Math 17 – Spring 2011 Name:______SOLUTIONS______________

March 3, 2011

Test # 1

Please complete the following problems. Be sure to ask me if you have any questions or anything is unclear. Partial credit will be given, so **please be sure to show all of your work. Please use complete sentences to answer ALL questions.**

The Z table and some binomial probability tables are included at the end of the test.

1. (7 pts) In 1964 there was a study that contrasted cholesterol levels between urban and rural Guatemalans. The data along with some summary statistics and graphs of the data are shown below.

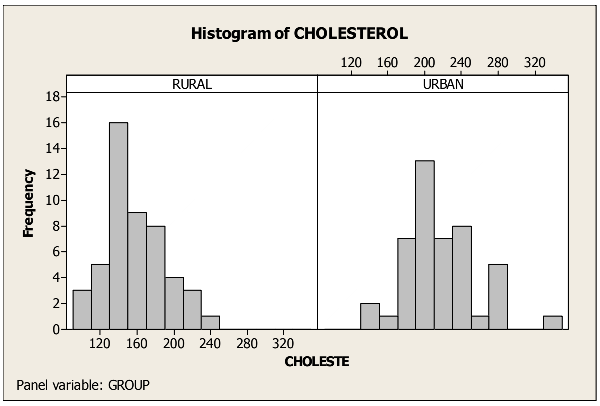
Cholesterol levels in urban and rural guatemalans

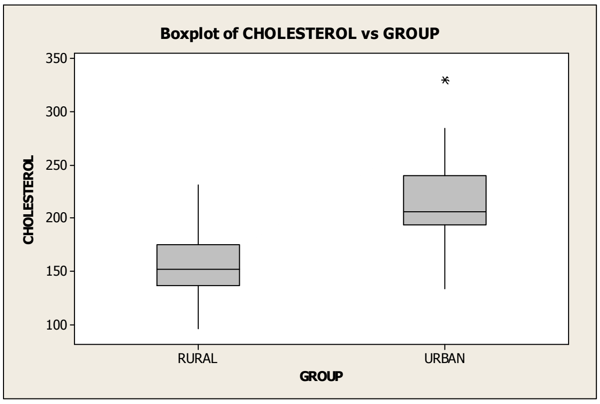
Serum total cholesterol (mg/l) levels among *urban* residents (n=45)
133 134 155 170 175 179 181 184 188 189 190 196 197 199 200 200 201 201 204 205
205 205 206 214 217 222 222 227 227 228 234 234 236 239 241 242 244 249 252 273
279 284 284 284 330

Serum total cholesterol (mg/l) levels among *rural* residents (n=49)
95 108 108 114 115 124 129 129 131 131 135 136 136 139 140 142 142 143 143 144
144 145 145 148 152 152 155 157 158 158 162 165 166 171 172 173 174 175 180 181
189 192 194 197 204 220 223 226 231

Descriptive Statistics: CHOLESTEROL

Variable	GROUP	n	Mean	StDev
CHOLESTEROL	RURAL	49	157.00	31.76
	URBAN	45	216.87	39.92

