Modeling Marriage Using an Option Framework

by Joseph Caissie

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Advisor: Geoffrey Woglom
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"Two roads diverged in a yellow wood,
(I determined my net present value, subtracted a risk premium, and added in the option value)
And that has made all the difference"

--Robert Frost

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Part I: Introduction

Marriage is an institution that permeates nearly every society on this planet. It has many forms, across both time and culture. Concern about the destruction of the institution of marriage is rampant among politicians and the general public today. A framework with which to predict trends in marriage would be an invaluable tool for economists and sociologists looking to investigate this complex subject. Such a framework would be particularly valuable in assessing which trends in marriage are of concern to our society, and which ones are simply efficient responses to changing underlying conditions.

I.1. **The Sociological Approach**

Many factors are implicated in the so-called decline of marriage. One of the common complaints is that "people don't take (the vows of) marriage seriously anymore." (Ahrons, 1995) A factor linked to this supposed trivializing of marriage vows is the institution of no-fault divorce laws. Another explanation offered is the entry of women into the workforce. One of the causes of women's ability to take a more active role in the workforce, the availability of reliable birth control, is also cited as a reason in itself for the increase in divorce rates. Generational effects are also suggested as a reason for trends in marriage; perhaps observing marriages of an older generation influences the choices made by the younger one.

Many of these suggested reasons for trends in marriage are not informative to economists. Simply asserting that people don't take marriage as seriously as they used to raises the question of what has caused that change in attitude. The fact that over 50% of all marriages now end in divorce undoubtedly motivates the intuition that marital vows are not taken as seriously as in the past, but according to interviews by sociologists (Orenstein, 2000), very few couples get married today expecting to get divorced. These diverging viewpoints on the current attitude towards marriage warrant investigation.

Another folk myth is that marital stability was the norm until the institution of no-fault divorce. While appealing, this explanation misses the possibility that the introduction of no-fault divorce laws was in response to a need for easier divorce, meaning that a force towards marital instability was already present.

As with no-fault divorce and changing social mores, the entry of women into the workforce and the availability of reliable birth control are complicated issues, with impacts on many different areas of marriage. To assert women's entry into the workforce as a general reason for marital instability is not helpful. Since it is widely held that ending gender discrimination in the workforce is a positive thing, we should determine more quantitative details of how this impacts marital stability. Since birth control and female entry into the labor force are so intertwined, we need a model that can separate the effects of the two.

Before investigating the specifics today, we should examine some of the past trends in the institution of marriage. According to sociologists, most of the general public feels that marriage was an unchanging institution until the 60's, when a bunch of slackers with irresponsible visions of free love wreaked havoc on the holy bonds. It turns out that on closer study, marriage has always been a dynamic institution; the 50's-style marriage that many envision as 'the way it used to be' was really only that way for the 50's, if at all. (Cherlin, 1981) 'The way it used to be' is the vision of the nuclear family, with a male breadwinner, a female engaged in domestic production, 2.4 kids, and no divorce.

Another myth about marriage is that is has always been and always should be about "love". In fact, it was not until past the turn of the 20th century that love even came into play in most marriages. Marriage was the standard building block of a society; the household was responsible for a large portion of production. Only when urbanization and industrialization arose in the 1920s did work become less household-centered, and the household became a source of other, less tangible things such as emotional intimacy and romance. (Ahrons, 1995)

Sociologists do not have a unified stance on the causes and effects of changes in the institution of marriage such as the high divorce rate. One theory is that love has become more important to a marriage, so when that love fades, the marriage is suddenly worth significantly less (Ahrons 1994). Another possible explanation is that couples wait to have children because marriage has become more uncertain. Others retort that marriage is uncertain because couples are waiting to have children. (Cramer, 1980) A common explanation for the decline in marital stability is that outside options have become more attractive, particularly to women, who now have the ability to support themselves by working in the market (Easterlin, 1987). This suggestion does not seem to fit with surveys that indicate that marriage is as important now as it ever has been to Americans (Waite and Gallagher, 2000). A model might help to resolve some of these paradoxes and direction-of-causation questions.

I. 2. **The Economic Approach**

Gary Becker is the father of economic thought on the family. In his seminal paper, "The Theory of Marriage: Part I" in 1973, he assumes that couples marry when the expected utility from marriage is greater than the utility from remaining single, and divorce when the utility from the marriage is less than that from being single or marrying someone else. In Part II of his work, Becker argues that couples "love" each other, where "A loves B" simply means that the cross-derivative of utility with respect to A and B is positive. Because of this, he argues, A will have less incentive to "steal" from B by shirking from his responsibilities toward common household production, and the household's utility will be jointly maximized (Becker, 1974).

Becker's work on divorce focuses on valuing marriage based on traits that make a match good or bad. He notes that the variance in these traits over time is important in deciding whether a couple will get divorced. Mostly, he focuses on individual marriages rather than overall trends when he assesses the likelihood of divorce or the benefit from a marriage. Using the tools Becker develops, we can say that couples derive an uncertain benefit from being married. He states "it is natural to consider the probability of divorce as a function of two factors: the expected gain from the marriage and the distribution of a variable describing unexpected outcomes." He goes on to show that the probability of divorce is smaller with a greater expected gain from marriage and a smaller variance in that expected gain (Becker, 1977). With this simplified view of a marriage, it is possible to take all of the suggested reasons for trends in marriage and ask of them: what do they do to the variance of outcomes and the expected gain from a marriage?

A more recent economic analysis of marriage (Allen, 1997) takes this dependence on variance in expected benefit of marriage and shows that an increase in the variance of the value of marriage is what led to demand for no-fault divorce laws in Canada. The emphasis of his analysis is that while societal changes, such as the entry of women into the workforce, may have decreased the expected gain of marriages, the culprit in the increase in the divorce rate is variance. To prove his point, he observes the divorce rate of groups with high levels of variance in the expected value of their marriages, such as the very young, or couples for whom it is unclear whether there will be one or two incomes.

In contrast, sociologists tend to think about marital phenomena in terms of larger socioeconomic and demographic changes, such as the entry of women into the workforce or a more lenient attitude towards divorce. Economists, on the other hand, attempt to translate these specific changes into more model-able language such as ‘variance’ and ‘expected value’.

I. 3. **Embedded Options**

In the same economic vein, Scott and Triantis (2004), discuss the general case of what I hope to apply specifically to marriage; embedded options in a contract. In their analysis, they introduce the concept that the price stated in a contract for purchase is not simply the price of the good, but the price of the good plus the value of the option of breach to the buyer (and consequently the cost to the supplier of providing that option), if that option is allowed. If the market price falls below what the buyer has contracted to pay for the good, he has the option to breach and buy the good for the market price. This sort of contract contains what Scott and Triantis call an embedded option. Another way of looking at the case described above is that the buyer has purchased a good, but when the market price drops, he exercises his option and sells the good back to the seller for the contract price. This sort of option is also known as a put.

