Math 11 Final Examination May 11, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{(x+1)^2-1}$

(b) $\displaystyle\lim_{x\to 3^-} \frac{x^2-8x+15}{1-8x+g(x+1)}$, where $g(x)=x^2+7$.

(c) $\displaystyle\lim_{x\to 8} \frac{8-x}{\sqrt{x+1}-3}$

(d) $\displaystyle\lim_{x\to 7} \frac{x^2-5x-14}{|7-x|}$

2. [30 Points] Compute each of the following derivatives. Simplify numerical answers. Do not simplify your algebraically complicated answers.

(a) $f'\left(\frac{\pi}{12}\right)$, where $f(x)=\sec^2(2x)+\sin(4x)$.

(b) $\displaystyle\frac{d}{dx}\ln\left(\frac{(x^2+1)^{\frac{3}{7}}\,e^{\tan x}}{\sqrt{1+\cos x}}\right)$

(c) $g'(x)$, where $g(x)=\sqrt{1+\cos^7\left(\frac{5}{x}\right)}$

(d) $\displaystyle\frac{dy}{dx}$, if $e^{xy^3}+\sin^3 x=\ln(xy)+\sin(e^9)$.

(e) $g''(x)$, where $g(x)=\displaystyle\int_x^{2011}\sqrt{\ln t}+\ln\sqrt{t}\,dt$.

(f) $\displaystyle\frac{d}{dx}\,x^{\cos x}$

3. [25 Points] Compute each of the following integrals.

(a) $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{9}}\tan(3x)\,dx$

(b) $\displaystyle\int\frac{\left(x^{\frac{5}{2}}+1\right)^2}{x}\,dx$

(c) $\displaystyle\int_e^{e^4}\frac{3}{x\sqrt{\ln x}}\,dx$

(d) $\displaystyle\int e^{x^2+\ln x+1}\,dx$

4. [10 Points] Give an ε–δ proof that $\displaystyle\lim_{x\to 2} 6-5x=-4$.

5. [10 Points] Let $f(x)=\dfrac{x+2}{x-3}$. Calculate $f'(x)$, using the **limit definition** of the derivative.

6. [15 Points] Compute $\int_0^8 x - 3\, dx$ using each of the following **three** different methods:

(a) Area interpretations of the definite integral,
(b) Fundamental Theorem of Calculus,
(c) Riemann Sums and the limit definition of the definite integral $***$.

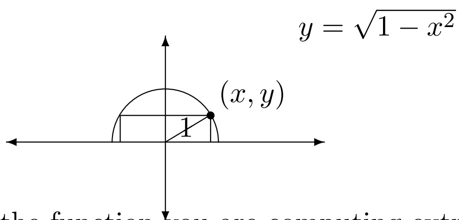
***Recall $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} 1 = n$

7. [10 Points] Find the equation of the tangent line to $y = \cos(\ln(x+1)) + \ln(\cos x) + e^{\sin x} + \sin(e^x - 1)$ at the point where $x = 0$.

8. [20 Points] Let $\boldsymbol{f(x) = \frac{x}{e^x} = xe^{-x}}$. For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve. Take my word that $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = -\infty$.

9. [15 Points] A conical tank, 14 feet across the entire top and 12 feet deep, is leaking water. The radius of the water level is decreasing at the rate of 2 feet per minute. How fast is the water leaking out of the tank when the radius of the water level is 2 feet? **Recall the volume of the cone is given by $V = \frac{1}{3}\pi r^2 h$

10. [15 Points] Let R be the region inside the top semicircle of radius one, centered at the origin, given by $y = \sqrt{1 - x^2}$. Find the area of the largest rectangle that can be inscribed in this region R. Two vertices of the rectangle lie on the x–axis. Its other two vertices lie on the semicircle.



(Remember to state the domain of the function you are computing extreme values for.)

11. [15 Points] Consider the region in the first quadrant bounded by $y = e^x + 1$, $y = 4$, and the y–axis. (a) Draw a picture of the region. (b) Compute the area of the region.

(c) Compute the volume of the three-dimensional object obtained by rotating the region about the horizontal line $\boldsymbol{y = -2}$

12. [15 Points] Consider an object moving on the number line such that its velocity at time t seconds is $\boldsymbol{v(t) = 4 - t^2}$ feet per second. Also assume that the position of the object at one second is $\frac{5}{3}$.

(a) Compute the acceleration function $a(t)$ and the position function $s(t)$.

(b) Compute the **total distance** travelled for $0 \le t \le 3$.