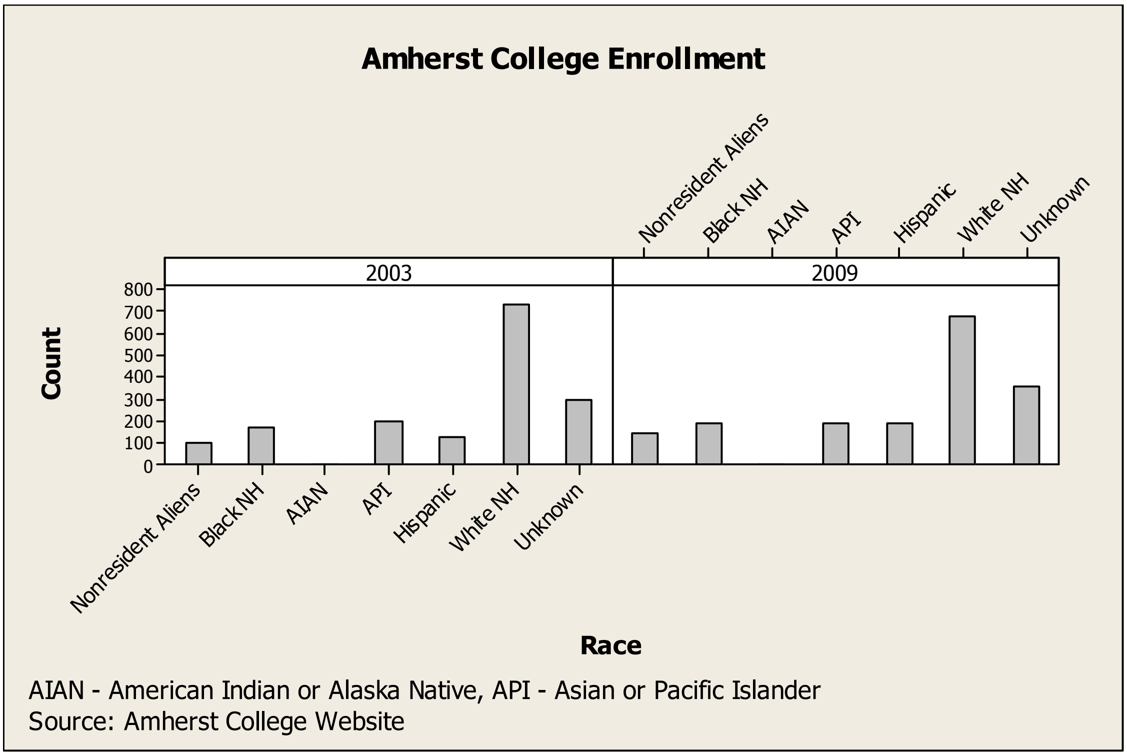




Compare the distributions of Rural and Urban cholesterol levels.

The distribution of cholesterol in rural Guatemalans is fairly symmetric and is centered at about 157 mg/l. There don't appear to be any outliers. For Urban Guatemalans, the distribution is centered at about 216 mg/l. There appears to be an outlier of 330 mg/l. Were it not for the outlier, the distribution would be fairly symmetric as well. Both group have similar standard deviations.

2. (4 pts) Amherst College compiles statistics about its enrollment. A graph of Race for the years 2003 and 2009 is given below.



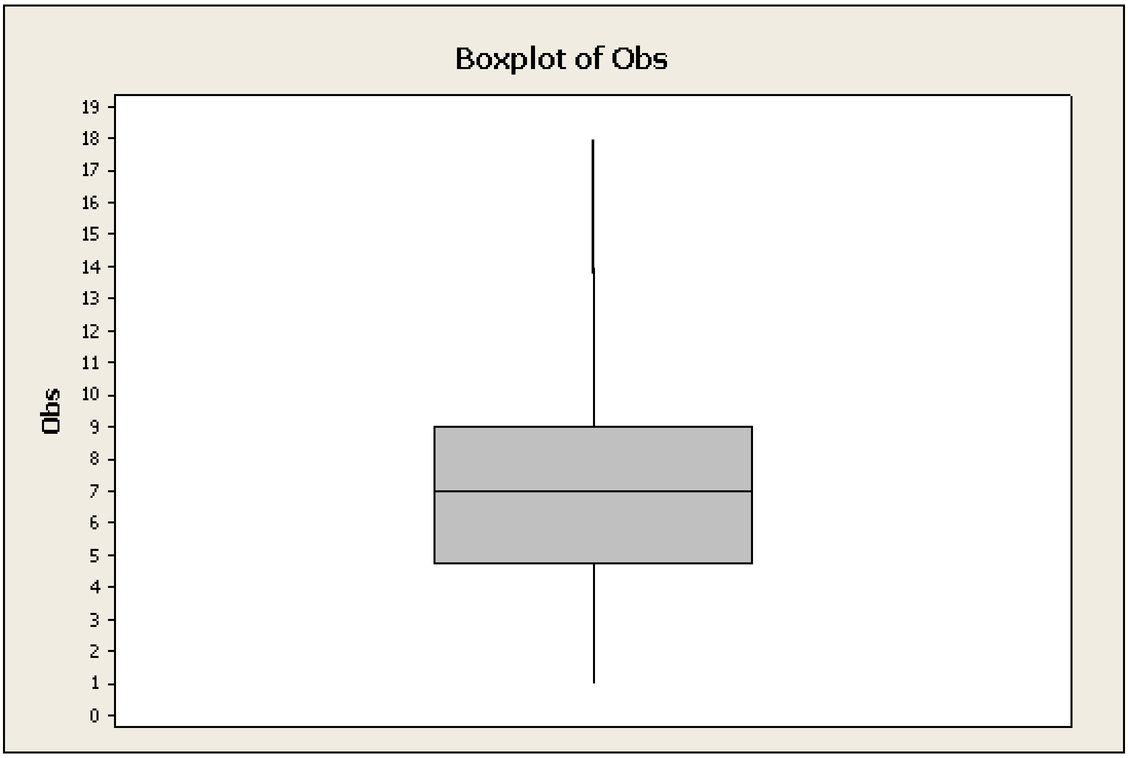
Describe the composition of Amherst's race for each year and compare the two.