I. 4. **The Embedded Option Model of Marriage**

It is with this perspective on contracts in mind that I model marriage. Since divorce is a breach of contract that a couple has as an option, it would make sense to model marriage in an option framework. As in Scott and Triantis, the value of the marriage to the couple is not just the expected value of the marriage, but also the value of the embedded option. This is more the case today than in times past, when divorce was difficult to obtain and socially costly, but the option model presented here can also accommodate a high penalty for breach. Alternatively, in cases where the cost of divorce is low enough so that divorce is a realistic option, the model shows that the expected utility from a marriage with the option to divorce is greater than the expected utility of the marriage without the option. This difference will be referred to as the "put utility" of the marriage. For the remainder of this paper, the marriage without the option to divorce will be referred to as the "underlying marriage"; "marriage" appearing by itself is intended to include the option value.

One of the somewhat counterintuitive results of this model is that many couples will marry with a rational expectation that the marriage will go badly and that they will get a divorce. How do we explain that, given the above surveys done by sociologists, where people universally indicate that they truly believe that their marriage will last? An explanation for this "expectation" of the marriage lasting is that engaged couples never think of divorce as an option when getting married because they envision the best-case scenario. It is true that given the way the model is constructed, the best-case scenario will never result in divorce, but the envisioned scenario is not necessarily the expected scenario, in the sense of a rationally expected value.

Another counterintuitive result of this model is that an increase in variance often increases the expected utility of the marriage. Financial options have the property that increasing variance of the underlying security, all other things equal, increases their value. (Ross, Westerfield, and Jaffe, 2005) I show that an increase in the variance in the benefits to the underlying marriage with no change in the expected value of the underlying marriage increases the expected utility to the couple from marrying. Since the couple often envisions the best-case scenario of a marriage, an increase in variability brings to mind an increased chance of the marriage significantly deviating from its envisioned, ideal value. In order to talk about the variance of the marriage changing without changing the expected value, however, larger magnitude downward fluctuations in a marriage must be balanced by some combination of a decrease in the chance of those fluctuations or larger magnitude upward fluctuations (or simply a higher 'envisioned' value of marriage).

The increased expected utility with an increase in variance allows couples with a low expected value of the underlying marriage but high variance to get married rationally. Other things equal, with increased variance, we expect more marriages should be started. A corollary to this increase in the number of marriages is that more marriages with low expected value will be started. The model predicts, reasonably, that these marriages will have a high divorce rate.

An assumption of this model is joint utility maximization. Many analyses (McElroy and Horney, Masner and Brown) since Becker have used a Nash-bargaining model in which the individuals in a couple have different utility functions. This type of interaction would affect both the value of the marriage and the assumption that a couple will get a divorce when it is efficient to do so. Becker's original reason for his assumption of joint utility maximization was that the couple "love" each other, and this will minimize policing costs that would be present in a loveless marriage. Love is implicitly allowed to vary in my model. Thus we can still say that as a couple, utility is maximized, *given* that the two members of the couple have fallen into a Nash-bargaining mode. This is a lower utility than would be experienced had the couple continued to love each other, but within a relationship, love is regarded as an exogenous stochastic variable. Furthermore, the couple is aware of the possibilities for love to fail, and will take this into account when making decisions. Regardless of the accuracy of modeling an entire couple as utility maximizing, individuals maximize utility, and this model could be adapted to individuals making nuptial decisions.

Part II: The Simple Model

II. 1. **Introduction to the model**

To show some basic results, I model marriage in three periods. In the text, the basic ideas are presented, with the technical details in the appendices for the interested reader. In each period n, the couple receives a "happiness dividend", equal in value to a certain amount of money, D^n. Thus benefits to a marriage accrue over time, as a flow. The dividend, D^n, represents this flow of utility, or more precisely, the cash payment the couple would exchange for this amount of utility. In each period, the dividend changes value to either an up state or a down state; $D^{n+1} = u*D^n$ or $d*D^n$ (see Figure 1). The magnitude of u and d are determined by how uncertain the benefits to the marriage are. In fact, when u and d are calculated for a traded stock in the binomial model, they are based on the past volatility of the underlying asset. (Copeland and Antikarov, 2001) There is no readily observable underlying marriage as in the case of a stock, although I do presume that the couple itself makes projections as to the variance. In simulations, the variance is set to a given level and the u and d are inferred from this assumption, but in analytical results, u and d are used as a proxy for the variance.

Another issue with having no underlying asset is that in a binomial model, the underlying asset's price determines the probability of the stock taking on the up or the down state. In all simulations, for simplicity, the probability of taking the up and the down state is set to ½.

Being in an up state or a down state represents some change in the quality of the couple's match as information is revealed. The individuals in the relationship may change traits unexpectedly, or their tastes may change. Looking again to the question of joint vs. individual utility maximization, changing tastes could include a lower cross-derivative of utility functions (less love in the marriage), which would hurt the joint utility by introducing policing costs.

A complication in making a direct analogy to a traded option is that marriage is a project with a termination date: the anticipated lifespan of the individuals in the couple. When the last period ends, there is no more underlying stock value. In addition, the difference between marriage and a traded option is that there is no indication of the underlying value of the marriage as there is in the traded option's case where this is equal to the expected present value of the future dividends. I presume that couples implicitly value the option to divorce when deciding to marry. Thus, the underlying value of the marriage to the couple cannot be determined from the value of some other relationship with no option for divorce, as is the case with a traded option. This is no different from a firm deciding on a new project with an embedded real option, such as the option to abandon a project. The firm is not interested as I am in what the project would be worth if there were no option to abandon. I define the value, V^{path} of the underlying marriage, as the present discounted value of the future dividends at a particular point of the path. I denote the initial underlying value of the marriage as V^0, and the value at any future point as V^{path}; for example, after finding the marriage has taken on the down state in period 1, the underlying marriage now has an expected value for the remainder of the marriage of V^d (see Figure 2). This value gives us a baseline for comparison; I can determine the value of the option to divorce. Were the couple not married, they would individually receive amounts that sum to R in each period, the reserve payoff. As a simplification, the reserve payoff is taken to be certain. While the expected value of remaining single is in reality not certain, the addition of another individual will likely increase the variance of expected value. Variance in the expected value of the marriage can be regarded as variance over and above the variance in the expected value of being single. If variance outside of the marriage were shown to be greater than variance within the marriage, the model would still be feasible; it would merely be necessary to subtract a risk premium from the reserve payoff, but all this does is affect the value of R at a given time.

