# Spring 2020 Math 355 - 02: Introduction to Analysis (Alvarado)

**Instructor:** Professor Alvarado **Office:** 303 Seeley Mudd **E-mail:** [rjalvarado@amherst.edu](mailto:rjalvarado@amherst.edu)

(he/him/his)

**Office Hours:** Mon, 3-4pm; Tues, 11am-12pm; Wed, 3-4pm; Fri, 3:30-5pm; or by appointment.

*No office hours on Thursday.*

# Basic information about the course

**Course Webpage:** Up-to-date course information can be found on the course moodle page:

<https://moodle.amherst.edu/course/view.php?id=19531>

## Class Meeting Times and Locations:

|  |  |  |
| --- | --- | --- |
| **Day of Week** | **Meeting Time** | **Location** |
| Monday | 10:00am - 10:50am | Science Center, Room E210 |
| Tuesday | 10:00am - 10:50am | Science Center, Room E210 |
| Wednesday | 10:00am - 10:50am | Science Center, Room E210 |
| Thursday | —– | —– |
| Friday | 10:00am - 10:50am | Science Center, Room E210 |

**Course Content:** Broadly speaking, in this course we will investigate ways in which we can answer the following central question from a mathematically rigorous perspective:

*How can we trust Calculus?*

To address this question, we will spend the semester exploring the foundations of analysis. Here is a brief outline of the topics we will investigate this semester:

1. ***Functions and Cardinality:*** We begin by discussing the notion of a function and learn about some important properties (injectivity, surjectivity, and bijectivity), as well as operations on functions, such as composition. These ideas are an essential in analysis and will be of particular importance for us in discussing cardinality of sets. The latter notion concerns ways in which we can answer questions like: “Just how many elements does an infinite set contain?”.
2. ***Number Systems:*** Why do we study functions defined on the real number line? Why not another number system? It turns out that the real number system possesses a very important property known as “completeness”. Without this property much of Calculus as you know it would not be possible. We will learn about this crucial property as well as some other important classes of numbers: the natural numbers, integers, and rational numbers.
3. ***Sequences and Series of Real Numbers:*** We then learn about infinite ordered lists of real numbers (sequences) and discuss important classes of sequences known as Cauchy sequences. We will also learn about series and answer questions like “what it means to add up infinite many numbers?”. Sequences and series will prove to be an invaluable tool as we move through the semester.
4. ***Topology of the Real Line:*** You most likely encountered intervals like [0*,* 1], [0*,* 1), and (0*,* 1) in a previous math course. It is visually clear that these intervals are different, but just how different are they? What properties do they possess that can help us distinguish them from one another? The answers to these questions will aid us in formulating many of the key results in analysis. In particular, they will help us better understand how the behavior of a given function can be affected by its domain.
5. ***Functional Limits and Continuity:*** We explore the notions of limits and continuity of a function. You may seen many of this ideas before in your calculus class; however, we shall use the deep properties of the real numbers that we have developed in this text to obtain some important new insights.
6. ***Differentiation:*** How can we quantify the rate at which the outputs of a function are changing versus its inputs? The central idea behind this question is the derivative. We will spend time establishing a rigorous and precise understanding of this concept and investigate the many ways that the derivative can be used as an analytic tool.
7. ***Sequences and Series of Functions:*** Having already explored the notions of infinite ordered lists and infinite sums of *real numbers*, we take our exploration to the next level by learning about infinite ordered lists and infinite sums of *functions*!
8. ***Riemann Integration:*** The remainder of the semester will be spent on Riemann Integration where we cover we prove the Fundamental Theorem of Calculus!

**Course Objectives:** The central objective of this course is to investigate the fundamental concepts of analysis for real functions of a single variable. This is supported by:

* Learning about the basic tools used in analysis.
* Using the above mathematical tools to tackle certain basic results in analysis.
* Learning how to formalize one’s approach to reading and analyzing mathematical proofs in analysis as well as communicating mathematics in both written and oral formats.
* Learning how topics in analysis relate to other fields within the discipline of mathematics.

