Math 130 – Fall 2011
November 15, 2011

Name: Solutions

# Test # 2

Please complete the following problems. Be sure to ask me if you have any questions or anything is unclear. Partial credit will be given, so **please be sure to show all of your work. Please use complete sentences to answer ALL questions.**

Unless specifically asked to check or comment, you may assume that any necessary assumptions are reasonably well satisfied.

The Z table and T table are included at the end of the test.

## General Theory

These first 7 questions relate to hypothesis testing and confidence intervals in general.

1. (5 pts) Which is true about a 98% confidence interval for a proportion based on a given sample? Circle the correct response.

   I. We are 98% confident that the sample proportion is in our interval.
   II. There is a 98% chance that our interval contains the population proportion.
   III. The interval is wider than a 95% confidence interval would be.

   A) I only    B) I and II    C) II only    [✓ D) III only]    E) None

2. (5 pts) We have calculated a confidence interval based on a sample of size $n = 100$. Now we want to get a better estimate with a margin of error that is only one-fourth as large. How large does our new sample need to be?

Either for mean    or Proportions

$ME = t^* \frac{s}{\sqrt{n}}$    $ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\frac{1}{4} ME = \frac{1}{4} t^* \frac{s}{\sqrt{n}}$    $\frac{1}{4} ME = \frac{1}{4} z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= t^* \frac{s}{\sqrt{16n}}$    $= z^* \sqrt{\frac{\hat{p}\hat{q}}{16n}}$

It would need to be 16 times larger, or 1600.

3. (5 pts) We have calculated a 95% confidence interval and would prefer that our next confidence interval has a smaller margin of error without losing any confidence. How can we achieve this?

We can achieve this by increasing the sample size.

4. (5 pts) Botanists observed 30 bristlecone pines and estimated their ages. A 95% confidence interval for the mean age of bristlecone pines was calculated to be (1775 , 4225) years.

   In addition, the botanists wanted to do a hypothesis test. Let $\mu$ be the mean age of bristlecone pines. The botanists want to test $H_0 : \mu = 4000$ versus $H_a : \mu \neq 4000$. They plan to use a significance level of $\alpha = 0.05$.

   Based on the information given, what can you say about the $p$-value for such a hypothesis test?

Since the interval contains 4000, we cannot Reject Ho

the p-value must be larger than 0.05.

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5. (10 pts) A diagnostic test for Alzheimer's disease tests the hypotheses

$H_0$: The patient does not have Alzheimer's disease
$H_A$: The patient does have Alzheimer's disease

Suppose researchers set $\alpha = 0.01$, and have calculated the power of the test to be 92%.

a. Describe a Type I and Type II error in the context of this hypothesis test. Which is worse?

A type I error occurs if we Reject Ho when true, OR we conclude someone ~~doesn't have~~ has Alzheimer's when they do. no

A type II error occurs if we decide a person does not have Alzheimer's when they really do. Here, a type II error is worse

b. Find the probability of both the Type I and Type II errors.

P(Type I) = $\alpha$ = 0.01

P(Type II) = $\beta$ = 1 - Power = 0.08

c. Define what is meant by the "power" of a hypothesis test.

The power is the probability that we correctly Reject Ho when the alternative is true.

6. (8 pts) Many people have trouble setting up all the features of their cell phones, so a company has developed what it hopes will be easier instructions. The goal is to have at least 96% of customers succeed. The company tests the new system on 200 people, of whom 188 were successful. Is this strong evidence that the new system fails to meet the company's goal? The work was given to a summer intern. Evaluate the student's test, pointing out any errors you find.