Figure 1: Dividend Tree

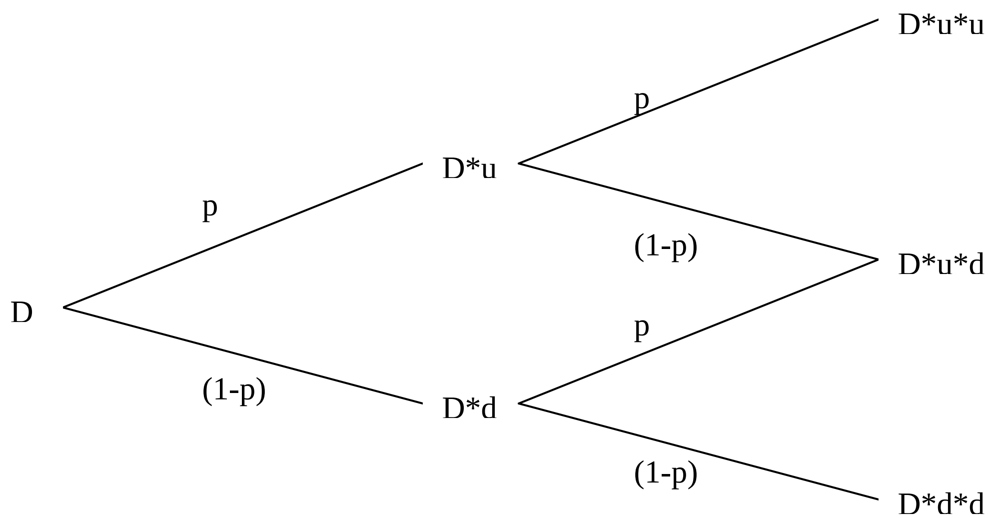
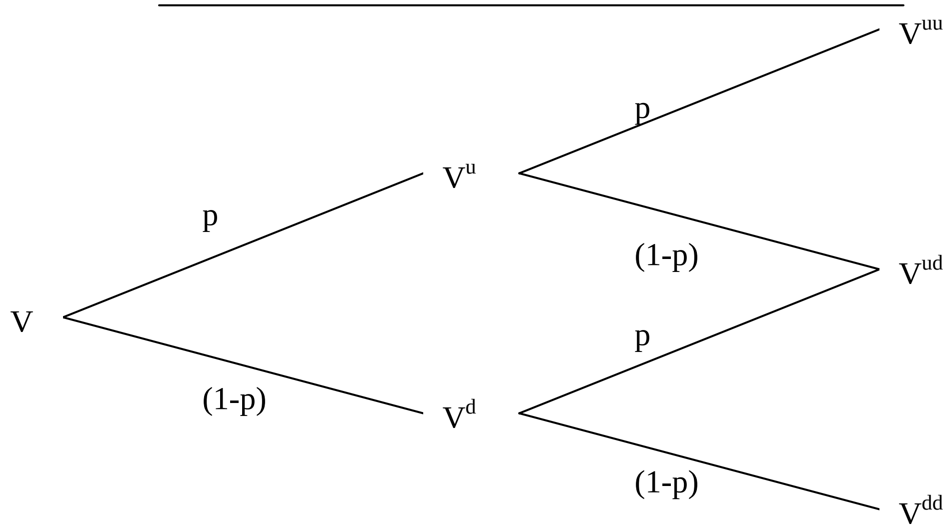


Figure 2: Stock Tree



In period n + 1, the dividend received is u*D^n with probability p, or d*D^n with probability (1-p). In all simulations done, u > 1 and d < 1, and within each individual simulation, I keep the p, u, and d constant through all periods. There is no reason to believe that either of these assumptions is true; this model cannot make any predictions about the timing of a divorce without better estimates of how the variance in the benefits to a marriage changes through time. Furthermore, it is entirely possible that all couples start off in period one happier than they will be in future periods, so it is not necessarily true that u > 1. The simple case is presented here, however, and many of the results of the simple case can be generalized.

The couple decides whether to get divorced in period n before receiving D^n, opting instead for R in period n, and R in all future periods. Again, it is assumed that the couple jointly maximizes utility when deciding to divorce. Zelder (1993) addresses this issue in more detail and show that in some cases under no-fault divorce, it is unimportant who is leaving whom; marriages only dissolve when they are inefficient overall. In this model, if the joint utility of the couple is higher than the sum of their reserve payoffs, then they will choose to marry/remain married. Regardless of whether utility is maximized individually or jointly, the option value of the marriage is present.

Table 1: A quick guide to notation

duu	Node at which the marriage sits after two up movements and one down (commutative)
(u, d, u)	Marriage goes up in period 1, down in period 2, and up in period 3 (not commutative)
D^{ud}	Dividend to be received at node du
V^{ud}	Value of underlying marriage at node du
$V^{ud,\ put}$	Value of marriage with option to divorce at node du
R^n	Reserve utility in period n

II. 2. **A simple underlying marriage**

For a numerical example, the simplest case is where p = .5, d = (2 – u) (Appendix D), and the reservation payoff R is the same as the initial dividend, D. (see Appendix A) The couple would be equally happy single or married in period zero. Assuming risk neutrality and a discount rate of 0, the sum of the future reservation utilities, R^0, is 3R.

The expected value of the marriage at any point on this tree, or the underlying value, is the sum along each future path times the probability of taking that path. To make things more concrete, D and R are set equal to 100, u = 1.5, and d = .5 in the dividend and underlying value trees in Appendix A.

In the analytical case, the value of the underlying marriage in period 0 is 3D, and since D = R, the underlying value is equal to the reserve utility at the outset of the marriage. Given the couple's risk-neutrality and discount rate of 0, if they were not allowed to divorce, they would be indifferent between remaining separate and marrying. (See Appendix A)

II. 3. **Divorce in a simple marriage**

Now the possibility of divorce is introduced to the model; the couple decides to divorce or remain married after learning the dividend in the coming period but before receiving it. If they divorce in period n, they receive R in all subsequent periods; the reserve utility in period n is denoted as R^n, the present value of the future reserve payments (see Figure 3). The condition for divorce is:

$$R^n > V^{node,\ put}$$

$V^{node,\ put}$ could include a divorce in a later period m, where the couple would receive R^m rather than a lower $V'^{node,\ put}$. To find the values of the nodes on the put tree, I work backwards from the final period. With one period left, the couple has a choice between D^{final} and R, so the final period dividend received by the couple is Max(D^{final}, R). This choice then changes the expected value of the period prior to this; in the three period model the value in period one in the down state,

$$V^{d,\ put} = d*D + p*Max(u*d*D, R^2) + (1-p)*Max(d*d*D, R^2)$$

The value in period zero is then

$$V^{0,\,put} = D + p*Max(V^{u,\,put}, R^1) + (1-p)*Max(V^{d,\,put}, R^1)$$

The couple chooses between $V^{0,\,put}$ and R^0 when deciding whether to get married. Figure 4 shows the numerical put tree that results from the trees in Appendix A.

