Math 13 Fall 2009: Exam 2

November 4, 2009

Name:

Instructions: There are 4 questions on this exam each scored out of 8 points for a total of 32 points. You may not use any outside materials(eg. notes or calculators). You have 50 minutes to complete this exam. Remember to fully justify your answers.

Score:

Problem 1. Define
$$f(x, y) = \begin{cases} \frac{2x^4 - x^3 + xy^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0). \end{cases}$$

(a) Determine where $f(x, y)$ is continuous.

(b) Use the limit definition of partial derivatives to compute $f_x(0, 0)$.

Problem 2.

(a) Given $z = 2x^2 - 3xy + 7y^2$ and $x = u \sin v, y = v \cos u$, find $\frac{\partial z}{\partial u}$ in terms of u and v.

(b) Given $f(x, y, z) = \sqrt{xyz}$ and two points $P = (2, 1, 2)$ and $Q = (-1, 1, 6)$. Find the directional derivative of f at P in the direction of Q.

Problem 3. Find the maximal volume of a rectangular box which has three faces in the coordinates planes and one vertex in the first octant on the paraboloid $z = 4 - x^2 - y^2$.

Problem 4. Classify the critical points of $f(x, y) = 6xy^2 - 2x^3 - 3y^4$.