Physics 16 Problem Set 10 Solutions

Y&F Problems

10.24. **IDENTIFY:** Apply conservation of energy to the motion of the marble.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}MR^2$. $v_{cm} = R\omega$ for no slipping. Let $y = 0$ at the bottom of the bowl. The marble at its initial and final locations is sketched in Figure 10.24.

EXECUTE: **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$ and $v = \sqrt{\frac{10}{7}gh}$.

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{rot} = mgh' + K_{rot}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

(b) $mgh = mgh'$ so $h' = h$.

EVALUATE: **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.

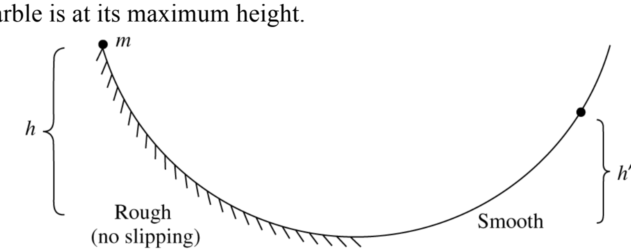

Figure 10.24

10.42. **IDENTIFY:** Apply conservation of angular momentum to the diver.

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2)$.

EXECUTE: If she had tucked, she would have made $(2\ \text{rev})(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2) = 0.40\ \text{rev}$ in the last 1.0 s, so she would have made $(0.40\ \text{rev})(1.5/1.0) = 0.60\ \text{rev}$ in the total 1.5 s.

EVALUATE: Untucked she rotates slower and completes fewer revolutions.

10.83. **IDENTIFY:** Use conservation of energy to relate the speed of the block to the distance it has descended. Then use a constant acceleration equation to relate these quantities to the acceleration.

SET UP: For the cylinder, $I = \frac{1}{2}M(2R)^2$, and for the pulley, $I = \frac{1}{2}MR^2$.

EXECUTE: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed *v*, the pulley has angular velocity *v*/*R* and the cylinder has angular velocity *v*/2*R*, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance *y*, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

EVALUATE: If the pulley were massless and the cylinder slid without rolling, $Mg = 2Ma$ and $a = g/2$. The rotation of the objects reduces the acceleration of the block.

10.90. **IDENTIFY:** Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$.

SET UP: For constant mass the moment of inertia is proportional to the square of the radius.

EXECUTE: $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = (R_0 + \Delta R)^2(\omega_0 + \Delta\omega) = R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega$, where the terms in $\Delta R\Delta\omega$ and $(\Delta\omega)^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives

$$\Delta R = -\frac{R_0}{2}\frac{\Delta}{\omega_0} = -1.1 \text{ cm}.$$

EVALUATE: $\Delta R/R_0$ and $\Delta\omega/\omega_0$ are each very small so the neglect of terms containing $\Delta R\Delta\omega$ or $(\Delta\omega)^2$ is an accurate simplifying approximation.

10.91. **IDENTIFY:** Apply conservation of angular momentum to the collision between the bird and the bar and apply conservation of energy to the motion of the bar after the collision.

SET UP: For conservation of angular momentum take the axis at the hinge. For this axis the initial angular momentum of the bird is $m_{\text{bird}}(0.500\text{ m})v$, where $m_{\text{bird}} = 0.500$ kg and $v = 2.25$ m/s. For this axis the moment of inertia is $I = \frac{1}{3}m_{\text{bar}}L^2 = \frac{1}{3}(1.50\text{ kg})(0.750\text{ m})^2 = 0.281\text{ kg}\cdot\text{m}^2$. For conservation of energy, the gravitational potential energy of the bar is $U = m_{\text{bar}}gy_{\text{cm}}$, where y_{cm} is the height of the center of the bar. Take $y_{\text{cm},1} = 0$, so $y_{\text{cm},2} = -0.375$ m.