Figure 3: Put Tree

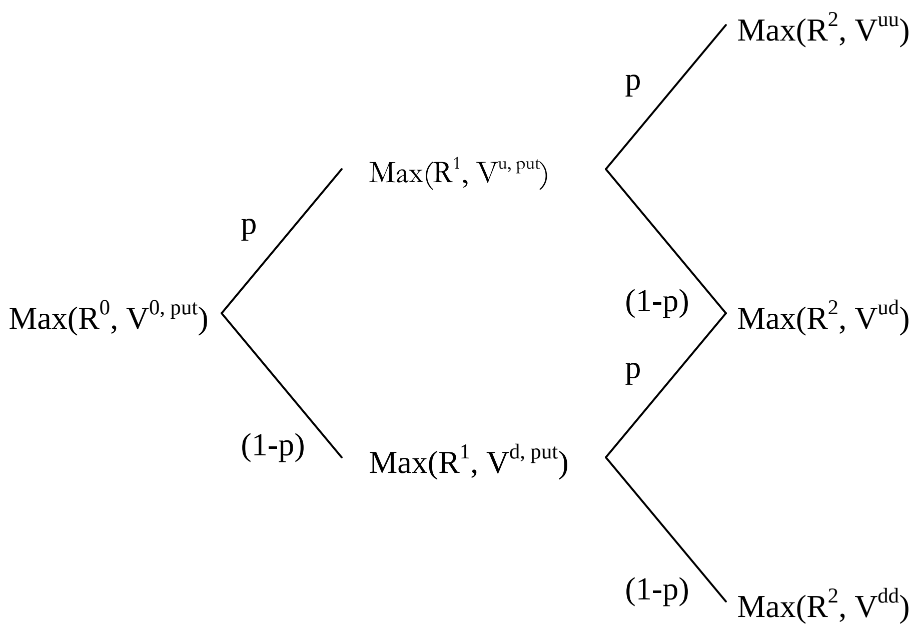
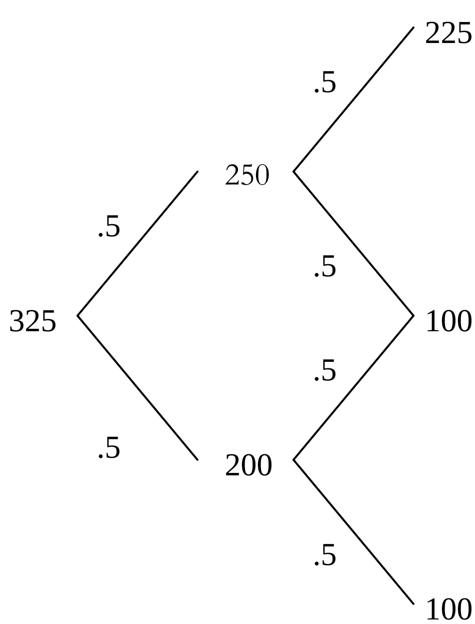


Figure 4: Numerical Put Tree



The marriage now has the same or higher expected value at the outset with the option to divorce, $V^{0, put}$. I refer to the difference as the put value of the marriage: $V^{0, put} – V^0$. With the parameters specified above, the option will always be exercised in the down state in period 1. (See Appendix B) In this case it is easy to see why the marriage would acquire a higher value with the possibility of divorce. The reserve payoff in period 0 is identical to the dividend, so the couple gets the same in that period whether they marry or not. In period 1, with probability ½ the payoff is u*D, which is greater than R, or with probability ½ the payoff is exactly R, when the couple gets divorced. The couple risked nothing in getting married with these parameters, and has the possibility of the relationship working well. What is interesting about this simple model is that, as in the case of a stock option, $dV^{put}/du > 0$; i.e. the value of a marriage increases as the variance[1] in the benefit from the marriage increases, even though the expected value of the underlying marriage is unaffected by equal changes in u and d. (Appendix B)

II. 4. **A basic result**

The simple case above can be made more interesting by allowing the underlying value to be below the reserve utility in the initial period, that is, $V^0 < R^0$. If the couple were not allowed to get divorced, they would on average be less happy getting married than they would be single. With the option to divorce, however, there are some cases where $V^0 < R^0$, but $V^{0, put} > R^0$, and the couple will decide to marry. The number of couples falling into this case depends on the size of the put value, which increases as the variance of V^0 goes up. These couples expect that the marriage will go bad, but get married

1. Since the variance depends solely on u and d, and d is a function of u, the derivative of the value of the put with respect to the parameter u is comparable to the derivative of the value of the put with respect to variance.

anyway on the chance that the marriage will not go bad, knowing that if it does, they can get out for a reasonable reserve payoff. As the put value increases with increasing variance, more marriages and more divorces would be expected under our set of assumptions.

Part III: The More Complex Model

III. 1. **New parameters**

The benefit to marriage is largely dependent on household public goods, whose value dissolves in the case of a divorce. A household public good is a good that, within the household, are non-rival, non-excludable, etc. Also decreasing joint utility in the case of a divorce are semi-public goods, such as a newspaper subscription or a sofa. It is unrealistic to ignore these costs of divorce. Divorces are socially and emotionally costly as well. The decrease in the cost of a divorce, in any of its forms, is also an exogenous change that is implicated in the rising divorce rate. To incorporate the cost of a divorce in this model, I keep the convention of a reserve payoff of R in each period, but for the period in which a divorce occurs, the dividend is $R - C_d$.

Another factor I have ignored is couple's risk aversion. Until now, I have been using payoffs in monetary equivalent units. It is necessary to deduct a premium from future uncertain payoffs based on the couple's risk aversion. To determine this premium, I assume preferences exhibit constant relative risk aversion, which makes the premium proportional to the variance and the wealth of the couple. Constant relative risk aversion will also allow the selection of arbitrary values for the initial dividend and reserve utility without changing the results, given that the ratio of the reserve utility to the dividend remains the same. (See Appendix C) The utility of the marriage with the risk premium subtracted is denoted $U(V^{node,\ put})$. (See Appendix E)

III. 2. **Determination of the marginal marriage**

Now that the value of the option, the cost of divorce, and the risk premium have been taken into account, it is possible to more realistically state how variance within a relationship might impact marriage and divorce decisions. In this more complex model, a fourth period has been added to the model to allow for more flexibility and variety of results; in the three period model for example, the only possibilities for the probability of divorce for a given marriage are 25, 50, and 75%. I again assume at first that the expected value of the marriage is independent of the variance, so $p = .5$, and $d = 2 – u$. This formula for the value of d as a function of u keeps the initial value of the underlying marriage constant when $p = .5$. In general, Excel's Solver function can be used to keep the initial value at the desired level when $p \neq .5$. The reserve utility is pegged at 100 per period, the discount rate on future periods is 10%, and the degree of relative risk aversion is 4. The model is set up in an Excel spreadsheet, available on request.

