Math 28 Spring 2009: Exam 1

Instructions: Each problem is scored out of 10 points for a total of 50 points. You may not use any outside materials(eg. notes or calculators). You have 50 minutes to complete this exam.

Problem 1. Using the definition of convergence show that a monotone sequence converges if and only if it is bounded.

Problem 2. Apply the Cauchy Criterion to each of the following series.

(a) $\sum_{n=1}^{\infty} \frac{1}{n!}$.

(b) $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$.

Problem 3. Provide an example (or state why it is not possible) for each of the following.

(a) A sequence of nested intervals whose intersection is empty.

(b) A convergent sequence with a subsequence which is strictly increasing and a subsequence which is strictly decreasing.

(c) A Cauchy sequence which is not monotone.

Problem 4. Determine the cardinality of the set $\{(x, y) \mid x, y \in \mathbb{Q}\}$.

Problem 5. Define a sequence recursively by

$$a_{n+1} = a_n^2 - a_n + 1 \quad \text{for all } n \in \mathbb{N}$$

with $a_1 = \frac{1}{2}$. Determine the convergence or divergence of this sequence. If it converges, find the value of convergence.