Physics 16 Problem Set 8 Solutions

Y&F Problems

8.20. **IDENTIFY:** In part (a) no horizontal force implies P_x is constant. In part (b) use the energy expression, Eq. 7.14, to find the potential energy intially in the spring.

SET UP: Initially both blocks are at rest.

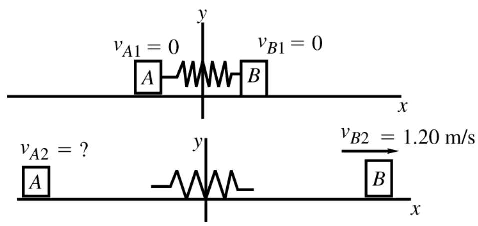

Figure 8.20

EXECUTE: **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right) v_{B2x} = -\left(\frac{3.00\text{ kg}}{1.00\text{ kg}}\right)(+1.20\text{ m/s}) = -3.60\text{ m/s}$$

Block *A* has a final speed of 3.60 m/s, and moves off in the opposite direction to *B*.

(b) Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2 = \tfrac{1}{2}(1.00\text{ kg})(3.60\text{ m/s})^2 + \tfrac{1}{2}(3.00\text{ kg})(1.20\text{ m/s})^2 = 8.64\text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64$ J

EVALUATE: The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

8.27. **IDENTIFY:** The horizontal component of the momentum of the system of the rain and freight car is conserved.

SET UP: Let +*x* be the direction the car is moving initially. Before it lands in the car the rain has no momentum along the *x* axis.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ says $(24{,}000\text{ kg})(4.00\text{ m/s}) = (27{,}000\text{ kg})v_{2x}$ and $v_{2x} = 3.56\text{ m/s}$.

(b) After it lands in the car the water must gain horizontal momentum, so the car loses horizontal momentum.

EVALUATE: The vertical component of the momentum is not conserved, because of the vertical external force exerted by the track.

8.38. **IDENTIFY:** Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

SET UP: Conservation of momentum applied to the collision between the bullet and the block: Let object A be the bullet and object B be the block. Let v_A be the speed of the bullet before the collision and let V be the speed of the block with the bullet inside just after the collision.

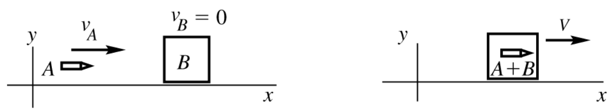

Figure 8.38a

P_x is constant gives $m_A v_A = (m_A + m_B)V$.
Conservation of energy applied to the motion of the block after the collision:

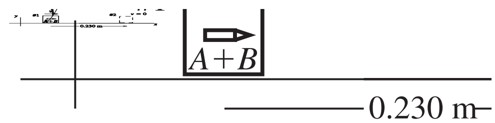

Figure 8.38b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80 \text{ m/s}^2)(0.230 \text{ m})} = 0.9495 \text{ m/s}$

Use this in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00\times10^{-3} \text{ kg} + 1.20 \text{ kg}}{5.00\times10^{-3} \text{ kg}}\right)(0.9495 \text{ m/s}) = 229 \text{ m/s}$$

EVALUATE: When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

8.41. **IDENTIFY:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Since the collision is elastic, Eqs. 8.24 and 8.25 give the final velocity of each block after the collision.

SET UP: Let $+x$ be the direction of the initial motion of A.

EXECUTE: **(a)** Momentum conservation gives $(2.00 \text{ kg})(2.00 \text{ m/s}) = (12.0 \text{ kg})V$ and $V = 0.333 \text{ m/s}$. Both blocks are moving at 0.333 m/s, in the direction of the initial motion of block A. Conservation of energy says the initial kinetic energy of A equals the total kinetic energy at maximum compression plus the potential energy U_b

stored in the bumpers: $\frac{1}{2}(2.00 \text{ kg})(2.00 \text{ m/s})^2 = U_b + \frac{1}{2}(12.0 \text{ kg})(0.333 \text{ m/s})^2$ and $U_b = 3.33 \text{ J}$.

(b) $v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\frac{2.00 \text{ kg} - 10.0 \text{ kg}}{12.0 \text{ kg}}\right)(2.00 \text{ m/s}) = -1.33 \text{ m/s}$. Block A is moving in the $-x$ direction at 1.33 m/s.

$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right)v_{A1x} = \frac{2(2.00 \text{ kg})}{12.0 \text{ kg}}(2.00 \text{ m/s}) = +0.667 \text{ m/s}$. Block B is moving in the $+x$ direction at 0.667 m/s.

EVALUATE: When the spring is compressed the maximum amount the system must still be moving in order to conserve momentum.

