Syllabus for the Analysis Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Mathematical Induction

The Real Numbers
- Rational and irrational numbers
- Real numbers and the axiom of completeness

Sequences
- Convergence
- Convergence of bounded monotone sequences
- Cauchy sequences
- The Bolzano-Weierstrass Theorem for sequences

Point-Set Theory
- Limit points (also called cluster points or accumulation points)
- The Bolzano-Weierstrass Theorem for sets
- Open and closed sets
- Compact sets and the Heine-Borel Theorem

Infinite Series
- Convergence
- p-series and geometric series
- Absolute and conditional convergence
- Comparison, ratio, and alternating series tests

Limits and Continuity
- The limit of a function
- The definition of continuity and relation to sequences
- Continuity of sums, products, quotients, and compositions
- The Intermediate Value Theorem
- Boundedness
- Attainment of extreme values
- Uniform continuity

Differentiability and Derivatives
- Limit definition of derivative
- Derivatives at local extreme points
- The Mean Value Theorem and Rolle's Theorem

Sequences of Functions
- Pointwise and uniform convergence
- Continuity of the limit function
- Proving uniform convergence

Series of Functions

- Pointwise and uniform convergence
- The Weierstrass M-test
- Power series
- Radius and interval of convergence, behavior at endpoints
- Continuity and differentiation of power series

Integration

- Definition of the Riemann integral
- Properties of the Riemann integral
- Integrability of a continuous function over $[a, b]$
- Integration of sequences and series
- Integration of power series