$H_0: \hat{p} = 0.96$
$H_A: \hat{p} \neq 0.96$ ← should be $H_0: p = .96$
$H_A$: ~~$p \neq .96$~~ $p < .96$

SRS, $0.96(200) > 10$ ← also check $(.04)(200) = 8 \not> 10$

$\frac{188}{200} = 0.94$, $SD(\hat{p}) = \sqrt{\frac{(0.94)(0.06)}{200}} = 0.017$ ← .04, .96

$SD(\hat{p}) = \sqrt{\frac{(.96)(.04)}{200}} = 0.014$

$z = \frac{\overset{.94}{\cancel{0.96}} - \overset{.96}{\cancel{0.94}}}{\overset{}{\cancel{0.017}}\ .014} = \cancel{1.18} = -1.43$

$P = P(z > 1.18) = 0.12$ $\quad P(z < -1.43) = 0.0764$

There is strong evidence that the new instructions do not work. ← wrong conclusion

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7. (5 pts) For the following scenarios, state which hypothesis test procedure should be used.

   a. You are comparing the job placement success of UMASS Business School graduates with those of Yale. You randomly sample 25 UMASS graduates and 15 Yale graduates, and record the starting salary of each graduate. What test could you use to determine whether the starting salary of UMASS Business School graduates is less than that of Yale graduates?

      two-sample t-test

   b. To report a mileage estimate to the EPA for a new sedan, a car manufacturer randomly selects 18 cars from their production line. They test each car under identical conditions, and record the mileage per gallon for each car. They wish to test the claim that the mean mileage for this sedan is over 30 mpg. What test would they use to do this?

      one-sample t-test

   c. Drug companies do a lot of clinical trials while researching their products. Early in drug development, these companies conduct trials of their drug on normal, healthy individuals. In a "crossover design," each sample person receives both the drug and a placebo. Measurements are made each time to record information such as blood pressure when using the placebo and blood pressure when using the drug. The company wishes to claim that their drug lowers blood pressure. What test could be used to test such a statement?

      matched pairs t-test

   d. Are Idaho's "famous potatoes" really better? The Idaho Potato Growers Association wants to find out. They randomly sample 50 potatoes from each state. The potatoes are baked, and a trained tester tastes each one, ranking their satisfaction from 1 to 10. The Association would like to claim that the mean satisfaction rating of Idaho potatoes is higher than that of Maine potatoes. Which test could be used to test this claim?

      two-sample t-test

      possibly matched pairs as well

   e. The state of Massachusetts is interested in comparing the proportion of households with broadband internet access for households west of the Quabbin reservoir versus households east of the reservoir. We sample 250 households from each area, and compare the percentage of households that have broadband internet access. Which test could be used to make this comparison?

      two-proportion z-test

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## Scenario 1 – The following scenario will be used to answer questions 8 - 10

A group of scientists is interested in studying air pollution. One component of air pollution is airborne particulate matter such as dust and smoke. To measure particulate pollution, a vacuum motor draws air through a filter for 24 hours. The filter is weighed at the beginning and at the end of the period. The weight gained over the 24 hour period is a measure of the concentration of particles in the air. This study made measurements in the center of a small city and at a rural location 10 miles southwest of the city. The data are shown below:

| **Location** | Particulate Level (grams) |
|---|---|
| Rural | 67, 42, 33, 46, 43, 54, 38, 88, 108, 57, 70, 42, 43, 39, 52, 48, 56, 44, 51, 21, 74, 48, 84, 51, 43, 45, 41, 47, 35 |
| City | 39, 68, 42, 34, 48, 82, 45, 60, 57, 39, 123, 59, 71, 41, 42, 38, 57, 50, 58, 45, 69, 23, 72, 49, 86, 51, 42, 46, 44, 42 |

The alternative hypothesis used in this analysis is the 2-sided (not equal) hypothesis. Some summary statistics are given below:

**Two-Sample T-Test and CI: Rural, City**

```
Two-sample T for Rural vs City

        N  Mean  StDev  SE Mean
Rural  29  52.1   18.2      3.4
City   30  54.1   19.4      3.5
```

You may assume that the degrees of freedom are $df = \dfrac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2+\frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} = 57.$

8. (8 pts) Is there a difference in particulate pollution between the rural and city locations? Carry out an appropriate inference procedure.

