Solutions

Lab 13 for Math 17 - Chi-square Investigations + Name that Scenario

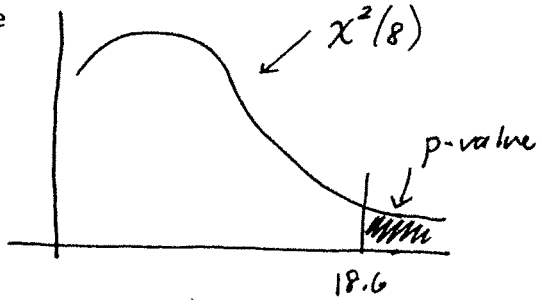
Handout is designed to give you lots of practice with Chi-square – 2 day duration – April 27 and 28.

1. A chi-square test of independence between a happiness rating and education level results in a test statistic of 18.6 where one variable (education) has 5 categories and the other (happiness) has 3 categories. Assume the conditions are met for the test to be appropriate.

a. Determine the appropriate df for the test. $r = 5 \quad c = 3$

$df = (4)(2) = 8$

b. Sketch and label the distribution used to find the p-value and shade in the area that represents the p-value.



17.535 20.090

.025 .01

c. What is the p-value? $.01 < \text{p-value} < .025$

d. Interpret the p-value in context.

If happiness rating and education level were indep., we would get a χ^2 of 18.6 or larger between 1 & 2.5% of the time.

e. What would your conclusion be at a significance level of .05?

(Decision: Reject H_0)

We have evidence that happiness rating and education level are NOT independent.

2. A chi-square test of homogeneity with 4 populations and a categorical variable with 3 levels results in a test statistic of 5.4. How can you tell you will have a large p-value without using the table?

$\chi^2 = 5.4 \qquad r = 4 \quad c = 3 \qquad df = 3 \cdot 2 = 6$

$\chi^2 < df \Rightarrow$ large p-value

3. American Religious Identification Survey (ARIS)

The ARIS 2008 survey was based on a sample of 54,461 adult Americans. Data consistent with the percents reported from 2008 is below for your observed counts. The survey asked basic questions about the religion of its participants.

	Christian	Other Religion	None	Don't Know
Count (2008)	41390	2097	8169	2805

Is there significant evidence that the religious makeup of the US has changed since 1990 when 86.2% were Christian, 3.3% were Other, 8.2% were None, and 2.3% Don't Know? **(Treat the 1990 values as "known" values)**. Perform the relevant hypothesis test, following all the steps, using the table below to help in your calculations.

	Christian	Other Religion	None	Don't Know
Observed Counts	41390	2097	8169	2805
Expected Counts	46945.382	1797.213	4465.802	1252.603
(O-E)^2	$(-5555.382)^2$	$(299.787)^2$	$(3703.198)^2$	$(1552.397)^2$
((O-E)^2)/E	657.41	50.01	3070.82	1923.94

H_0: $p_{Christian} = .862$ $p_{Other} = .033$ $p_{None} = .082$ $p_{DK} = .023$

H_A: Not H_0

Expected Counts = $np_i \Rightarrow 54461\, p_i \Rightarrow$ see chart all ≥ 5 ✓

Assume rep. sample of adult Americans.

$\chi^2 = 657.41 + 50.01 + 3070.82 + 1923.94 = 5702.18$

Reject H_0 @ any α basically

Conclusion: We have evidence that the religious makeup of the U.S. has changed from 1990.

4. Roadside Survey – In the early 1970s, it was legal in Oklahoma to sell 3.2% beer to females under 21 but not to males under 21. A study in Oklahoma during the early 1970s asked young drivers (under 20 years of age) whether or not they had consumed alcohol in the previous 2 hours and also recorded their gender. The study was stated to be "random" but probably was not since police stopped cars at pre-determined locations in order to ask the questions. The data collected was ultimately used in a Supreme Court case – Craig vs. Boren.

The following table summarizes the results:

Gender/Alcohol	Yes	No	Total
Males	77 (72.27)	404 (408.73)	481
Females	16 (20.73)	122 (117.27)	138
Total	93	526	619

If you chose one of the sampled drivers randomly, what is the probability he or she had consumed alcohol within the previous 2 hours? 93/619

What proportion of males had consumed alcohol? What proportion of females had consumed alcohol?