The composition of race at Amherst is very similar for the two years. The largest group is the white non-Hispanics, while the number of black non-Hispanics, Asian and Pacific Islanders, and Hispanics are similar. There appear to be a few more Hispanics in 2009 than 2003. The smallest group are the American Indian/Alaska natives, with negligible counts in both years.

3. (5 pts) Give an example of some data that would be skewed to the right. (Don't just draw a picture of a skewed histogram.) Give the scenario and the variable.

Lets consider weekly mileage of runners. If we have a sample of training logs of runners, and one of the runners is a training Olympian, they will likely have a much higher weekly mileage total than our other runners, skewing the distribution to the right.

4. (4 pts) A simple boxplot of a data set is given below.



The two largest observations in this data set are 14 and 18. Use a mathematical rule to determine if they are outliers.

We'll use the 1.5 IQR rule. From the boxplot, it appears that the first quartile, Q1, is 5 and the third quartile, Q3, is 9. The IQR is

$$IQR = Q_3 - Q_1 = 9 - 5 = 4$$

Upper Fence: $Q_3 + 1.5IQR = 9 + 1.5(4) = 15$
Lower Fence: $Q_1 - 1.5IQR = 5 - 1.5(4) = -1$

The observation of 18 is an outlier, but 14 is not.

5. (5 pts) What general features are evident in a box plot of data from a normal distribution? How do these features differ when the data come from a skewed distribution?

For a normal distribution, the boxplot's whiskers will be the same length. The line representing the median will lie in the center of the box. Also, there won't be any flagged outliers. A skewed distribution may have uneven whiskers, the median line may be closer to one of the 1st/3rd quartiles than the other, and there may be flagged outliers.

6. (6 pts) Suppose that the number of traffic stops per day on Route 9 is normally distributed with a mean of 12 and a standard deviation of 2. One day the Amherst police made 19 traffic stops.

   a. Calculate the Z-score corresponding to x = 19 traffic stops.

$$Z = \frac{X - \mu}{\sigma} = \frac{19 - 12}{2} = 3.5$$

   b. Would you consider the observation x = 19 to be an outlier? Please justify your response.

   Yes I would consider the observation of 19 to be an outlier. It is more than 3 standard deviations above the mean.

7. (6 pts) Describe three different strategies for drawing a random sample.

   Four strategies are:

   Simple random sample – The equivalent of putting everyone's name into a hat, and drawing out your sample

   Stratified sample – To ensure you get sample from several groups of interest, split your population into groups, and take a simple random sample from within each group.

   Cluster sample – To save on costs, split the population into clusters, usually based on location. Randomly sample a location, and then sample all members at that location.

   Systematic sample – Enumerate the population. Select a random start point, and then sample every kth subject after that.

8. (6 pts) A study found that individuals who lived in houses with more than two bathrooms tended to have higher blood pressure than individuals who lived in houses with two or fewer bathrooms. Can cause-and-effect be determined from this? (Please justify) If not, list a possible confounding variable that might explain this result.

   Cause and effect cannot be established from this. It is an observational study. Size of the house may be a confounding variable. Large houses have more bathrooms. They also have higher mortgages and upkeep, which could elevate the blood pressures of those who live there.