I'll use a two proportion t-test

$H_0: \mu_R = \mu_c$

$H_A: \mu_R \neq \mu_c$

Note: A confidence interval could also be used here

$$t = \frac{\bar{x}_R - \bar{x}_c}{\sqrt{\frac{s_R^2}{n_R}+\frac{s_c^2}{n_c}}} = \frac{52.1-54.1}{\sqrt{\frac{18.2^2}{29}+\frac{19.4^2}{30}}} = \frac{-2}{4.8957} = -0.4085$$

Look at Row 60. $\overset{t}{\cancel{p}}$-value is off chart to left, so p-value > 0.20

We do not have evidence of a difference in pollution levels for Rural vs City.

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9. (6 pts) Name the assumptions the data must satisfy in order for the conclusions based your inference procedure to be valid.

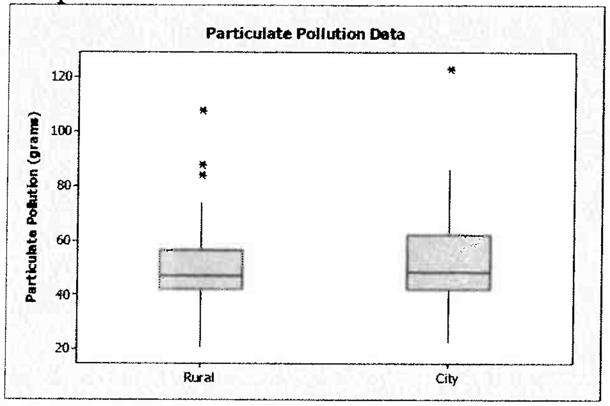
We need observations between and within groups to be independent.

We also need the groups to be nearly normal

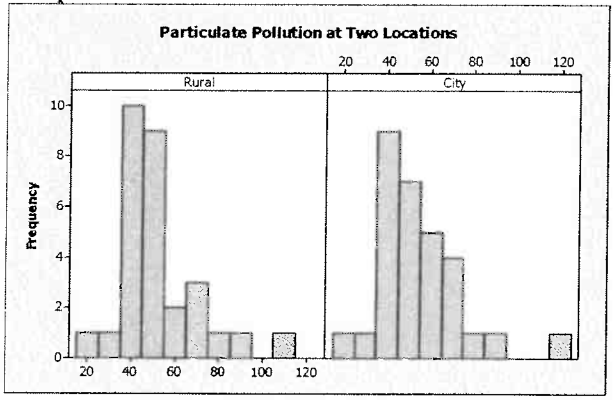
10. (5 pts) Additional graphical output is given below. Discuss the validity of the assumptions you listed in Question 9 on the basis of the graphical output. Remember to cite the graph number you are referring to.

There may be a problem with the normality assumption we made. all four graphs show outliers. As both graphs are skewed in the same direction, though, our t-procedure is probably okay.

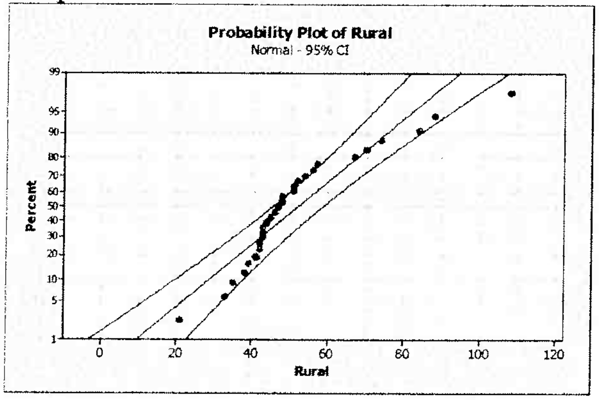
Graph #1:



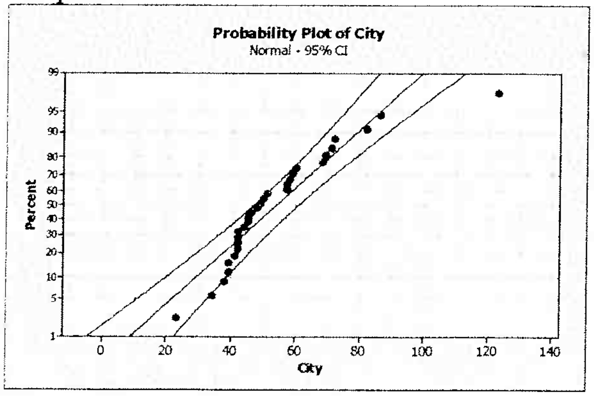
Graph #2:



Graph #3:



Graph #4:



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## Scenario 2 – The following scenario will be used to answer questions 11 - 14

A study on crew teams analyzed the weights of randomly selected rowers from the Oxford and Cambridge crew teams. From data collected over past years, 8 Oxford and 8 Cambridge rowers were randomly selected, and their weight their senior year on the team was recorded. A curious crew fan wants to know if Oxford rowers weigh more on average than Cambridge rowers.