It would be informative to find the limit of how unhappy a couple can expect to be, yet still get married. To determine the marginal marriage at different variances (See Appendix G) and costs of divorce, I have the solver decrease the initial dividend while adjusting the value of the parameters u and d to keep the marriage at the desired level of variance until $U(V^{0,\ put}) = R^0$. At this point, the couple is indifferent between marrying and staying single. In order to decide whether the couple will divorce at various points in time, the utility at each node, $U(V^{node,\ put})$, is compared to R^n; if is it less than the reserve, then the couple will opt for a divorce at that point. I thus can also determine the divorce rate of the marginal marriage, which is equal to the sum of the probabilities of taking paths that lead to divorce.

Table 2: Marginal marriage

$C_d = 0$			$C_d = 10$			$C_d = 20$			$C_d = 30$		
σ	D	DR(%)	σ	D	DR(%)	σ	D	DR(%)	σ	D	DR(%)
50	89.8	75	50	94.5	75	50	98.2	63	50	101.3	50
100	81.7	88	100	87.2	75	100	93.0	75	100	98.0	62.5
150	75.4	88	150	81.8	88	150	87.8	75	150	94.2	75

The value of the parameters C_d, D, and σ are arbitrary, but the function is homogenous of degree one with respect to these variables, so it is proper to talk about them as ratios. In this model, if divorce were costless, a rational couple looking ahead and expecting 75.4% of the outside utility within a marriage with a standard deviation of 150 would marry, even though in 88% of cases, they will get a divorce (see Figure 6). Since p is fixed at .5 for now, the standard deviation is dependent solely on u, d, and D. A standard deviation of 150 in this example is equivalent to a 61% increase or decrease (u = 1.61, d = .39) in the dividend from one period to the next when D = 74.5.

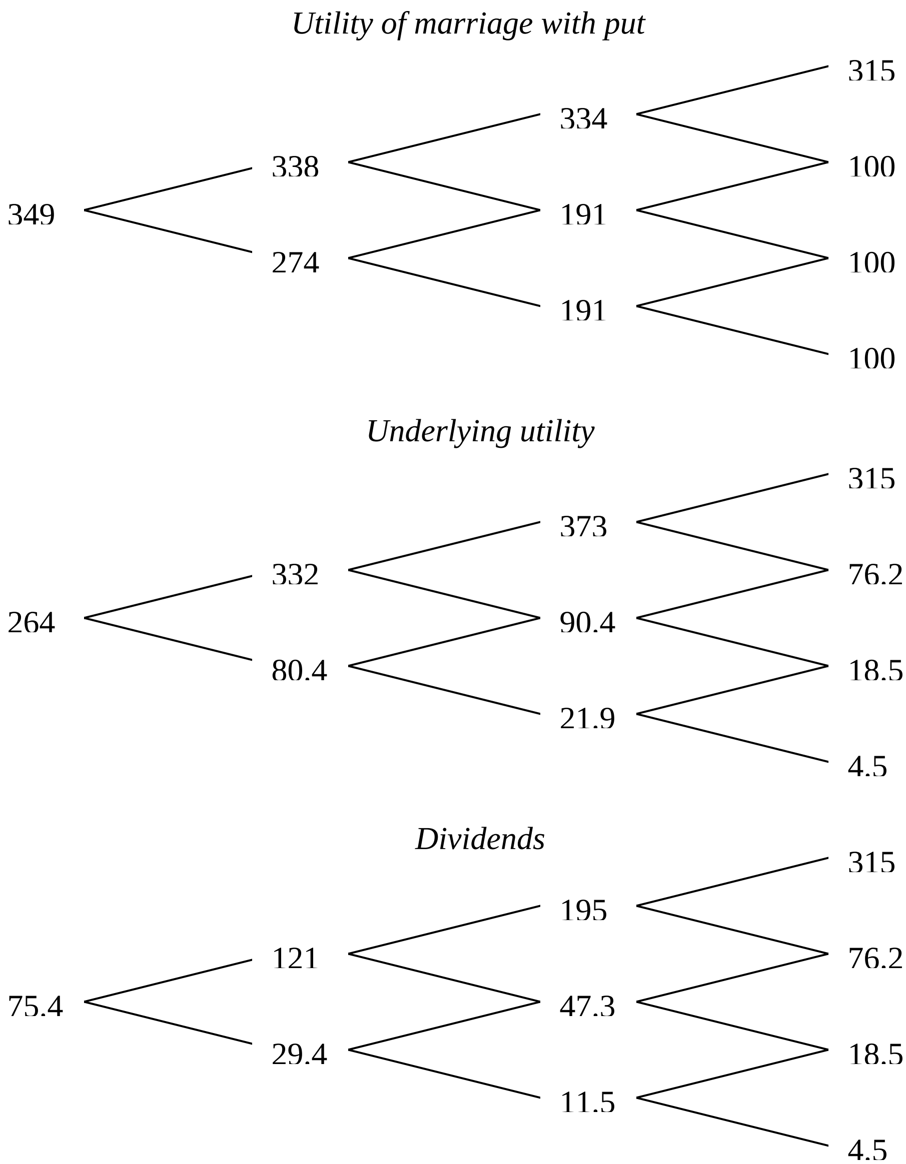
When the cost of divorce is 30% of the reserve utility and the standard deviation is low, the expected value of the marriage must actually be higher than the reserve utility before the couple decides to get married. At this point, the cost of a divorce has reduced the value of the option to such an extent that the risk premium associated with the extra variance of a marriage outweighs the value of the option. The value of the marriage in this case still increases with increasing variance since the marginal marriage is less valuable with higher variance. Note, however, that the value of the marginal marriage changes by only about 3% on a jump from $\sigma = 50$ to $\sigma = 100$ at this cost of divorce, as opposed to an 8% change when divorce is costless. At sufficiently high levels of C_d, the required expected value of marriage will actually increase with increasing variance. When divorce is so expensive that it is essentially not an option, variance only serves to increase the risk premium.

