that a function is continuous to calculate limits
  - know that if f and g are continuous at a, then $f+g, f-g, fgandf/g$ are continuous at a (for f/g, $g(a)$ must be not zero), and be able to use these facts
  - know that all polynomial, power and trig functions are continuous at all points in their domains
  - know what it means for a function to be continuous from the left, or from the right, at a particular point, and what it means to say that a function is continuous on an interval

- general limit problems
  - be able to combine the ideas from chapter 1 to calculate limits of lots of functions using by:
    * using the knowledge that certain functions are continuous
    * using the limit laws
    * by rearranging or manipulating a formula so that limit laws, or continuity can be applied
  - specifically be able to deal with limits of functions of the form f/g where both $f(a)$ and $g(a)$ are zero by factoring, cancelling or otherwise rearranging the formula

- definition of derivative (2.1)
  - know the limit definition of the derivative, or rate of change of a function at a particular point
  - be able to calculate the derivative of a function at a given point using the limit definition and the techniques for finding limits from chapter 1
  - specifically be able to calculate derivatives of constant, linear, quadratic and cubic functions (polynomials of degree up to 3)
  - specifically be able to calculate derivatives of functions involving square-roots and reciprocals (the power functions x^{-1} and $x^{1/2}$) [here I mean that you should definitely have seen and practiced calculating derivatives for these types of functions - I could ask you to do other functions too, but would give you more time]

  - know and understand why the derivative tells us the slope of the tangent line to the graph of the function
  - know how to interpret the meaning of the derivative as the rate of change when the function represents some real-world situation

- the derivative as a function (2.2)
  - be able to sketch the graph of f' based on the graph of f, or the graph of f based on the graph of f'
  - be able to use the graph of f' to interpret which intervals the graph of f has positive or negative slope, or at which points the graph of f has a horizontal tangent line
  - be able to give examples of functions that are not differentiable at a particular point and explain why, both using the graph of the function, and using the limit definition of derivative
  - know that differentiable functions are continuous and be able to use this fact to show that a given function is not differentiable at some point

This is not a complete list of what you might have to do on the test but it covers most of the ideas involved. In particular, you may have to combine several of these ideas or techniques, and you may to think to decide what to use to solve a problem.

Preparing for the test

The best way to prepare for the test is to do practice tests. This means you should sit down, without a textbook or your notes, and try to do as much as you can of the practice test in 50 minutes, as though it were the real thing. This will give you an idea of how well prepared you are, what topics you might need to review, and how you react under test conditions. This is especially important if you don't have much experience taking timed tests, or if you have had anxiety problems with tests in the past. The more practice you do, the better prepared you will be.

You should also go back over past homework problems, especially those for which the grader has written a comment or deducted points and make sure you understand the comment or why you lost points. If you can't work this out or have any other questions about the grading, please come and ask me about it. I'll have extra office hours on Wednesday and Thursday next week to help you prepare.

You should also just work through more practice problems. If you didn't do the practice problems assigned for the homeworks, now would be a good time to do those. If you did, you can make up some more problems on your own (which is also a good exercise to see if you understand the material) and try to solve them. You can always ask me if you are unsure of something.

Beyond that, please let me know how else I can help you prepare, and good luck!