9. (10 pts) In recent election years, political scientists have analyzed whether a "gender gap" exists in political beliefs and party identification. The table below shows data collected from the 2002 General Social Survey on gender and party identification:

	Party Identification			
Gender	Democrat	Independent	Republican	Total
Male	356	460	369	1,185
Female	567	534	395	1,496
Total	923	994	764	2,681

a. If a voter is chosen at random, what is the probability that the voter is a Democrat?

$$P(\text{Democrat}) = \frac{923}{2681} = 0.3443$$

b. What is the probability that the voter is a female and a Republican?

$$P(\text{Female and Republican}) = \frac{395}{2681} = 0.1473$$

c. What is the probability that the voter is a male or an Independent?

$$P(\text{Male or Independent})$$
$$= P(\text{Male}) + P(\text{Independent}) - P(\text{Male and Indpendent})$$
$$= \frac{1185}{2681} + \frac{994}{2681} - \frac{460}{2681} = \frac{1719}{2681} = 0.6412$$

d. Given that the voter is female, what is the probability that the voter is an Independent?

$$P(\text{Independent}|\text{Female}) = \frac{P(\text{Independent and Female})}{P(\text{Female})} = \frac{\frac{534}{2681}}{\frac{1496}{2681}} = \frac{534}{1496}$$
$$= 0.3570$$

e. Are the events "male" and "Republican" mutually exclusive? Are they Independent?

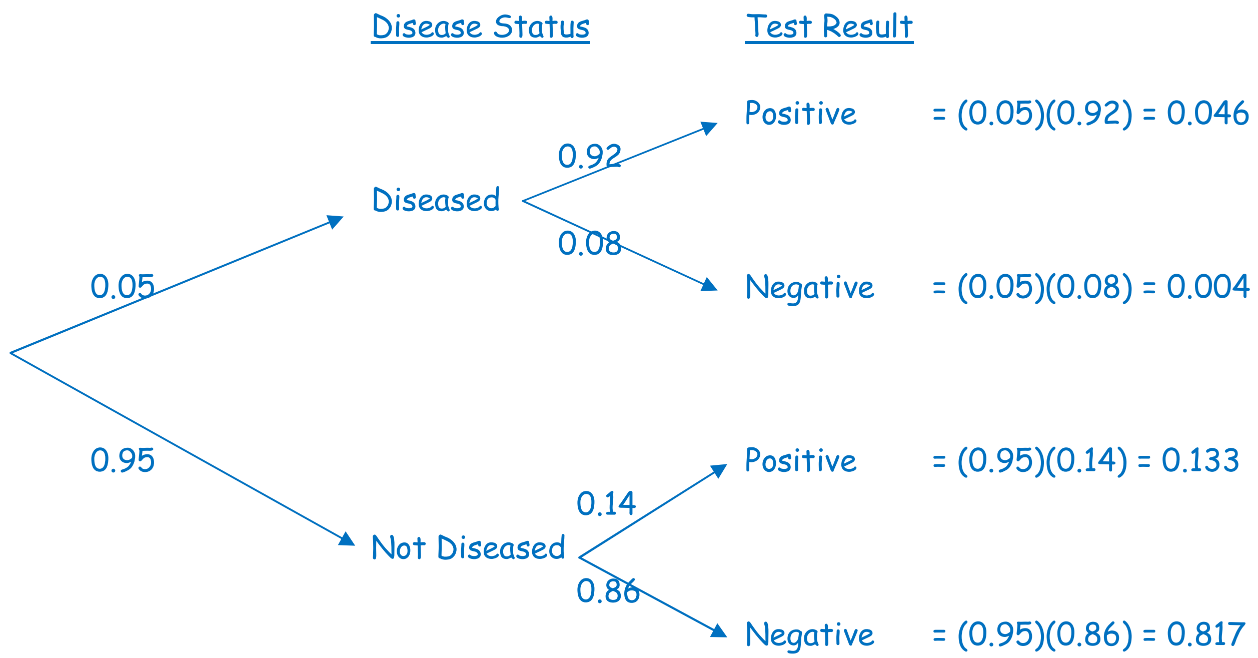
There are 369 people who are male and republican. They overlap, so are not mutually exclusive. To check independence, we find $P(\text{Male and Republican}) = \frac{369}{2681} = 0.1376 \neq 0.1260 = \left(\frac{1185}{2681}\right)\left(\frac{764}{2681}\right) = P(\text{Male})P(\text{Republican})$. So the events are not independent either.

10. (6 pts) In medicine a diagnostic test's accuracy is measured using the sensitivity and the specificity.

Sensitivity = P(Test result is positive | Subject has the disease)

Specificity = P(Test result is negative | Subject doesn't have the disease)

For a given test, suppose the probability that a subject has the disease is 0.05. Suppose the sensitivity is 0.92 and the specificity is 0.86. Find the probability that someone has the disease, given that they have a positive test result.



$$P(\text{Disease}|\text{Positive}) = \frac{P(\text{Diseased and Positive})}{P(\text{Positive})} = \frac{0.046}{0.046+0.133} = 0.2570$$

Given a positive test result, there is a 25.70% chance that they actually have the disease.