III. 3. **Interpretation**

What do these results mean for marriages today? Douglas Allen (1997) points to the increase in the female labor participation rate as a force towards more variance in the benefits to marriage. Before women worked in large numbers, a fairly certain benefit to marriage was specialization; the man worked in the market, and the woman specialized in home production. In more recent times, according to surveys, men and women of marrying age expect that both will be part of the labor force to some extent. (Ganong and Coleman 1992) The absence of this specialization aspect decreases the expected benefit to marriage, but in addition, future wage is significantly more certain than other factors that impact the utility of the couple, such as love or physical attractiveness. (Ahrons, 1994) My model suggests that the increase in female labor force participation rate will have a positive impact on the number of divorces. If the variance had not changed relative to the expected value of the marriage, we would expect a significant decline in the marriage rate as well. However, the relative increase in variance means that couples need rationally expect a lower fraction of their reserve utility to jump into a marriage.

Another exogenous change that is implicated in the changing nature of marriage is the ability of abortion and birth control since the 1960's. It would be possible to include children in a more complex option model, but it is still possible to frame the issue in

Figure 6: Trees for marginal marriage with
σ = 150, R = 100, D = 75.4, p = .5, rra= 4, r = .1



terms of the current model parameters. It is assumed that any children are "loved" in the technical sense of the word, i.e., the family makes a joint decision to divorce. Each additional person will have their own friction associated with divorce, so to simplify the issue, the presence of children increases the cost of a divorce. If married couples with no birth control expect a certain number of children, they enter marriage with the expectation of a higher cost of divorce than a couple with birth control, other things equal. Variance is less of a positive effect on the value of the marriage in this case, and the marginal marriage must have a higher value. The divorce rate is lower in the case of no birth control. Other things equal, the marriage rate is also lower.

From Table 2, observe that when going from high cost of divorce to low cost of divorce, the critical expected value of the underlying marriage decreases, or in other words, marriages become more valuable. Note that marriages become more valuable faster at high variance than at low variance as the cost of divorce decreases. (See Appendix F) For example, going from $C_d = 30$ to $C_d = 20$ at $\sigma = 50$ results in a roughly 3% decrease in the requirement for a marriage. However, at $\sigma = 150$, this same change in the cost of divorce results in a 6% decrease. I do not address the cost to society or to the couple of lowering C_d in this model, but if the cost of divorce has been set to maximize overall social utility, then the marginal cost of lowering C_d has been set equal to the marginal benefit of lowering it. Increasing variance increases the marginal benefit of lowering the cost of divorce, so an optimal C_d would be lower in a society where the benefits to marriage have a high variability. This result agrees with Allen's claim that no-fault divorce laws were enacted as the result of a demand for more divorce as opposed to a root cause of the increasing divorce rate. This result is speculative, however, and cannot be verified without looking more deeply into the social benefits of a high level of commitment in marriage.

Part IV: Conclusions and Extensions

This analysis attempts to assess some of the changes in marital patterns over the past half-century using an option framework. I first find that marriage becomes more valuable, and thus more likely to occur, when the option to divorce is present. Then it is shown that an increase in variance of the benefits to marriage coupled with a low (zero) cost of divorce actually increases the value of a marriage when the underlying value of the marriage remains the same and the couple is risk-neutral.

When the cost of divorce is high, increasing variance increases the value of the marriage more slowly and even decreases the value of the marriage due to the risk-aversion of the couple outweighing the additional option value. This effect would also hold true in any circumstance where divorce is not likely; a couple with an initial valuation of marriage high enough relative to the reserve utility will not get divorced. In this case, variance would only decrease the couple's utility by increasing the risk premium.

Besides changing the effect of variance on the expected utility of a marriage, a decrease in the cost of divorce in general increases the divorce rate for two reasons. One is the frequently cited reason that people are more willing to give up on a marriage if it is easy to divorce. The other effect, however, is that decreasing the cost of divorce increases the marriage rate by increasing the put utility of marriages and thus making them more valuable. The additional marriages have a low underlying value, however, so they tend to end in divorce more frequently.

It is with these effects in mind that we can assess the accuracy of some of the posited reasons for the high divorce rate, or other changes in marital patterns. The availability of birth control and the institution of no-fault divorce laws decrease the cost of divorce, so their predicted effect is to increase both the number of marriages and the number of divorces. The entry of women into the workforce will increase the variance within marriages, and decrease the expected gain from the marriage, thus causing an increase in the divorce rate, and a decrease in the number of marriages. The decline in the marriage rate is not as significant as it would be had a more variable factor contributing to the value of a marriage had been diminished, however.

This model confirms the popular notion that "people don't take marriage seriously anymore", but it suggests that this notion can be ruled out as an underlying cause of an increase in the divorce rate. Changes in other factors have made it rational to marry with lower expectations for the marriage, but there is no need to assert some sort of mob mentality or generational lapse in morals in order to explain the increase in divorce since the 1950's.

This model is not yet strong enough to provide a completely accurate picture of marriage. It suggests some interesting conclusions, but makes some assumptions that may or may not be justified. A glaring omission is the possibility of children. Future work with this model could include an option to bear children, which would significantly increase the value of a marriage by giving the couple the opportunity to receive the utility of a child at the price of a higher cost of divorce. A factor alluded to above, the value of commitment to both a couple and to third parties, is also not included in this model. The simplification that the variance of a marriage stays constant over time is probably not accurate; it likely decreases over time as fewer and fewer unexpected traits emerge. Taking this changing variance into account would allow for a prediction of the timing of divorces. More information on the distribution of the value of marriage across couples would allow the model to predict a divorce rate. It also might be possible to extend this model to infinite periods in order to derive more analytical results. A model such as this, however, is a first step towards accurately taking into account the impact of changes in variance, expected value, and the cost of divorce on marital trends.

Appendix A

In general in the three-period model to calculate the underlying value, we sum the expected value of all future dividends and discount them:

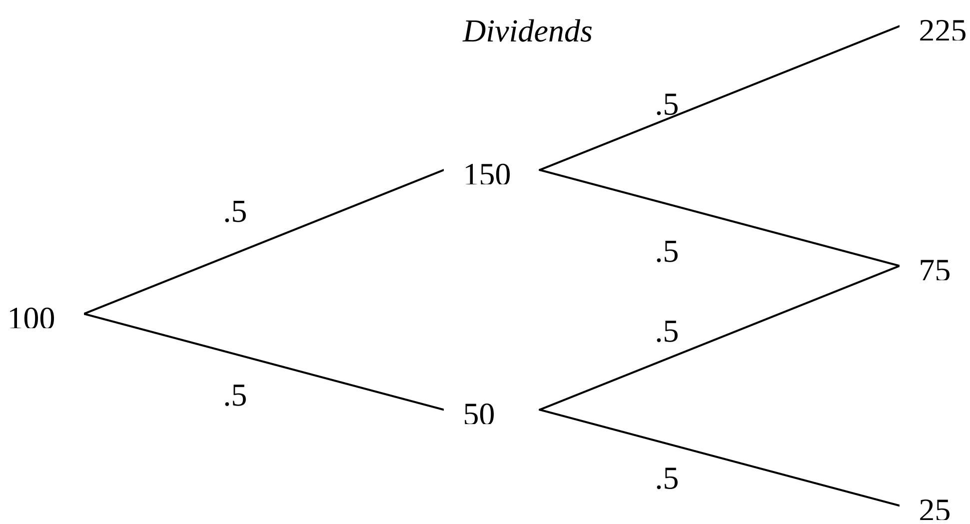
$$V = p^2*(u^2D/(1+r)^2 + uD/(1+r) + D) + p(1-p)*(udD/(1+r)^2 + uD/(1+r) + D) +$$
$$p(1- p)*(duD/(1+r)^2 + dD/(1+r) + D) +(1-p)^2*(ddD/(1+r)^2 + dD/(1+r) + D)$$