$\hat{p}_m = \frac{77}{481} = .1601$ $\hat{p}_f = \frac{16}{138} = .1159$

If you wanted to know whether or not the proportions of males and females who had consumed alcohol were the same, what 2 analyses are appropriate?

χ^2 Homogeneity

Test for $p_1 - p_2$

Perform the appropriate chi-square analysis. You should write your expected counts in the table (convention: put them in parentheses after the observed counts).

H_0: The levels of alcohol states are the same for males & females. H_A: Not H_0

Conditions: 1. Randomization and $\perp$ for sample? Have to assume representative sample.

2. Expected counts all ≥ 5? Yes, min is 20.73.

$\chi^2 = \sum \left(\frac{(O-E)^2}{E}\right) = .3096 + .0547 + 1.0793 + .1908 = 1.6344$

$df = (r-1)(c-1) = (2-1)(2-1) = 1$ $\chi^2(1)$ p-value > .10 b/c lowest χ^2 on chart is 2.706

What is your conclusion?

We do not have evidence to suggest the dist. of alcohol states is different for males and females.

If you had wanted to know if a higher proportion of males had consumed alcohol compared to females, is this chi-square analysis still appropriate? Explain. (Think about this, it's in the reading, but I didn't explicitly state it in class.)

No. χ^2 tests cannot have directed hypotheses.

5. **People Said to Believe in Aliens and Ghosts More Than God**

livescience.com **Mon Nov 24, 9:48 am ET**

More people believe in aliens and ghosts than in God, a new survey finds, according to a British newspaper.

The survey, however, was done by a marketing firm in conjunction with the release of an X-Files DVD, and details of how the poll was conducted were not reported in the Daily Mail. Survey questions, depending on how they are written, can greatly skew results, along with how subjects are sampled.

That said, the poll of 3,000 people found that 58 percent believe in the supernatural, including paranormal encounters, while 54 percent believe God exists. Women were more likely than men to believe in the supernatural and were also more likely to visit a medium. Indeed, humans are prone to believing in things they can neither see nor find logical evidence for.

A survey of U.S. college students done in 2006 found 23 percent of freshmen had a general belief in paranormal concepts - from astrology to communicating with the dead. Interestingly, the number jumped to 31 percent among seniors and 34 percent among graduate students.

Researchers who have compared various human belief systems say our tendency to believe is deeply rooted.

"While it is difficult to know for certain, the tendency to believe in the paranormal appears to be there from the beginning," said Christopher Bader, a Baylor University sociologist. "What changes is the content of the paranormal. For example, very few people believe in faeries and elves these days. But as belief in faeries faded, other beliefs, such as belief in UFOs, emerged to take their place."

Religion and belief in the paranormal are not linked as one might imagine. A handful of surveys show just the opposite, in fact.

"Paranormal beliefs are very strongly negatively related to religious belief," said Rod Stark, another Baylor researcher. Some scientists think this is so because religions tend to discourage paranormal beliefs, and indeed most devout practitioners of a religion have been shown to be the least likely to believe in Bigfoot, ghosts or aliens.

What are some possible issues relating to the results described in this article?

seniors bias due to DVD release

Suppose a new study in 2008 found that 26% of freshmen, 35% of seniors, and 31% of graduate students had a general belief in paranormal concepts. Would it be appropriate to use a chi-square goodness of fit test (based on the corresponding counts) to see if the percents have changed from the 2006 results reported? Why or why not?

No. Diff. proportions and $\Sigma p_i \neq 1$ for diff. populations

For a goodness of fit test, do all proportions stated in the null hypothesis need to be equal to each other? What is the restriction on the proportions?

No, they can be diff $\sum_{i=1}^{k} p_i = 1$ is only restriction.

