## Physics 16 Problem Set 4 Solutions

4.43. Identify: Use Newton's 2nd law to relate the acceleration and forces for each crate.
(a) SET UP: Since the crates are connected by a rope, they both have the same acceleration, $2.50 \mathrm{~m} / \mathrm{s}^{2}$.
(b) The forces on the 4.00 kg crate are shown in Figure 4.43a.


## Execute:

$\sum F_{x}=m a_{x}$
$T=m_{1} a=(4.00 \mathrm{~kg})\left(2.50 \mathrm{~m} / \mathrm{s}^{2}\right)=10.0 \mathrm{~N}$.

Figure 4.43a
(c) Set Up: Forces on the 6.00 kg crate are shown in Figure 4.43 b


Figure 4.43b
(d) ExECUTE: $\quad \sum F_{x}=m a_{x}$ gives $F-T=m_{2} a$

$$
F=T+m_{2} a=10.0 \mathrm{~N}+(6.00 \mathrm{~kg})\left(2.50 \mathrm{~m} / \mathrm{s}^{2}\right)=10.0 \mathrm{~N}+15.0 \mathrm{~N}=25.0 \mathrm{~N}
$$

Evaluate: We can also consider the two crates and the rope connecting them as a single object of mass $m=m_{1}+m_{2}=10.0 \mathrm{~kg}$. The free-body diagram is sketched in Figure 4.43c.


$$
\sum F_{x}=m a_{x}
$$

$$
\bar{F}=m a=(10.0 \mathrm{~kg})\left(2.50 \mathrm{~m} / \mathrm{s}^{2}\right)=25.0 \mathrm{~N}
$$

This agrees with our answer in part (d).
4.56. Identify: Apply $\sum \overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}$ to the balloon and its passengers and cargo, both before and after objects are dropped overboard.

SET UP: When the acceleration is downward take $+y$ to be downward and when the acceleration is upward take $+y$ to be upward.
ExECUTE: (a) The free-body diagram for the descending balloon is given in Figure 4.56.
$L$ is the lift force.
(b) $\sum F_{y}=m a_{y}$ gives $M g-L=M(g / 3)$ and $L=2 M g / 3$.
(c) Now $+y$ is upward, so $L-m g=m(g / 2)$, where $m$ is the mass remaining.
$L=2 M g / 3$, so $m=4 M / 9$. Mass $5 M / 9$ must be dropped overboard.
Evaluate: In part (b) the lift force is greater than the total weight and in part (c) the lift force is less than the total weight.


Figure 4.56
5.15. Identify: Apply Newton's first law to the ball. Treat the ball as a particle.

SET UP: The forces on the ball are gravity, the tension in the wire and the normal force exerted by the surface. The normal force is perpendicular to the surface of the ramp. Use $x$ and $y$ axes that are horizontal and vertical.
Execute: (a) The free-body diagram for the ball is given in Figure 5.15. The normal force has been replaced by its $x$ and $y$ components.
(b) $\sum F_{y}=0$ gives $n \cos 35.0^{\circ}-w=0$ and $n=\frac{m g}{\cos 35.0^{\circ}}=1.22 m g$.
(c) $\sum F_{x}=0$ gives $T-n \sin 35.0^{\circ}=0$ and $T=(1.22 \mathrm{mg}) \sin 35.0^{\circ}=0.700 \mathrm{mg}$.

Evaluate: Note that the normal force is greater than the weight, and increases without limit as the angle of the ramp increases towards $90^{\circ}$. The tension in the wire is $w \tan \phi$, where $\phi$ is the angle of the ramp and $T$ also increases without limit as $\phi \rightarrow 90^{\circ}$.


Figure 5.15
5.63. Identify: Apply $\sum \overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}$ to the rope.

SET UP: The hooks exert forces on the ends of the rope. At each hook, the force that the hook exerts and the force due to the tension in the rope are an action-reaction pair.
Execute: (a) The vertical forces that the hooks exert must balance the weight of the rope, so each hook exerts an upward vertical force of $w / 2$ on the rope. Therefore, the downward force that the rope exerts at each end is $T_{\text {end }} \sin \theta=w / 2$, so $T_{\text {end }}=m g /(2 \sin \theta)$.
(b) Each half of the rope is itself in equilibrium, so the tension in the middle must balance the horizontal force that each hook exerts, which is the same as the horizontal component of the force due to the tension at the end; $T_{\text {end }} \cos \theta=T_{\text {middle }}$. so $T_{\text {middle }}=m g /(2 \tan \theta)=m g \cot \theta / 2$.
(c) Mathematically speaking, $\theta \neq 0$ because this would cause a division by zero in the equation for $T_{\text {end }}$ or $T_{\text {midde }}$. Physically speaking, we would need an infinite tension to keep a non-massless rope perfectly straight. (d) At $90^{\circ}$, there is no tension in the middle of the rope because all forces are in the up/down direction. Each end of the rope supports half the weight, so $T_{\text {end }}=\mathrm{mg} / 2$.
Evaluate: The tension in the rope is not the same at all points along the rope.
5.67. Identify: Apply $\sum \overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}$ to each block. Use Newton's $3^{\text {rd }}$ law to relate forces on $A$ and on $B$.