Since $d = (2-u)$, $r = 0$, and $p = .5$,

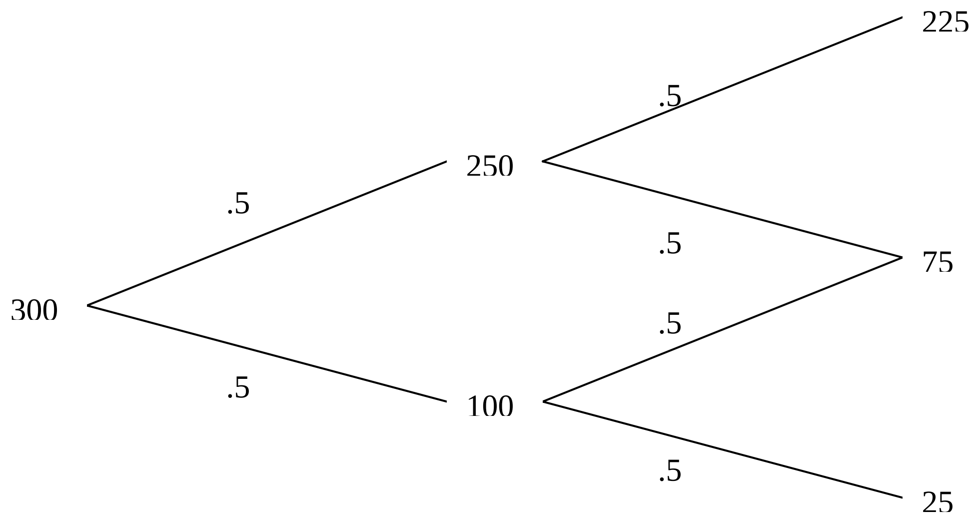
$$V = .25*(u^2D + uD + D) + .25*(u(2-u)D + uD + D) + .25*((2-u)uD + (2-u)D + D)$$
$$+.25*((2-u)^2D + (2-u)D + D),$$

which simplifies to 3D, or 3R.

Numerical example of dividend and underlying marriage trees:



Underlying marriage:



Appendix B

The option to divorce is only valuable when a divorce occurs. Thus looking ahead from period 0, the put value can be calculated by finding the gain from divorce in each situation and weighting it by the probability of being in that situation. A demonstration of the put value calculation here is based on the simple model specified in Appendix A.

After a down movement in the first period (probability = .5), the couple can expect:

$$D*d + p*D*d*u + (1-p)*D*d^2$$

when p = .5, and d = (2-u), the expectation is

$$D[(2-u) + .5(u*(2-u) + (2-u)^2)]$$

$$= D[2(2-u)] = 2D*d$$

Since d < 1, the couple would opt for 2R, and divorce.

After an up movement in the first period and then a down movement (probability = .25), the couple is at the final node with a dividend of u(2-u)*D, which is less than R when u is greater than 1, so the couple will divorce.

A path of (d) and (u, d) are the only two situations in which the couple will divorce. It is now possible to calculate the put value:

$$\text{Put value} = (2R - 2D*d)*.5 + (R - D*u*d)*.25$$

substituting D for R, and (2-u) for d,

$$= D[u^2/4 + u/2 + 3/4]$$

differentiating with respect to u,

$$\partial \text{Put}/\partial u = D(1+u)/2$$

This is positive for u > -1, as it is in this model.

Appendix C

The following is derived from Nicholson's Microeconomic Theory (1992) based on Pratt (1964):

If we want to subtract a risk premium P from the expected value of the marriage in order to determine the certainty-equivalent value of the marriage for a risk-averse couple. The variable h represents a fair bet of h "dollars":

$$E[U(V + h)] = U(V – P)$$

Assuming the utility function is differentiable, use the Taylor series for approximations:

$$U(V-P) = U(V) – P*U'(V),$$

$$E[U(V + h)] = E[U(V) + h*U'(V) + (h^2/2)*U''(V)]$$

$$= U(V) + E(h^2/2)*U''(V)$$

Solving for P,

$$P = -[U''(V)/U'(V)]*E(h^2)/2$$

Since $\sigma^2 = E(h^2) – E(h)^2$, and E(h) = 0 (by the definition of a fair bet),

$$P = -[U''(V)/U'(V)]*\sigma^2/2$$

In order to make the risk premium proportional to wealth, I assume preferences exhibit constant relative risk aversion:

$$rr(V) = -[U''(V)/U'(V)]*V$$

Putting this into the equation for P:

$$P = rr(V)*\sigma^2/(2V)$$

The beneficial feature of this formula for the risk premium is that it remains a constant percentage of wealth. For example, if D = R = 100, u = 1.5, d = .5, and p = .5, I want that to give the same proportional risk premium as if D = R = 1,000. Suppose a two-period model with rr(V) = 2:

D = R = 100, u = 1.5, d = .5, p = .5

$$V = 100 + .5*1.5*100 + .5*.5*100 = 200$$

$$\sigma^2 = 2{,}500$$

$$P = 2*2500/(2*200) = 12.5$$

Now with the same parameters, only now D = R = 1,000

$$V = 1{,}000 + .5*1.5*1{,}000 + .5*.5*1{,}000 = 2{,}000$$

$$\sigma^2 = 250{,}000$$

$$P = 2*250{,}000/(2*2000) = 125$$

When the expected value is multiplied by 10, the risk premium is multiplied by 10, which is what I wanted to show. In order to err on the more risk-averse side in the simulations, a value of 4 for the relative risk aversion is used. The generally accepted estimate for the degree of relative risk aversion is something greater than 2. (Friend and Blume, 1975).