8.71. IDENTIFY: The horizontal component of the momentum of the sand plus railroad system is conserved.
SET UP: As the sand leaks out it retains its horizontal velocity of 15.0 m/s.
EXECUTE: The horizontal component of the momentum of the sand doesn't change when it leaks out so the speed of the railroad car doesn't change; it remains 15.0 m/s. In Exercise 8.27 the rain is falling vertically and initially has no horizontal component of momentum. Its momentum changes as it lands in the freight car. Therefore, in order to conserve the horizontal momentum of the system the freight car must slow down.
EVALUATE: The horizontal momentum of the sand does change when it strikes the ground, due to the force that is external to the system of sand plus railroad car.

8.100. IDENTIFY: There is no net horizontal external force so v_{cm} is constant.
SET UP: Let $+x$ be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_{\text{A}} = 45.0 \text{ kg}$, $m_B = 60.0 \text{ kg}$. The center of mass of the canoe is at its center.
EXECUTE: Initially, $v_{\text{cm}} = 0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}} = \dfrac{m_A x_{A1} + m_B x_{B1}}{m_A + m_B}$.
After she walks, $x_{\text{cm2}} = \dfrac{m_A x_{A2} + m_B x_{B2}}{m_A + m_B}$. $x_{\text{cm1}} = x_{\text{cm2}}$ gives $m_A x_{A1} + m_B x_{B1} = m_A x_{A2} + m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2} = x_{B2} + 1.50 \text{ m}$.

$$(45.0 \text{ kg})(1.00 \text{ m}) + (60.0 \text{ kg})(2.50 \text{ m}) = (45.0 \text{ kg})(x_{B2} + 1.50 \text{ m}) + (60.0 \text{ kg})x_{B2}.$$

$(105.0 \text{ kg})x_{B2} = 127.5 \text{ kg}\cdot\text{m}$ and $x_{B2} = 1.21 \text{ m}$. $x_{B2} - x_{B1} = 1.21 \text{ m} - 2.50 \text{ m} = -1.29 \text{ m}$. The canoe moves 1.29 m to the left.
EVALUATE: When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00 \text{ m} - 1.29 \text{ m} = 1.71 \text{ m}$ to the right relative to the water. Note that this distance is $(60.0 \text{ kg}/45.0 \text{ kg})(1.29 \text{ m})$.

8.102. IDENTIFY: Conservation of x and y components of momentum applies to the collision. At the highest point of the trajectory the vertical component of the velocity of the projectile is zero.
SET UP: Let $+y$ be upward and $+x$ be horizontal and to the right. Let the two fragments be A and B, each with mass m. For the projectile before the explosion and the fragments after the explosion. $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$.
EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$ gives that the maximum height of the projectile is $h = -\dfrac{v_{0y}^2}{2a_y} = -\dfrac{([80.0 \text{ m/s}]\sin 60.0^\circ)^2}{2(-9.80 \text{ m/s}^2)} = 244.9 \text{ m}$. Just before the explosion the projectile is moving to the right with horizontal velocity $v_x = v_{0x} = v_0 \cos 60.0^\circ = 40.0 \text{ m/s}$. After the explosion $v_{Ax} = 0$ since fragment A falls vertically. Conservation of momentum applied to the explosion gives $(2m)(40.0 \text{ m/s}) = mv_{Bx}$ and $v_{Bx} = 80.0 \text{ m/s}$. Fragment B has zero initial vertical velocity so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives a time of fall of $t = \sqrt{-\dfrac{2h}{a_y}} = \sqrt{-\dfrac{2(244.9 \text{ m})}{-9.80 \text{ m/s}^2}} = 7.07 \text{ s}$. During this time the fragment travels horizontally a distance $(80.0 \text{ m/s})(7.07 \text{ s}) = 566 \text{ m}$. It also took the projectile 7.07 s to travel from launch to maximum height and during this time it travels a horizontal distance of $([80.0 \text{ m/s}]\cos 60.0^\circ)(7.07 \text{ s}) = 283 \text{ m}$. The second fragment lands $283 \text{ m} + 566 \text{ m} = 849 \text{ m}$ from the firing point.
(b) For the explosion, $K_1 = \frac{1}{2}(20.0 \text{ kg})(40.0 \text{ m/s})^2 = 1.60 \times 10^4 \text{ J}$.
$K_2 = \frac{1}{2}(10.0 \text{ kg})(80.0 \text{ m/s})^2 = 3.20 \times 10^4 \text{ J}$. The energy released in the explosion is $1.60 \times 10^4 \text{ J}$.
EVALUATE: The kinetic energy of the projectile just after it is launched is $6.40 \times 10^4 \text{ J}$. We can calculate the speed of each fragment just before it strikes the ground and verify that the total kinetic