11. (5 pts) State the hypotheses you would test (be sure to define your order of subtraction) to address the fan's question.

Let $\mu_o$ be mean weight of Oxford Rowers, and $\mu_c$ be mean weight of Cambrigde Rowers. We'll test

$H_o: \mu_o \neq \mu_c$

$H_A: \mu_o > \mu_c$

12. (4 pts) You can assume the 16 rowers selected are a representative sample of rowers from the schools and that the weights are independent. What ***other*** assumption needs to be valid to perform your hypothesis test, and what graphs (specifically) would you make to check that assumption? (i.e. if you say bar chart - bar chart of what?)

We need the measurements to be nearly normal for each group. Ahistogram or Normal probability plot of the weights for each group will help establish this.

13. (6 pts) We will assume the conditions hold. Complete your test procedure, using the appropriate R output below. Provide the numeric values of the test statistic and p-value for ***your*** hypotheses.

```
Paired t-test (Cambridge-Oxford)
t = 0.7501, df = 7, p-value = 0.2388
alternative hypothesis: true difference in means is greater than 0

Welch Two Sample t-test  (Cambridge- Oxford)
t = 0.4259, df = 13.775, p-value = 0.3384
alternative hypothesis: true difference in means is greater than 0
```

Test statistic= $t = 0.4259$ $p$-value = 0.3384

Interpret your $p$-value in terms of its definition (not in terms of whether we reject the null hypothesis).

Assuming the means of the two groups are equal, there's a 33.84 % of observing our difference or something greater.

we fail to reject the null hypothesis.

14. (5 pts) What type of error might you have made?

**Type I** [✓ **Type II**] **No Error**

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**Scenario 3 – The following scenario will be used to answer questions 15 - 17**

Bullying has become a "hot topic" due to recent media attention and events. A study on secondary school bullying in England found that out of 92 short students, 42 had experienced bullying. Out of 117 not-short students, 30 had experienced bullying. The determination of short vs. not-short was made based on an unknown but likely reasonable height cutoff. Researchers want to know what the difference in bullying rates is for short vs. not-short students.

15. (5 pts) If the percentages of bullying experiences for short and not-short students were equal, what would your best estimate of that percentage be?

Pooled $\hat{p} = \frac{42+30}{92+117} = \frac{72}{209} = 0.3445$

I'd estimate that 34.45% of students had experienced bullying.

16. (8 pts) Address the researcher's question by creating a 98% confidence interval, showing your work, and checking conditions.

$\hat{p}_s = \frac{42}{92} = .4565 \quad \hat{p}_{NS} = \frac{30}{117} = .2564$

check

SRS ? Not sure

92(.4565) = 42>10

117(.2564) = 30>10

others are ok, too.

$$(\hat{p}_s - \hat{p}_{NS}) \pm z^* \sqrt{\frac{\hat{p}_s \hat{q}_s}{n_s} + \frac{\hat{p}_{NS} \hat{q}_{NS}}{n_{NS}}} = (.4565 - .2564) \pm 2.326 \sqrt{\frac{(.4565)(1-.4565}{92} + \frac{(.2564)(1-.2564)}{117}}$$

$$= 0.2001 \pm 2.326\,(0.0658)$$

$$= 0.2001 \pm .1530 = (0.047, 0.353)$$

We are 98% confident the difference in bullying rates is between 4.7 and 35.3 %

17. (5 pts) Based on the CI, is it reasonable to conclude that a higher percentage of short students have experienced bullying compared to not-short students? Explain in one sentence. At what significance level can you state that conclusion?