SET UP: Constant speed means $a=0$.
ExECUTE: (a) Treat $A$ and $B$ as a single object of weight $w=w_{A}+w_{B}=4.80 \mathrm{~N}$. The free-body diagram for this combined object is given in Figure 5.67a. $\sum F_{y}=m a_{y}$ gives $n=w=4.80 \mathrm{~N}$.
$f_{\mathrm{k}}=\mu_{\mathrm{k}} n=1.44 \mathrm{~N} . \sum F_{x}=m a_{x}$ gives $F=f_{\mathrm{k}}=1.44 \mathrm{~N}$
(b) The free-body force diagrams for blocks $A$ and $B$ are given in Figure 5.67b. $n$ and $f_{\mathrm{k}}$ are the normal and friction forces applied to block $B$ by the tabletop and are the same as in part (a). $f_{\mathrm{k} B}$ is the friction force that $A$ applies to $B$. It is to the right because the force from $A$ opposes the motion of $B . n_{B}$ is the downward force that $A$ exerts on $B . f_{\mathrm{k} A}$ is the friction force that $B$ applies to $A$. It is to the left because block $B$ wants $A$ to move with it. $n_{A}$ is the normal force that block $B$ exerts on $A$. By Newton's third law, $f_{\mathrm{k} B}=f_{\mathrm{k} A}$ and these forces are in opposite directions. Also, $n_{A}=n_{B}$ and these forces are in opposite directions.
$\sum F_{y}=m a_{y}$ for block $A$ gives $n_{A}=w_{A}=1.20 \mathrm{~N}$, so $n_{B}=1.20 \mathrm{~N}$.
$f_{\mathrm{k} A}=\mu_{\mathrm{k}} n_{A}=(0.300)(1.20 \mathrm{~N})=0.36 \mathrm{~N}$, and $f_{\mathrm{k} B}=0.36 \mathrm{~N}$.
$\sum F_{x}=m a_{x}$ for block $A$ gives $T=f_{\mathrm{k} A}=0.36 \mathrm{~N}$.
$\sum F_{x}=m a_{x}$ for block $B$ gives $F=f_{\mathrm{k} B}+f_{\mathrm{k}}=0.36 \mathrm{~N}+1.44 \mathrm{~N}=1.80 \mathrm{~N}$
Evaluate: In part (a) block $A$ is at rest with respect to $B$ and it has zero acceleration. There is no horizontal force on $A$ besides friction, and the friction force on $A$ is zero. A larger force $F$ is needed in part (b), because of the friction force between the two blocks.




Figure 5.67a-c
5.92. Identify: Apply $\sum \overrightarrow{\boldsymbol{F}}=m \overrightarrow{\boldsymbol{a}}$ to each block.

SET UP: Use coordinates where $+x$ is directed down the incline.
EXECUTE: (a) Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; i.e., the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block,
$(4.00 \mathrm{~kg}) g\left(\sin 30^{\circ}-(0.25) \cos 30^{\circ}\right)-T=(4.00 \mathrm{~kg}) a$, or $11.11 \mathrm{~N}-T=(4.00 \mathrm{~kg}) a$, and similarly for the larger, $15.44 \mathrm{~N}+T=(8.00 \mathrm{~kg}) a$. Adding these two relations, $26.55 \mathrm{~N}=(12.00 \mathrm{~kg}) a, a=2.21 \mathrm{~m} / \mathrm{s}^{2}$.
(b) Substitution into either of the above relations gives $T=2.27 \mathrm{~N}$.
(c) The string will be slack. The $4.00-\mathrm{kg}$ block will have $a=2.78 \mathrm{~m} / \mathrm{s}^{2}$ and the $8.00-\mathrm{kg}$ block will have $a=1.93 \mathrm{~m} / \mathrm{s}^{2}$, until the $4.00-\mathrm{kg}$ block overtakes the $8.00-\mathrm{kg}$ block and collides with it.
Evaluate: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The $8.00-\mathrm{kg}$ block would have a smaller acceleration even though it has a larger mass, since it has a larger $\mu_{\mathrm{k}}$.