**Learning Outcomes:** Upon successful completion of this course, students will be able to:

* Read and analyze a given mathematical proof so as it determine its validity.
* Provide clear, mathematically rigorous arguments for the validity of a given theorem using fundamental tools in analysis.

**Textbook Information:** The required textbook for this course is *Understanding Analysis* by S. Abbott. Information on places where you can purchase a hard copy of the book can be found [HERE](https://www.amherst.edu/academiclife/departments/courses/1819S/MATH/MATH-355-1819S). You can also get a free PDF copy of the book on [SpringerLink](https://link.springer.com/book/10.1007/978-1-4939-2712-8) (be sure to be on the Amherst network when visiting this link). All assignments and exams will be based on lecture notes and this book. Not that there are several books which address the topics discuss in this course, and that you can use for reference. Each text has a unique approach to the topic so do not hesitate to consult multiple texts. Here are a couple books that may be of help:

* *Real Analysis for the Undergraduate* by M.A. Pons. [Free PDF available on SpringerLink.](https://link.springer.com/book/10.1007/978-1-4614-9638-0)
* *How to Think About Analysis* by L. Alcock (good for looking at the big picture)

Note: Although the texts above are freely available online in PDF format, you may also have the books printed for a small free through Amherst College’s partnership with SpringerLink.

**What to Expect During a Typical Week:** We will have four days of lecture. (During some weeks, one of these days will be dedicated to working in groups on problem sets.) In addition to one take-home problem set, each Sunday you will submit a Theorem/Definition sheet that contains all theorems and definitions covered that week in lecture.

**Necessary Mathematical Background:** MATH 211 and either MATH 271 or 272, or consent of the instructor. Students with a grade of B+ or lower in linear algebra are encouraged to take another 200-level course with proofs before taking MATH 355. Outside of these prereqs, the most important thing you need to ensure that you have successful/enjoyable time in this course is to have the right attitude. Being enthusiastic and having a bit of curiosity about mathematics beyond what you may have seen in other math courses is important. Some of these new topics may be hard to grasp at first; however, by staying motivated and being willing to work through the class materials *every* day, these ideas will become second nature and you will be well on your way to having a fantastic time! You cannot procrastinate in this course and you need to be willing to ask for help if/when you need it! If, upon reading this, you feel overwhelmed then please ask for help (see “Getting Help” section, below).

# Course Assessments

**Grading Scheme:** Your course grade will be determined as follows:

|  |  |
| --- | --- |
| Participation | 5% |
| Homework | 20% |
| Theorem/Definition Sheet Checks | 5% |
| Three Midterm Exams | 60% (20% each) |
| Analysis Comp Style Exam | 10% |

**Participation:** *Your engagement in the class is essential, for both your learning and the development of our community in the classroom.* 5% may not seem like much but it can be the deciding factor in borderline cases say, between an A- and B+.

The breakdown of the 5% is as follows: 3% will be based on asking *and* answering questions in class, actively engaging during group work days, and exit tickets. Coming to class on time and fully prepared with all of your materials is also factored into this 3% (it is also a courtesy and sign of respect for others). The remaining 2% will be based on successfully completing two items from the following list.

* Attend office hours a minimum of 10 times during the semester and participate in asking *and* answering questions. (Any combination of Q-fellow’s, the math 355 fellow’s, or my office hours count.)
* Attend a Math Colloquium talk and write a *short* report on something you saw/learned. I will announce appropriate events throughout the semester. Feel free to suggest events as well!
* Interview an Amherst math professor to learn about their story of how they got into mathematics! OR interview an Amherst STEM professor to learn about how Mathematics is used in their line of work. (Please see me for details)
* Give a five minute presentation related to mathematics/proofs. Please come talk to me if you are interested in doing this. Presentations will occur in March/April.
* Watch a movie/play, or read a book/paper/blog/etc about mathematics or mathematicians, and write a response paper. The paper should be about a page long and should describe the movie/play, something you learned, something you thought, etc. For a movie/play please include some proof like a ticket stub, etc.