Appendix D

In general, I want to be able to change the variance and the probability of an up movement without changing the expected value of the marriage; this requires d to be a function of u and p. For this to be the case in the three period model, I set the derivative of expected value with respect to u equal to zero and solve the resulting differential equation for d as a function of p and u:

$E(V) = D + p*u*D + (1-p)*\mathbf{d(u, p)}*D + p^2*u^2*D + 2*p*(1-p)*u*\mathbf{d(u, p)}*D + (1-p)^2*\mathbf{d(u, p)}^2$

$\vartheta E(V)/\vartheta u = D*p + D*(1-p)*\mathbf{d'(u, p)} + 2*D*p^2*u + 2*D*p*(1-p)*[\mathbf{d(u, p)} + u*\mathbf{d'(u, p)}] + 2*D*(1-p)^2*\mathbf{d(u, p)}*\mathbf{d'(u, p)}$

Setting this equal to 0 and solving for **d'(u, p)**:

$\mathbf{d'(u, p)} = -[p + 2p^2u + 2p(1-p)*\mathbf{d(u, p)}]/[(1-p) + 2p(1-p)u + 2(1-p)^2*\mathbf{d(u, p)}]$

Using Mathematica,

d(p, u) = (1 - pu)/(1-p). In the case where p = .5, d = 2 – u.

In the four-period model, the differential equation obtained is exceedingly complex, so the constraint function in Excel's Solver is used to numerically keep the expected value constant.

Appendix E: Finding risk premiums for stock and put trees

The variance at each node of the stock tree is the probability-weighted average of the square of the value of each path minus the probability-weighted average of the value of each path squared. From this, the risk premium P is determined at each node, and the expected value is reduced to yield the utility at each node.

The risk premium on the put tree is not quite as straightforward as the risk premium on the stock tree. On the stock tree, there are a defined number of different paths, each with a probability and a value, so the variance and value are easily calculable. On the put tree, since the decision to exercise the option is dependent on the utilities at future nodes, which themselves depend on whether the option is exercised or not, there are not neatly defined paths from which to calculate variance. Instead, I determine the risk premium for each node at the end of the decision tree and work backwards. In the second-to-last period, the last point at which there is uncertainty, there are two possible outcomes, neither of which depends on future risk premiums. For example, in the four-period model:

$V^{uu,\, put}$ = u*u*D + Max(u*u*u*D, R) with probability p, or

u*u*D + Max(d*u*u*D, R) with probability (1-p)

We can find the variance of this situation, calculate the risk premium at node uu, P(uu), and subtract it from the expected value to give the utility at node uu. The variance in the period prior to this can now be calculated because subtracting a risk premium from the two possible outcomes has made them “certain”:

$V^{u,\, put}$ = uD + Max($V^{uu,\, put}$ – P(uu), R + R) with probability p

or uD + Max($V^{ud,\, put}$ – P(ud), R + R) with probability (1-p)

The risk premium determined from this variance is not a risk premium for the rest of the marriage, just the period in which it is calculated.

Appendix F: Calculation of $\partial Put/\partial C_d$

As before, the value of the put is the probability-weighted sum of the value of the put in the situations in which it will be exercised. Since there are many maximization functions in the general equation for the value of the put in a four-period model, it is necessary to find $\partial Put/\partial C_d$ for six different cases. These cases represent different divorcing strategies that the couple would undertake. These strategies are based on the initial parameters of the marriage.

In cases I and II, the dividend in the final period is lower than $R - C_d$ only when the marriage takes the down value in each period; D^{ddd} is this dividend. In case I, the couple would get divorced only in the final period, at node ddd. In case II, the couple divorces at node dd because D^{ddu} is not high enough to make up for the low D^{dd} that would be obtained in period 2. The couple opts instead for $R - C_d$ in period 2 and R in period 3.

For cases III, IV, and V, in the final period divorce occurs at D^{ddd} and D^{ddu}. In case III, the couple divorces at node dd, and also at node ddu, when the marriage takes the path (u, d, d) or (d, u, d). In case IV, the couple divorces at node d, and also at ddu when the dividend takes the path (u, d, d). In case V, the couple divorces at node d, and also at node du when the marriage takes the path (u, d). In case VI, only D^{uuu} is high enough to keep the couple from getting divorced in the final period. The couple divorces after path (d), path (u, d), and path (u, u, d).

For example, to calculate $\partial Put/\partial C_d$ for period VI:

$$Put = [(1-p)/(1+r)]*[R - C_d + R/(1+r) + R/(1+r)^2 - E(V^d)] + [p*(1-p)/(1+r)^2]*[R - C_d + R/(1+r) - E(V^{ud})] + [p^2*(1-p)/(1+r)^3]*[R - C_d - D^{uud}]$$

$$\partial Put/\partial C_d = -[(1-p)/(1+r)] - [p*(1-p)/(1+r)^2] - [p^2*(1-p)/(1+r)^3]$$

∂ Put/ ∂ C_d for all six cases:

Case	∂ Put/ ∂ C_d
I	$-[(1-p)/(1+r)]^3$
II	$-[(1-p)/(1+r)]^2$
III	$-[(1-p)^2/(1+r)^2*(1 + 2p/(1+r))]$
IV	$-[(1-p)/(1+r)*(1 + p(1-p)/(1+r)^2]$
V	$-[(1-p)/(1+r)*(1 + p/(1+r))]$
VI	$-[(1-p)/(1+r)*(1 + p/(1+r) + (p/(1+r)^2)]$

With $r > 0$ and $0 < p < 1$, the "cross-derivative" of the value of the put with respect to the cost of divorce and the case number is negative; that is, in a higher case number, the value of the put decreases faster as the cost of divorce increases. In this model, a higher case number corresponds to a higher variance. This depends on the fact that the variance is the same in each period, however, so a more careful analysis is necessary.

Appendix G: Justification of variance parameters

The amount by which an individual marriage can vary from period to period is not at all well known, so in deciding on variance parameters in numerical examples, I make some crude estimates. An analysis by Blanchflower and Oswald (2000) estimates that the value of a lasting marriage yields happiness equivalent to $100,000 of additional income each year. These simulations suggest that a marriage will not end if it is yielding more than the reserve utility, so a marriage that ends in divorce is likely yielding the equivalent of a negative income each year. Roughly, then, the annual value of a marriage can vary by at least $100,000; twice median household income, which in this model would be analogous to the reserve utility. If the values for variance yield a marriage that is in the neighborhood of twice the reserve utility greater in a good state than in a bad state, they are probably reasonable. When $\sigma = 150$ (u = 1.48, d = .52), D = R = 100, the marriage in the best state is worth 650 total, and in the worst state is worth 180, a difference of 470, about 1.3 times the total reserve utility of ~350, so 150 is a reasonable parameter for the standard deviation. This is not a precise analysis, but it suggests that the variance used is at least on the right order of magnitude.

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