Physics 16 Problem Set 2 Solutions

Part 1 of written

2.15. **IDENTIFY** and **SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \dfrac{dx}{dt} = 2.00\text{ cm/s} - (0.125\text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0$ cm, $v_x = 2.00$ cm/s, $a_x = -0.125\text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for t: $t = 16.0$ s.

(c) Set $x = 50.0$ cm and solve for t. This gives $t = 0$ and $t = 32.0$ s. The turtle returns to the starting point after 32.0 s.

(d) Turtle is 10.0 cm from starting point when $x = 60.0$ cm or $x = 40.0$ cm.
Set $x = 60.0$ cm and solve for t: $t = 6.20$ s and $t = 25.8$ s.

At $t = 6.20$ s, $v_x = +1.23$ cm/s.

At $t = 25.8$ s, $v_x = -1.23$ cm/s.

Set $x = 40.0$ cm and solve for t: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).

At $t = 36.4$ s, $v_x = -2.55$ cm/s.

(e) The graphs are sketched in Figure 2.15.

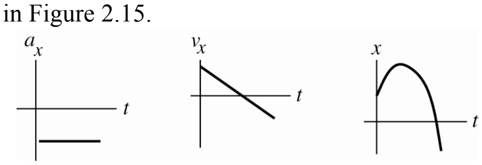


Figure 2.15

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

2.34. **IDENTIFY:** Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60\text{ m/s}^2)(14.0\text{ s})^2 = 157\text{ m}$.

At $t = 14.0$ s, the speed is $v_x = v_{0x} + a_x t = (1.60\text{ m/s}^2)(14.0\text{ s}) = 22.4\text{ m/s}$. In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4\text{ m/s})(70.0\text{ s}) = 1568\text{ m}$.

For the interval during which the train is slowing down, $v_{0x} = 22.4\text{ m/s}$, $a_x = -3.50\text{ m/s}^2$ and $v_x = 0$.

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4\text{ m/s})^2}{2(-3.50\text{ m/s}^2)} = 72\text{ m}$.

The total distance traveled is $157\text{ m} + 1568\text{ m} + 72\text{ m} = 1800\text{ m}$.

EVALUATE: The acceleration is not constant for the entire motion but it does consist of constant acceleration segments and we can use constant acceleration equations for each segment.

2.44. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y = -9.80\ \text{m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00\ \text{m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0\ \text{m}$. At its maximum height the sandbag has $v_y = 0$.

EXECUTE: **(a)** $t = 0.250\ \text{s}$:

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = (5.00\ \text{m/s})(0.250\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(0.250\ \text{s})^2 = 0.94\ \text{m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_yt = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.250\ \text{s}) = 2.55\ \text{m/s}$.

$t = 1.00\ \text{s}$: $y - y_0 = (5.00\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 0.10\ \text{m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_yt = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(1.00\ \text{s}) = -4.80\ \text{m/s}$.

(b) $y - y_0 = -40.0\ \text{m}$, $v_{0y} = 5.00\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives

$-40.0\ \text{m} = (5.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$. $(4.90\ \text{m/s}^2)t^2 - (5.00\ \text{m/s})t - 40.0\ \text{m} = 0$ and

$t = \frac{1}{9.80}\left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)}\right)\text{s} = (0.51 \pm 2.90)\ \text{s}$. t must be positive, so $t = 3.41\ \text{s}$.

(c) $v_y = v_{0y} + a_yt = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(3.41\ \text{s}) = -28.4\ \text{m/s}$

(d) $v_{0y} = 5.00\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (5.00\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 1.28\ \text{m}$. The maximum height is 41.3 m above the ground.

(e) The graphs of a_y, v_y, and y versus t are given in Fig. 2.44. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

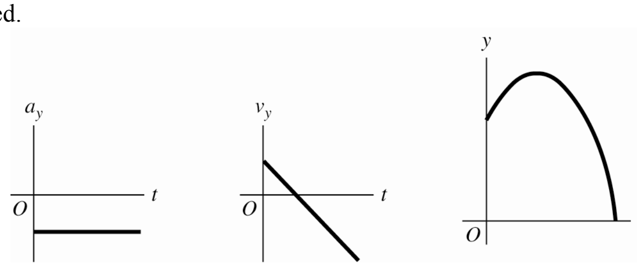


Figure 2.44

2.95. **IDENTIFY:** Apply constant acceleration equations to the motion of the two objects, the student and the bus.

SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: **(a)** Setting $x_1 = x_2$ and solving for the times t gives $t = \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{(0.170\ \text{m/s}^2)}\left((5.0\ \text{m/s}) \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) = 9.55\ \text{s and } 49.3\ \text{s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

(b) The speed of the bus is $(0.170\ \text{m/s}^2)(9.55\ \text{s}) = 1.62\ \text{m/s}$.

(c) The results can be verified by noting that the x lines for the student and the bus intersect at two points, as shown in Figure 2.95a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\ \text{m/s}^2)(49.3\ \text{s}) = 8.38\ \text{m/s}$.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at $3.5\ \text{m/s}$, Figure 2.95b shows that the two lines do *not* intersect:

(f) For the student to catch the bus, $v_0^2 > 2ax_0$. and so the minimum speed is $\sqrt{2(0.170\ \text{m/s}^2)(40\ \text{m/s})} = 3.688\ \text{m/s}$. She would be running for a time $\dfrac{3.69\ \text{m/s}}{0.170\ \text{m/s}^2} = 21.7\ \text{s}$, and covers a distance $(3.688\ \text{m/s})\ (21.7\ \text{s}) = 80.0\ \text{m}$.

However, when the student runs at $3.688\ \text{m/s}$, the lines intersect at *one* point, at $x = 80\ \text{m}$, as shown in Figure 2.95c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{\text{tot}} - t_1$

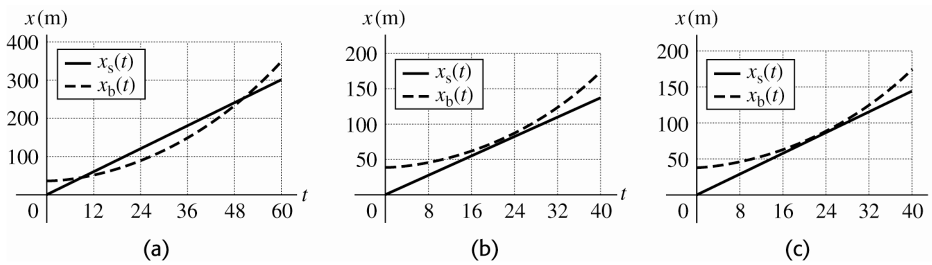


Figure 2.95

3.4. **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$-angle with both axes when its *x*- and *y*-components are equal.

SET UP: $\dfrac{d(t^n)}{dt} = nt^{n-1}$.

EXECUTE: $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

EVALUATE: Both components of $\vec{v}$ change with *t*.

3.13. **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\ \text{m} - 1.80\ \text{m} = 19.5\ \text{m}$ downward during the time it travels 61.0 m horizontally.

SET UP: Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Use the vertical motion to find the time in the air:

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(19.5\ \text{m})}{9.80\ \text{m/s}^2}} = 1.995\ \text{s}$$

Then $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{61.0\ \text{m}}{1.995\ \text{s}} = 30.6\ \text{m/s}$.

(b) $v_x = 30.6\ \text{m/s}$ since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.6\ \text{m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 36.3\ \text{m/s}$.

EVALUATE: We calculate the final velocity by calculating its *x* and *y* components.

3.56. **IDENTIFY:** The water moves in projectile motion.

SET UP: Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

EXECUTE: The equations of motions are $y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$ and $x = (v_0 \cos\alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2$.

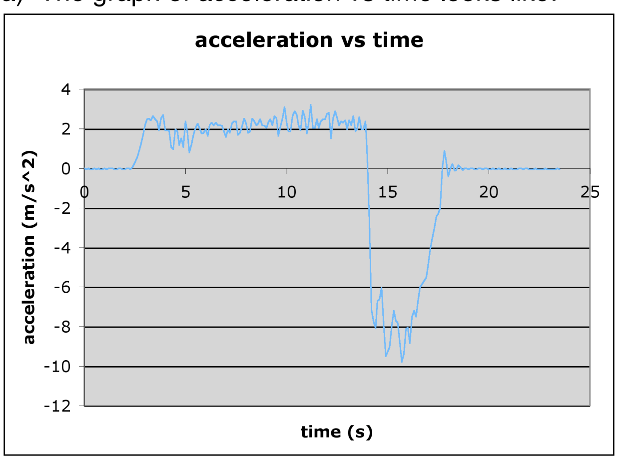
To reach the *minimum* distance: $6D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for t gives $t = \frac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\frac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for t gives $t = \frac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\frac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