**Theorem/Definition Sheet Check:** Every week we will cover several theorems and definitions during lecture. It is crucial to your success that you write down relevant theorems, and definitions on a separate sheet as these often get lost when they are buried in your day-to-day lecture notes. This sheet will be extremely helpful when working on homework/worksheets and when studying for exams. Every Sunday you will submit your theorem/definition sheets. These will be graded for completion.

**Homework:** Each week, you will have one assignment (typically due on Wednesday). These assignments are easily the most important activity for learning in this course, since this is where you get a chance to transform the passive knowledge that you gained by listening to lectures or reading the text into active knowledge of the material. Mathematics is not a spectator sport! *These assignments are intended to provide you vital feedback on your progress towards meeting the learning outcomes.* Such assignments are sometimes referred to as a formative assessment[1](#_bookmark0). You will submit your work using an online system called Gradescope. Collaboration with your peers is welcomed, and I encourage you to work on take-home assignments together, in pairs or small groups; however, each student must write up each problem in their own words. Failure to abide by the principle is considered in violation of Amherst’s Honor Code (see below). If you wish, you may typeset your homework solutions in LATEX for extra credit. An information sheet about how to get started with LATEX will be posted on the course moodle page.

In addition, each week you will be provided a set of practice problems. *You are expected to solve these problems, although they will not be collected and graded*. Note: I will look at these problems when I write your exams.

**Midterm Exams:** You will have three exams. *Your exams will evaluate your proficiency with the course material as it pertains to the learning outcomes.* Exams are sometimes referred to as summative assessments[2](#_bookmark1). Here are the tentative dates for your exams:

Exam 1: The evening of Thursday, March 5th Exam 2: The evening of Thursday, April 9th Exam 3: The evening of Thursday, April 30th

Note: If you have any conflict with these test dates, you must let me know by at least one week in advance (see also the late work/make-up policy for exams, below).

**Analysis Comp Style Exam:** Your last exam will be in the style of the [math department’s comprehensive](https://www.amherst.edu/academiclife/departments/mathematics-statistics/major/compexam) [exam](https://www.amherst.edu/academiclife/departments/mathematics-statistics/major/compexam). Details such as the date/time and topic list of this exam will be announced during the semester.

## Late Work/Make-Up Policy:

* ***Take-home Assignments:*** Your lowest homework/worksheet score will be dropped at the end of the semester. For this reason, late work for homework/worksheets will not be accepted. Students that will be traveling due to a college sanctioned event should hand in homework before leaving.

You may also request one 24 hour homework extension over the course of the semester. You do not need to provide an excuse or reason for your extension request; just ask. To claim an extension, you must:

* 1. Not have previously taken an extension, and
  2. Request the extension by email BEFORE the assignment’s due date. *Subject: HW Extension.*

1Formative assessments are tools intended to monitor a students’ progress towards the course objectives, providing ongoing feedback that can be used by students to improve their learning. (Emphasis is on providing feedback)

2Summative assessments evaluate student learning, knowledge, proficiency, or success at the conclusion of an instructional period. (Emphasis is on providing a specific grade)

**To Submit Late Work:** Gradescope will not let you submit late work. Therefore, to submit your late assignment you will need send me an email containing a single pdf of your scanned work, as well as a list of which pages contain which problems so that I can upload it manually to gradescope.

* ***Exams:*** Exam dates are firm and missed exams cannot be made up unless missed due to a conflict with an official college sponsored event, and I have been notified one week in advance. In the event of an extreme emergency, please contact me as soon as possible and we will work together to find an arrangement which is fair to both you and your colleagues.

**Email Policy:** If you happen to have a question outside of class/office hours, then the best way to reach me is by email. Barring any unforeseen circumstances (or unless otherwise stated) I will reply to emails within 24 hours; however, I often do not reply to emails after 8pm or on weekends.

# Intellectual Responsibility

As detailed in the Amherst College Honor Code: *“Amherst College cannot educate those who are unwilling to submit their own work and ideas to critical assessment.”* The default penalty for students that have violated the above Statement of Intellectual Responsibility is failure in the course.

