Amherst College, DEPARTMENT OF MATHEMATICS

Math 11, Final Examination, May 14, 2010

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers.

1. [20 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2-4x}{x^2+2x-8}$

(b) $\displaystyle\lim_{x\to 7} \frac{1}{|x-7|}$

(c) $\displaystyle\lim_{x\to 1^-} \frac{x^2+6x-7}{x^2-2x+1}$

(d) $\displaystyle\lim_{x\to -5} \frac{\frac{5}{x}-\frac{1}{x+4}}{x+5}$

2. [30 Points] Compute each of the following derivatives. Do not simplify your answers.

(a) $\displaystyle\frac{d}{dx}\left(\frac{e^{5x}-x^{\frac{3}{7}}}{\tan\sqrt{x}}\right)$

(b) $\displaystyle\frac{dy}{dx}$, if $\sin^3 x + 5e^y = 7 + \ln(xy)$.

(c) $\displaystyle\frac{d}{dx}\left(\int_x^7 \frac{e^t}{\ln t}\,dt\right)$

(d) $\displaystyle\frac{d}{dx}\left[\ln\left(\frac{\sqrt{x^7+1}\;e^{\sin x}}{(x^5+9)^3}\right)\right]$ (hint: you might want to simplify first)

(e) $f''(x)$, where $f(x) = e^{\cos x} + \dfrac{1}{x^7}$.

(f) $f'(x)$, where $f(x) = x^x$.

3. [25 Points] Compute each of the following integrals.

(a) $\displaystyle\int \frac{\left(x^{\frac{5}{2}}+1\right)^2}{x}\,dx$

(b) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \sin(2x)\cos(2x)\,dx$

(c) $\displaystyle\int \frac{e^{5x}}{7+e^{5x}}\,dx$

(d) $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$

(e) $\int_e^{e^4} \frac{3}{x\sqrt{\ln x}}\,dx$

4. [10 Points] Give an ε-δ proof that $\lim_{x\to\,-1} 5x-4=-9$.

5. [10 Points] Let $f(x)=\sqrt{5x-2}$. Calculate $f'(x)$, using the **limit definition** of the derivative.

6. [10 Points] Suppose that $f(x)=\cos(e^x)$. Write the **equation** of the tangent line to the curve $y=f(x)$ when $x=\ln\left(\frac{\pi}{2}\right)$.

7. [20 Points] Let $\boldsymbol{f(x)=xe^{-x}}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word for it that

$$\lim_{x\to\infty} xe^{-x}=0 \quad \text{and} \quad \lim_{x\to-\infty} xe^{-x}=-\infty$$

8. [15 Points] Suppose the top of a 50 foot ladder is sliding down a vertical wall at a rate of 10 feet per second. Consider the angle formed between the ground and the base of the ladder. At what rate is this angle changing when the top of the ladder is 30 feet above the ground?

9. [15 Points] A box with a square base and a (flat) top is to be made to hold a volume of 27 cubic feet. Determine the **dimensions** that minimize the amount of material used.

(Remember to state the domain of the function you are computing extreme values for.)

10. [20 Points]

(a) Compute the **area** bounded by $y=|x|$, $y=2-x^2$. **Sketch** the region.

(b) Consider the region in the plane bounded by $\boldsymbol{y=e^x+1}$, $\boldsymbol{y=1}$, $\boldsymbol{x=0}$ and $\boldsymbol{x=\ln 4}$. Compute the **volume** of the 3-dimensional object obtained by rotating the region about the $\boldsymbol{x}$–axis. **Sketch** the region.

11. [15 Points] Consider an object moving on the number line, starting at position 0, such that its acceleration at time t is $a(t)=2$ feet per square second. Also assume that the object has initial velocity equaling -6 feet per second.

(a) Compute the velocity function $\boldsymbol{v(t)}$ and position function $\boldsymbol{s(t)}$.

(b) Compute the **total distance** that it travels between time $t=0$ and $t=4$ seconds.