Physics 16 Problem Set 6 Solutions

Y&F Problems

1.55. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle ϕ as $\phi = \arccos\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\frac{A_x B_x + A_y B_y}{AB}\right)$.

SET UP: Eq.(1.14) shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22,\ A = \sqrt{40},\ B = \sqrt{13}$, and so $\phi = \arccos\left(\frac{-22}{\sqrt{40}\,\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60,\ A = \sqrt{34},\ B = \sqrt{136},\quad \phi = \arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0$, $0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0$, $90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0$, $\phi = 90°$ and the two vectors are perpendicular.

1.90. **IDENTIFY:** Calculate the scalar product and use Eq.(1.18) to determine ϕ.

SET UP: The unit vectors are perpendicular to each other.

EXECUTE: The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1)+(1)(-1)+(1)(-1)=2\ 1$, so from Eq. (1.18) the angle between the bonds is

$$\arccos\left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\frac{1}{3}\right) = 109°.$$

EVALUATE: The angle between the two vectors in the bond directions is greater than $90°$.

6.3. **IDENTIFY:** Each force can be used in the relation $W = F_{\parallel}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{\text{worker}} = f_k = \mu_k n$.

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{\text{worker}} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 74\ \text{N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}}\cos\phi)s = [(74\ \text{N})(\cos 0°)](4.5\ \text{m}) = +333\ \text{J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k\cos\phi)s = [(74\ \text{N})(\cos 180°)](4.5\ \text{m}) = -333\ \text{J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0$ J.

(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333\ \text{J} + 0.0\ \text{J} + 0.0\ \text{J} - 333\ \text{J} = 0.0\ \text{J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.48. **IDENTIFY** and **SET UP:** Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.

EXECUTE: The rate at which work is being done against gravity is

$P = Fv = mgv = (700\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m/s}) = 17.15\text{ kW}.$

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is

$17.15\text{ kW}/75\text{ kW} = 0.23.$

EVALUATE: The power we calculate for making the airplane climb is considerably less than the power output of the engine.

6.67. **IDENTIFY:** Calculate the work done by friction and apply $W_{\text{tot}} = K_2 - K_1$. Since the friction force is not constant, use Eq.(6.7) to calculate the work.

SET UP: Let x be the distance past P. Since μ_k increases linearly with x, $\mu_k = 0.100 + Ax$. When $x = 12.5\text{ m}$, $\mu_k = 0.600$, so $A = 0.500/(12.5\text{ m}) = 0.0400/\text{m}$

EXECUTE: **(a)** $W_{\text{tot}} = \Delta K = K_2 - K_1$ gives $-\int \mu_k mg\,dx = 0 - \frac{1}{2}mv_1^2$. Using the above expression for μ_k, $g\int_0^{x_2}(0.100 + Ax)dx = \frac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\frac{x_2^2}{2}\right] = \frac{1}{2}v_1^2$.

$(9.80\text{ m/s}^2)\left[(0.100)x_f + (0.0400/\text{m})\frac{x_f^2}{2}\right] = \frac{1}{2}(4.50\text{ m/s})^2$. Solving for x_2 gives $x_2 = 5.11\text{ m}$.

(b) $\mu_k = 0.100 + (0.0400/\text{m})(5.11\text{ m}) = 0.304$

(c) $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3\text{ m}$.

EVALUATE: The box goes farther when the friction coefficient doesn't increase.

6.82. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let h be the distance the 6.00 kg block descends. The work done by gravity is $(6.00\text{ kg})gh$ and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))\,(9.80\text{ m/s}^2)\,(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\frac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

Extra Problem

1. In raising a mass m to a height h, the work done against gravity is $W=mgh$. If the total distance traveled is s (along the incline), then $h = s\sin(\theta)$, giving $W = mgs\sin(\theta)$. The work per meter per kg is then $W/ms = g\sin(\theta)$.
2. This is the function that is plotted in the figure. Since the angles are all pretty small, $\sin(\theta)$ looks pretty linear.
3. The difference between the walking curve (the lower curve) and the line $g\sin(\theta)$ is the physiological inefficiency.
    a. You can eyeball the point where the curves are closest together, or notice that the minimum gap occurs when their slopes are equal. (Displace the straight line upward, keeping the slope the same, to convince yourself of this).
    b. A grade of about -2^{o} or -2.5^{o} gives the smallest inefficiency.
    c. At -2.5^{o}, the gap is about 1.5 J/kg.m.

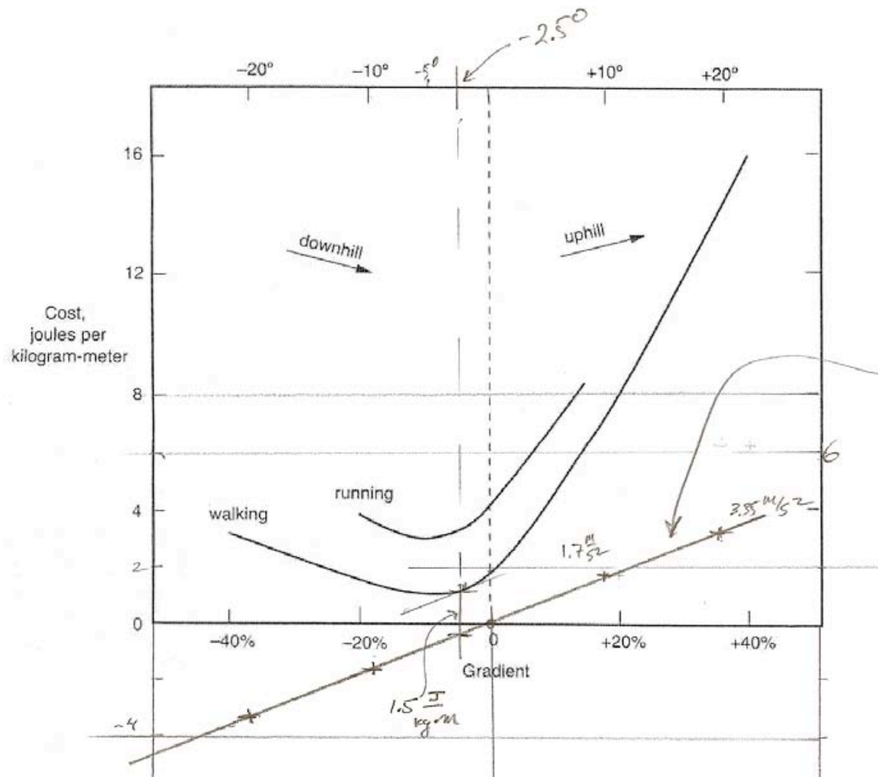

FIGURE 24.8 How our cost of transport varies with grade. The data refer to the energy relative to mass and distance needed to move along the grades at the optimal (cheapest) speed for each.

g sin θ