The Statement of Intellectual Responsibility applies to this course in the following manner:

* ***Homework:*** You are strongly encouraged to work with other students, consult online resources, and learn from as many people as possible. However,

**all of your submitted work must be written in your own words.**

Copying solutions from other students or internet sources is plagiarism and is in direct violation of the Amherst College Honor Code which carries with it a default penalty of failure in the course. You are expected to list each person with whom you worked and any online source you have used on the front of your take-home assignment.

* ***Exams:*** Aids (such as books, notes, or cheat sheets) are NOT permitted during exams. Use of such materials is considered cheating and is in direct violation of the Amherst College Honor Code which carries with it a default penalty of failure in the course.

In summary, cheating includes (but is not limited to):

* Using books, notes, cellphone, etc. when taking exams.
* Submitting work copied from your peers, tutors, internet sources, etc.
* Allowing another person to copy your work.

# Getting Help

There are many resources on campus where you can find help and support if you are having difficulty with the course material or just want to simply double-check your thought process. Some include:

1. ***My Office Hours:*** Please stop by to see me (in Seeley Mudd 303) during office hours – you can stop by unannounced during these times! If you have scheduling conflicts with my office hours then you are also welcome to make appointments to see me (outside of my regularly scheduled office hours) at a time which is mutually convenient. To schedule an appointment simply send me an email!
2. ***Math Fellow Office Hours:*** Please stop by to see our math fellows during office hours! Our fellows are students just like you that are VERY experienced with the course you are taking. They hold a total of 8 office hours each week! Their times and locations can be found on our Moodle page.
3. ***The QCenter:*** The Moss Quantitative Center provides drop-in help during the afternoons and evenings, as well as some one-on-one tutoring. The location and contact information for those in charge can be found on the course moodle page. [See here for more information.](https://www.amherst.edu/academiclife/support/moss_quantitative_center/for-students)
4. ***Each Other:*** Work together with your fellow classmates and form study groups. Just be mindful of the policy regarding collaborations on coursework.
5. ***Tutoring:*** If you feel that the above resources are not meeting your needs then you might want to consider getting a peer tutor. Please talk to me about it first.

# Accommodations and Inclusivity

**Inclusivity Statement:** I put great value in welcoming each and every student into the classroom, regardless of their sex, race, nationality, gender identity, socioeconomic status, ability (intellectual or physical), religious beliefs, or sexual orientation. Each student brings with them to the classroom a unique set of experiences and I expect everyone to contribute to providing an inclusive environment. If, at any time, you experience a situation within this course that you feel challenges your sense of inclusion or accurate assessment of achievement, then please notify me as soon as possible.

**Accessibility Services:** If you have a documented disability for which you are or may want to be requesting an accommodation, you are encouraged to contact the Accessibility Services via either email at [accessibil-](mailto:accessibility@amherst.edu) [ity@amherst.edu](mailto:accessibility@amherst.edu), or phone at 413-542-2337. Once you have your accommodations in place, I will be glad to meet with you privately to discuss the best implementation of your accommodations. Note that seeking out accommodations and notifying me of them is the responsibility of the student. [See here for more information.](https://www.amherst.edu/offices/student-affairs/accessibility-services)

# Statement on Classroom Recording

To ensure the free and open discussion of ideas, students may not record classroom lectures, discussion and/or activities without the advance written permission of the instructor, and any such recording properly approved in advance can be used solely for the student’s own private use. Taking photos of the blackboard is acceptable but please ask me before doing so.