Yes, it is reasonable to conclude that a higher percentage of short students have experienced bullying, because the entire confidence interval is above zero.

Since we did a 98% confidence interval, this conclusion is at an $\alpha = 0.02$ significance level.

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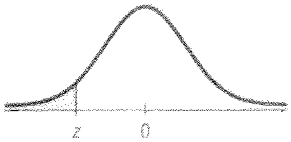
## Table Z

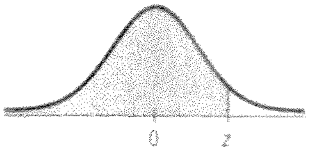
Areas under the standard Normal curve



| Second decimal place in $z$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.00 | $z$ |
| | | | | | | | | | 0.0000[†] | −3.9 |
| 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | −3.8 |
| 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | −3.7 |
| 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | −3.6 |
| 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | −3.5 |
| 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | −3.4 |
| 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 | −3.3 |
| 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 | −3.2 |
| 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0010 | −3.1 |
| 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 | −3.0 |
| 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 | −2.9 |
| 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 | −2.8 |
| 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 | −2.7 |
| 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 | −2.6 |
| 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 | −2.5 |
| 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 | −2.4 |
| 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 | −2.3 |
| 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 | −2.2 |
| 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 | −2.1 |
| 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 | −2.0 |
| 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 | −1.9 |
| 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 | −1.8 |
| 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 | −1.7 |
| 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 | −1.6 |
| 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 | −1.5 |
| 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 | −1.4 |
| 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 | −1.3 |
| 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 | −1.2 |
| 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 | −1.1 |
| 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 | −1.0 |
| 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 | −0.9 |
| 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 | −0.8 |
| 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 | −0.7 |
| 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 | −0.6 |
| 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 | −0.5 |
| 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 | −0.4 |
| 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 | −0.3 |
| 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 | −0.2 |
| 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 | −0.1 |
| 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 | −0.0 |

[†] For $z \leq -3.90$, the areas are 0.0000 to four decimal places.

**Table Z (cont.)**
Areas under the standard Normal curve

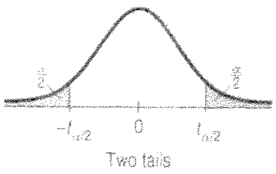


|  | Second decimal place in z |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|
| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.9 | 1.0000† |  |  |  |  |  |  |  |  |  |

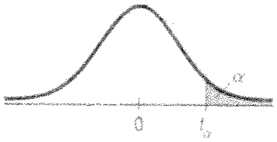
†For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

### Table T

Values of $t_\alpha$



Two tails



One tail

| **Two-tail probability** | | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | |
|---|---|---|---|---|---|---|---|
| **One-tail probability** | | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | |
| | df | | | | | | df |
| | 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 |
| | 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 |
| | 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 |
| | 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4 |
| | 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 |
| | 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 |
| | 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 |
| | 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 |
| | 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 |
| | 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 |
| | 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 |
| | 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 |
| | 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 |
| | 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 |
| | 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 |
| | 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 |
| | 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 |
| | 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 |
| | 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 |
| | 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 |
| | 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 |
| | 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 |
| | 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 |
| | 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 |
| | 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 |
| | 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 |
| | 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 |
| | 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 |
| | 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 |
| | 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 30 |
| | 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 32 |
| | 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.725 | 35 |
| | 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 40 |
| | 45 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 45 |
| | 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 50 |
| | 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 60 |
| | 75 | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 | 75 |
| | 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 100 |
| | 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 120 |
| | 140 | 1.288 | 1.656 | 1.977 | 2.353 | 2.611 | 140 |
| | 180 | 1.286 | 1.653 | 1.973 | 2.347 | 2.603 | 180 |
| | 250 | 1.285 | 1.651 | 1.969 | 2.341 | 2.596 | 250 |
| | 400 | 1.284 | 1.649 | 1.966 | 2.336 | 2.588 | 400 |
| | 1000 | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 | 1000 |
| | ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | ∞ |
| **Confidence levels** | | 80% | 90% | 95% | 98% | 99% | |