energy of the fragments just before they strike the ground is $6.40\times10^4\ \text{J}+1.60\times10^4\ \text{J}=8.00\times10^4\ \text{J}$. Fragment A has speed 69.3 m/s just before it strikes the ground, and hence has kinetic energy $2.40\times10^4\ \text{J}$. Fragment B has speed $\sqrt{(80.0\ \text{m/s})^2+(69.3\ \text{m/s})^2}=105.8\ \text{m/s}$ just before it strikes the ground, and hence has kinetic energy $5.60\times10^4\ \text{J}$. Also, the center of mass of the system has the same horizontal range $R=\frac{v_0^2}{g}\sin(2\alpha_0)=565\ \text{m}$ that the projectile would have had if no explosion had occurred. One fragment lands at $R/2$ so the other, equal mass fragment lands at a distance $3R/2$ from the launch point.

8.105. **IDENTIFY:** No external force, so $\vec{P}$ is conserved in the collision.
SET UP: Apply momentum conservation in the x and y directions:

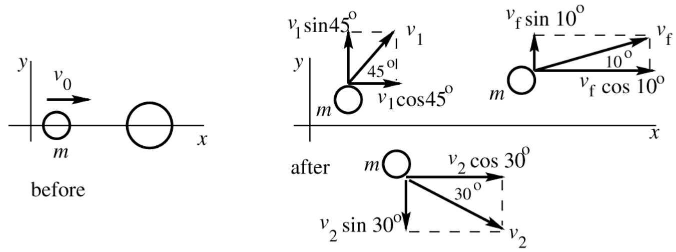

Figure 8.105

Solve for v_1 and v_2.

EXECUTE: P_x is conserved so $mv_0=m\left(v_1\cos45^\circ+v_f\cos10^\circ+v_2\cos30^\circ\right)$.

$v_0-v_f\cos10^\circ=v_1\cos45^\circ+v_2\cos30^\circ$.

$1030.4\ \text{m/s}=v_1\cos45^\circ+v_2\cos30^\circ$.

P_x is conserved so $0=m\left(v_1\sin45^\circ-v_2\sin30^\circ+v_f\sin10^\circ\right)$.

$v_1\sin45^\circ=v_2\sin30^\circ-347.3\ \text{m/s}$.

$\sin45^\circ=\cos45^\circ$ so

$1030.4\ \text{m/s}=v_2\sin30^\circ-347.3\ \text{m/s}+v_2\cos30^\circ$.

$$v_2=\frac{1030.4\ \text{m/s}+347.3\ \text{m/s}}{\sin30^\circ+\cos30.0^\circ}=1010\ \text{m/s}.$$

And then $v_1=\dfrac{v_2\sin30^\circ-347.3\ \text{m/s}}{\sin45^\circ}=223\ \text{m/s}$. Then two emitted neutrons have speeds of 223 m/s and 1010 m/s. The speeds of the Ba and Kr nuclei are related by P_z conservation.

P_z is constant implies that $0=m_{\text{Ba}}v_{\text{Ba}}-m_{\text{Kr}}v_{\text{Kr}}$

$$v_{\text{Kr}}=\left(\frac{m_{\text{Ba}}}{m_{\text{Kr}}}\right)v_{\text{Ba}}=\left(\frac{2.3\times10^{-25}\ \text{kg}}{1.5\times10^{-25}\ \text{kg}}\right)v_{\text{Ba}}=1.5v_{\text{Ba}}.$$

We can't say what these speeds are but they must satisfy this relation. The value of v_{Ba} depends on energy considerations.

EVALUATE: $K_1=\frac{1}{2}m_n\left(3.0\times10^3\ \text{m/s}\right)^2=\left(4.5\times10^6\ \text{J/kg}\right)m_n$.

$$K_2=\tfrac{1}{2}m_n\left(2.0\times10^3\ \text{m/s}\right)^2+\tfrac{1}{2}m_n\left(223\ \text{m/s}\right)^2+\tfrac{1}{2}m_n\left(1010\ \text{m/s}\right)^2+K_{\text{Ba}}+K_{\text{Kr}}$$
$$=\left(2.5\times10^6\ \text{J/kg}\right)m_n+K_{\text{Ba}}+K_{\text{Kr}}.$$

We don't know what K_{Ba} and K_{Kr} are, but they are positive. We will study such nuclear reactions further in Chapter 43 and will find that energy is released in this process; $K_2>K_1$. Some of the potential energy stored in the ^{235}U nucleus is released as kinetic energy and shared by the collision fragments.