11. (8 pts) You wish to study the size of households in the United States. Let X be the number of people in a randomly selected U.S. household. Based on the 2000 Census, X has the following probability distribution:

X	1	2	3	4	5	6	7+
$P(X)$	0.258	0.326	0.165	0.142	0.066	0.025	0.018

a. Find the probability that a randomly selected household has more than 4 people.

$$P(X > 4) = 0.066 + 0.025 + 0.018 = 0.109$$

b. Find the probability that a randomly selected household has at most 3 people.

$$P(X \leq 3) = 0.258 + 0.326 + 0.165 = 0.749$$

c. What is the expected value and standard deviation of X? For this part, treat the "7+" category as just "7".

$E[X] = \sum XP(X) = 1(0.258) + 2(0.326) + 3(0.165) + 4(0.142) + 5(0.066) + 6(0.025) + 7(0.018) = 2.579$

$Var[X] = \sum (X - \mu)^2 P(X) = 2.0998$

The standard deviation is $\sqrt{2.0998} = 1.4491$.

12. (6 pts) Mark Twain invested in several projects. The Paige Compositor was one of them. It was used for setting newspaper type for printing. Unfortunately, Mark Twain's investments went bad and he was forced to go on a worldwide speaking tour to raise money. Suppose that you would like to invest some money of your own. You choose seven promising inventions. It is safe to assume that the successes of the inventions are independent. Based on past experience, the probability of an individual invention getting a patent is 0.23. Let X be the number of inventions you invest in that earn a patent.

a. Is X a binomial random variable? Justify your answer using the conditions required to have a binomial random variable.

Yes, X is a binomial random variable. There are two possible outcomes for each trial (patent, no patent), the trials are independent, there is a fixed probability of success on each trial ($p = 0.23$), we have $n = 7$ trials, and X is the number of successes.

b. What are the possible values of X?

The possible values of X are: 0, 1, 2, 3, 4, 5, 6, 7

13. (9 pts) You are inspecting underground gas tanks at service stations. From previous experience, it is estimated that 12% of all tanks leak. You examine 15 tanks chosen at random and record whether or not the tank leaks. It is safe to assume that testing one tank is independent of testing another. Let X be the number of defective tanks.

a. Find the probability that exactly 4 tanks leak.

$$P(X = 4) = \binom{15}{4} 0.12^4 (1 - 0.12)^{15-4} = \frac{15!}{4!\,(11!)} (0.00020736)(0.2451)$$
$$= 0.069369$$

b. Find the probability that more than 4 tanks leak.

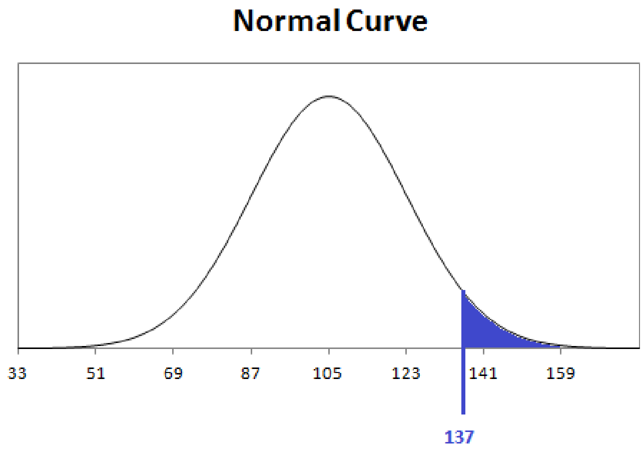
$$P(X > 4) = 1 - P(X \leq 4) = 1 - 0.97350 = 0.0265$$

c. What is the mean number of leaky tanks you'd expect to find?

We'd expect to find a mean of $E[X] = np = 15(0.12) = 1.8$ leaky tanks.

14. (6 pts) Suppose the heights of Eastern White pine trees in the Amherst College Wildlife Sanctuary are normally distributed with a mean height of 105 feet and a standard deviation of 18 feet.

a. Find the probability that the height of a randomly selected Eastern white pine tree is greater than 137 feet.



$$P(X > 137) = P\left(Z > \frac{137-105}{18}\right) = P(Z > 1.78) = 1 - 0.9625 = 0.0375$$

b. How tall would an Eastern white pine tree in the Amherst College Wildlife Sanctuary need to be to be in the tallest 10% of trees?

