

Math 28 Spring 2008: Exam 2

Instructions: Each problem is scored out of 10 points for a total of 40 points. You may not use any outside materials(eg. notes or books). You have 50 minutes to complete this exam.

Problem 1.

(a) Let $f : A \rightarrow \mathbb{R}$ where $A \subset \mathbb{R}$. State the definition for f to be uniformly continuous on A .

(b) Which of the following functions are uniformly continuous on $[0, \infty)$?

(i) $f(x) = \sin(x^2)$

(ii) $f(x) = \frac{1}{x+1}$

Problem 2. Let C be the Cantor set on $[0, 1]$ obtained in the standard way by successively removing the middle third of each interval. Define $g : [0, 1] \rightarrow \mathbb{R}$ by

$$g(x) = \begin{cases} 1 & x \in C \\ 0 & x \notin C. \end{cases}$$

(a) Show that g is discontinuous at every point in C .

(b) Show that g is continuous at every point not in C .

Problem 3.

(a) State the Generalized Mean Value Theorem.

(b) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function and suppose that f' is bounded. Show that f is uniformly continuous.

Problem 4.

(a) State the definition for a function $f : A \rightarrow \mathbb{R}$ to be differentiable on an interval A .

(b) Let $f : [-1, 1] \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x^2} & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Show that f is differentiable, but that its derivative is unbounded.