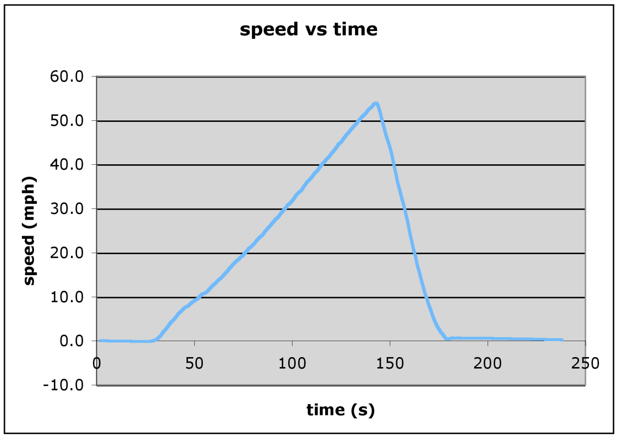
EVALUATE: A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of $6D$. The minimum speed required is greater than this, because the water must be at a height of at least $2D$ when it reaches the front of the tank.

Part 2 of written

(a) The graph of acceleration vs time looks like:

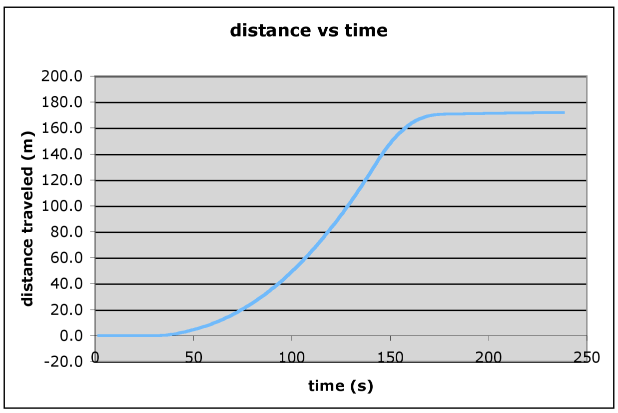


Integrating once gives:



(notice I've changed the units from m/s to mph)

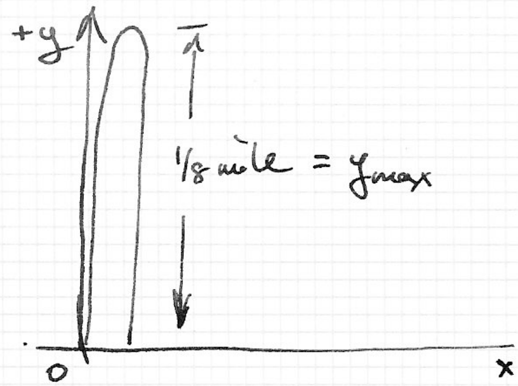
Integrating again gives:



(b) According to this data, I got up to about 53 mph; by my speedometer it was a little over 50 mph. Note that the final speed is, according to the integral, about 0.2 mph, while I told you that I braked to a stop. This makes me think that the accuracy of the speed data is not much better than +/- 0.2 mph (0.1 m/s) by the time we've integrated for a while.

(c) According to this data, I went about 170 m.

3. Superman has to leap 1/8 mile vertically:



Ignoring the width of the building, he needs to produce V_0 large enough so that his height above the ground

$$y(t) = V_0 t - \frac{1}{2} g t^2$$

clears 1/8 mile. Since $V_y = V_0 - gt$, & $V_y = 0$ at the top of the trajectory $t_{max} = \frac{V_0}{g}$, and

$$y_{max} = y(t_{max}) = V_0\left(\frac{V_0}{g}\right) - \frac{1}{2} g \left(\frac{V_0}{g}\right)^2 = \frac{1}{2}\frac{V_0^2}{g}, \text{ so}$$

$$V_0 = \sqrt{2g\, y_{max}}$$ is the minimum takeoff speed required to clear y_{max}.

$$V_0 = \sqrt{2\left(\frac{9.81\ m}{s^2}\right)\left(\frac{1}{8}\ mile\right)\left(\frac{1609\ m}{mile}\right)} = \sqrt{3947\ \frac{m^2}{s^2}} = 63\ m/s.$$

(Using the conversion factor $1\ mile = 5280\ ft \times \frac{12\ in}{ft} \times \frac{2.54\ cm}{in} \times \frac{10^{-2}\ m}{cm} = 1609\ m$)

$$63\frac{m}{s} \times \frac{1\ mile}{1609\ m} \times \frac{3600\ s}{h} = 141\ mph.$$ Pretty fast.