The Z corresponding to the tallest 10% is 1.28. Then, we use the Z-score equation to solve for X.

$$Z = \frac{X - \mu}{\sigma}$$

$$1.28 = \frac{X - 105}{18}$$

$$X = 1.28(18) + 105 = 128.04$$

A tree needs to be over 128.04 feet tall to be in the tallest 10% of trees.

15. (6 pts) Again, suppose the heights of pine trees in the Amherst College Wildlife Sanctuary forest are normally distributed with a mean height of 105 feet and a standard deviation of 18 feet. You choose a sample of 20 trees and calculate the average height of your sample.

   a. Describe the sampling distribution of $\bar{x}$.

      The sampling distribution of $\bar{x}$ is normal, with a mean of $\mu_{\bar{x}} = \mu = 105$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{20}} = 4.4721$.

   b. Find the probability that the average height of 20 trees is greater than 115 feet.

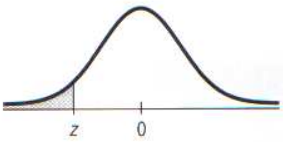
$$P(\bar{X} > 115) = P\left(Z > \frac{115-105}{4.4721}\right) = P(Z > 2.24) = 1 - 0.9875 = 0.0125$$

16. (6 pts) Suppose that the true proportion of defective light bulbs in a factory is p = 0.04. We'd like to sample 500 light bulbs at random, and calculate the sample proportion, $\hat{p}$. Do you think it would be reasonable to get a sample proportion of 0.06 or more? Be sure to justify your answer.

We have a binomial random variable here. We'd expect to get $np = 500(0.04) = 20$ defective bulbs, with a standard deviation of $\sqrt{np(1-p)} = \sqrt{500(0.04)(0.96)} = 4.38$.

A sample proportion of 0.06 or more means a numerator of 30 defective bulbs. This has a Z-score of $Z = \frac{X-\mu}{\sigma} = \frac{30-20}{4.38} = 2.28$. Such a point is not really a huge outlier. It seems reasonable to get a sample proportion of 0.06.

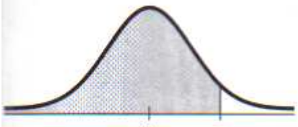
Table Z
Areas under the standard Normal curve



Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	*z*
									0.0000†	−3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	−3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	−3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

†For $z \leq -3.90$, the areas are 0.0000 to four decimal places.

Table Z (cont.)
Areas under the standard Normal curve



	Second decimal place in z									
z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000†									

†For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

Binomial with n = 15 and p = 0.12

x	P(X = x)
0	0.146974
1	0.300628
2	0.286963
3	0.169569
4	0.069369
5	0.020811
6	0.004730
7	0.000829
8	0.000113
9	0.000012
10	0.000001
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000

Binomial with n = 15 and p = 0.12

x	P(X <= x)
0	0.14697
1	0.44760
2	0.73457
3	0.90413
4	0.97350
5	0.99431
6	0.99904
7	0.99987
8	0.99999
9	1.00000
10	1.00000
11	1.00000
12	1.00000
13	1.00000
14	1.00000
15	1.00000

Binomial with n = 15 and p = 0.88

x	P(X = x)
0	0.000000
1	0.000000
2	0.000000
3	0.000000
4	0.000000
5	0.000001
6	0.000012
7	0.000113
8	0.000829
9	0.004730
10	0.020811
11	0.069369
12	0.169569
13	0.286963
14	0.300628
15	0.146974

Binomial with n = 15 and p = 0.88

x	P(X <= x)
0	0.00000
1	0.00000
2	0.00000
3	0.00000
4	0.00000
5	0.00000
6	0.00001
7	0.00013
8	0.00096
9	0.00569
10	0.02650
11	0.09587
12	0.26543
13	0.55240
14	0.85303
15	1.00000

Binomial with n = 4 and p = 0.12

x	P(X = x)
0	0.599695
1	0.327107
2	0.066908
3	0.006083
4	0.000207

Binomial with n = 4 and p = 0.88

x	P(X = x)
0	0.000207
1	0.006083
2	0.066908
3	0.327107
4	0.599695