# Expectations and Advice

The main objective of this course is to bring you on a mathematically rigorous journey through some of the notions that you may have learned about in a Calculus course. In our adventure, emphasis in this course will be on the theory/concepts and understanding the ideas behind certain mathematical structures. Listed below are just some pieces of advice to help you along the way (please feel free to let me know what has/has not worked for you):

* **Attend *every* class:** This class moves very quickly, and the material repeatedly builds on itself daily. In addition, I rely heavily on my lecture notes to write your homeworks and exams. Missing (or even being late to) just one class can have a huge impact on your success. I will not reteach material for unexcused absences during office hours.
* **Visit me during office hours:** I truly want you to succeed so please, never ever be embarrassed to come ask me for help. I am here because I enjoy helping you work through the material and I expect to see all of you in my office working hard and asking questions at some point during the semester.
* **Visit our math fellows during office hours:** Our math fellows are students just like you, and they are very experienced with the material in the course you are taking. They were hand selected by the

department, and they want to help you! They offer office hours in the evening to give you additional opportunities for help after classes are done for the day. I expect that you will working hard and asking questions at some point during the semester.

* **Read the book:** I often do not present in class the exact proof given in the book so you should spend time going over those proofs carefully and making sure you understand every step.
* **Make a concept sheet:** On a separate sheet, write down relevant definitions, theorems, and examples. These often get lost when they are buried in your day-to-day lecture notes. This sheet can be very helpful when working on homework/worksheets and in studying for exams.
* **Seek out online sources:** Get on the internet! There are plenty of places where you can see fully solved example problems (e.g., other professor’s websites, YouTube, Kahn Academy, etc.). Two warnings: first, be sure that the solutions you find are in-line with the instructor’s expectations and second, be careful not to violate the Amherst Honor Code by submitting work that is not in your own words.
* **Do *all* of the practice problems:** Every problem that I assign for you to complete (whether it be for a grade or for practice) plays an important role in helping you achieve the learning outcomes and has been carefully chosen by me. I cannot stress enough, read your notes and do all of the problems! Forming study groups with your classmates to work on these problems is a fantastic idea and I strongly encourage you to do so.
* **Do not procrastinate:** Start working on your homework the day it is assigned. Some assignments can take longer than expected to complete.
* **Be patient and come see me:** If you feel like you are struggling to keep up with the material, come see me – we can figure out a plan to help you get back on track. If you feel like you are bored because the class is moving too slowly, talk to me. You may belong in another class; however, be cautious of feeling too comfortable since the difficultly of material and pace of the class that could change at any time.
* **Listen first, take notes later:** Lecture notes are synced to an online server in real-time so that you can access them during and after lecture. This is meant to eliminate the added pressure on you having to simultaneously copy both what I am writing and saying during lecture. You can take notes in lecture on important comments I make, and then fill in your notebook later by copying the synced notes.
* **Participate in class:** Class participation is encouraged and expected. Ask questions if you have them

– even if just to clarify a statement that I made during lecture. Never worry about “sounding stupid” or stopping me during classtime. You may be surprised that other students share the same question. If I pose a question to the class, answer it! I have never remembered a student for being wrong. I always remember those that try.

* **Be respectful:** Show respect for other people’s questions, answers, and ideas. Moreover, the use of cellphones during lecture time is distracting and disrespectful to both me and your peers. Please refrain from using these devices during lecture.
* **Do not be afraid to ask for help:** This class can be HARD at times. Do not feel embarrassed. If you are struggling with the material then come to office hours or head over to the QCenter. Most of all, do not wait to get help. The sooner you do it the faster you can get back on track.

*In the Fall of 2019, I taught two sections of Math 355 On the last day of class, I asked each of them to reflect on the semester and think about what worked for them and maybe what they could have done differently to ensure a successful time in math 355. I then asked them: “What advice, if any, would you give to the students taking Math 355 in the Spring?” Listed below are the most popular of their responses:*

## Work every single day, reading over notes, to understand each definition/theorem/topic as it is presented. (20 students said this)

They emphasized “DO NOT WAIT”

## Attend professor/Qfellow/math fellow office hours OFTEN (16 students said this)

Many students emphasized the importance of working on HW before coming to office hours.

## Start homework early (before Friday) (10 students said this)

HW assignments are assigned on Wednesday. Many students would start working on the homework Wednesday night/Thursday then attend Friday office hours. This way, several of the students would have over half of the assignment done before heading into the weekend!

## Make study groups with other students in the class (5 students said this)

Many students made several new friends in the process