6. A company inspector is looking into production of parts for new fuel-efficient car engines. The company has a new part being produced at 3 different manufacturing plants which has very specific design parameters. Not all produced new parts are usable however (i.e. some parts are defective). The inspector wants to know if all three plants are contributing equally to the problem of defective parts. The inspector randomly selects some parts from each plant's production and determines if each part is defective or not in order to help answer this question.

	Plant A	Plant B	Plant C	Total
Good Parts	450 (451.67)	687 (677.5)	218 (225.83)	1355
Defective Parts	50 (48.33)	63 (72.5)	32 (24.17)	145
Total	500	750	250	1500

a. If a randomly selected part examined by the inspector was chosen, what is the probability it was from Plant C? $250/1500 = .1667$

b. If a randomly selected part from Plant B was chosen from this sample, what is the probability the part was defective? $63/750 = .084$

c. What test should you perform to determine if all three plants are contributing equally to the problem of defective parts? (Be specific.) χ^2 Homogeneity

d. Determine the expected counts for your chosen test and write them in the table in parentheses after the observed counts.

use cross product rule

e. State and check the conditions necessary for your chosen test.

1. Need 3 $\perp$ samples → reasonable 3 diff plants
2. Need randomization and indep. for each plants sample ⇒ Stated
3. Need all expected count ≥ 5. Yes smallest is 24.17.

f. The chi-square test statistic is 4.25. What distribution does the test statistic have assuming the null hypothesis is true? $r=2$ $c=3$ $df = 1(2) \Rightarrow$ $\chi^2(2)$ dist.

g. Compute bounds for the p-value for your test. p-value $> .10$ from chart

h. State the conclusion for your test (in context) using a .05 significance level.

We do not have evidence to suggest the 3 plants are NOT all contributing equally to the problem of defective parts.

7. Name that Scenario (Cumulative – Practice for both Third Midterm and Final Exam)

For this problem we will be considering a data set on bulls sold at an auction. The variables in the data set include:

Breed: 1= Angus; 5= Hereford; 8 = Simental C
Salepr: sale price at auction Q
Yrhgt: yearling height at shoulder (inches) Q
Ftfrbody: fat free body (pounds) Q
prct_b: percent fat-free body (as a percent, i.e. 70% = 70) Q
frame: scaled from 1-8 (1 = small, 8 = large), integer values only C
bkfat: back fat (inches) Q
saleht: sale height at shoulder (inches) Q
salewt: sale weight (pounds) Q

Determine whether each variable is categorical or quantitative. Just make a notation above next to each as C or Q. This should help with the questions below.

Name that Scenario: For each scenario, determine the appropriate statistical procedure. If you end up suggesting a hypothesis test or CI, give the parameter(s) and hypotheses/or say CI. If regression, specify which variable is response and which is explanatory. For ANOVA and chi-square, state your hypotheses.

1. Are more than 50% of bulls sold at this type of auction Angus breed?

Hyp test for p = prop. of Angus bulls
$H_0: p = .5$
$H_A: p > .5$

2. Do heavier bulls tend to be taller than lighter bulls? (Note: no specification is made as to what is "heavy" vs. what is "light").

Regression
explanatory : sale weight
response : sale height

3. Are there differences between the breeds in terms of yearling height at shoulder?

ANOVA
$H_0: \mu_1 = \mu_2 = \mu_3$ H_A: At least one μ_i is diff. (Not all means are =)
μ_i = mean yrling height for breed (1=Angus, 2 = Here. 3 = Sim.)

4. Is there a relationship between frame and breed?

χ^2 Independence
H_0: Frame & Breed are ⊥.
H_A: Not H_0

5. Are Hereford bulls heavier than Simental bulls at time of auction on average?

Hyp Test
$\mu_1 - \mu_2$
$H_0: \mu_1 = \mu_2$
$H_A: \mu_1 > \mu_2$
μ_1 = avg. Hereford sale weight
μ_2 = " Simental "

6. Can back fat (in inches) be used to accurately predict fat free body (pounds)?

Regression → Explanatory
↓ Response

7. Can it be said that 40% of the bulls sold at auction are Angus, 20% are Hereford, and the rest are Simental?

χ^2 GOF
$H_0: p_A = .4, p_H = .2, p_S = .4$
H_A: Not H_0