EXECUTE: **(a)** $L_1 = L_2$ gives $m_{\text{bird}}(0.500\text{ m})v = (\frac{1}{3}m_{\text{bar}}L^2)\omega$.

$$\omega = \frac{3m_{\text{bird}}(0.500\text{ m})v}{m_{\text{bar}}L^2} = \frac{3(0.500\text{ kg})(0.500\text{ m})(2.25\text{ m/s})}{(1.50\text{ kg})(0.750\text{ m})^2} = 2.00\text{ rad/s}.$$

(b) $U_1 + K_1 = U_2 + K_2$ applied to the motion of the bar after the collision gives

$\frac{1}{2}I\omega_1^2 = m_{\text{bar}}g(-0.375\text{ m}) + \frac{1}{2}I\omega_2^2$. $\omega_2 = \sqrt{\omega_1^2 + \frac{2}{I}m_{\text{bar}}g(0.375\text{ m})}$.

$$\omega_2 = \sqrt{(2.00\text{ rad/s})^2 + \frac{2}{0.281\text{ kg}\cdot\text{m}^2}(1.50\text{ kg})(9.80\text{ m/s}^2)(0.375\text{ m})} = 6.58\text{ rad/s}$$

EVALUATE: Mechanical energy is not conserved in the collision. The kinetic energy of the bar just after the collision is less than the kinetic energy of the bird just before the collision.

10.93. **IDENTIFY** and **SET UP:** Apply conservation of angular momentum to the system consisting of the disk and train.

SET UP: $L_1 = L_2$, counterclockwise positive. The motion is sketched in Figure 10.93.

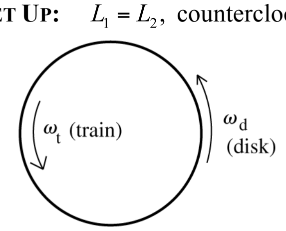

Figure 10.93

$L_1 = 0$ (before you switch on the train's engine; both the train and the platform are at rest)

$L_2 = L_{\text{train}} + L_{\text{disk}}$

EXECUTE: The train is $\frac{1}{2}(0.95\text{ m}) = 0.475$ m from the axis of rotation, so for it

$I_t = m_tR_t^2 = (1.20\text{ kg})(0.475\text{ m})^2 = 0.2708\text{ kg}\cdot\text{m}^2$

$\omega_{\text{rel}} = v_{\text{rel}}/R_t = (0.600\text{ m/s})/0.475\text{ s} = 1.263\text{ rad/s}$

This is the angular velocity of the train relative to the disk. Relative to the earth $\omega_t = \omega_{\text{rel}} + \omega_d$.

Thus $L_{\text{train}} = I_t\omega_t = I_t(\omega_{\text{rel}} + \omega_d)$.

$L_2 = L_1$ says $L_{\text{disk}} = -L_{\text{train}}$

$L_{\text{disk}} = I_{\text{d}}\omega_{\text{d}}$, where $I_{\text{d}} = \frac{1}{2}m_{\text{d}}R_{\text{d}}^2$

$\frac{1}{2}m_{\text{d}}R_{\text{d}}^2\omega_{\text{d}} = -I_{\text{t}}(\omega_{\text{rel}} + \omega_{\text{d}})$

$$\omega_{\text{d}} = -\frac{I_{\text{t}}\omega_{\text{rel}}}{\frac{1}{2}m_{\text{d}}R_{\text{d}}^2 + I_{\text{t}}} = -\frac{(0.2708\ \text{kg}\cdot\text{m}^2)(1.263\ \text{rad/s})}{\frac{1}{2}(7.00\ \text{kg})(0.500\ \text{m})^2 + 0.2708\ \text{kg}\cdot\text{m}^2} = -0.30\ \text{rad/s}.$$

EVALUATE: The minus sign tells us that the disk is rotating clockwise relative to the earth. The disk and train rotate in opposite directions, since the total angular momentum of the system must remain zero. Note that we applied $L_1 = L_2$ in an inertial frame attached to the earth.