Physics 16 Problem Set 8 Solutions

Y&F Problems

8.20. **IDENTIFY:** In part (a) no horizontal force implies P_x is constant. In part (b) use the energy expression, Eq. 7.14, to find the potential energy intially in the spring.
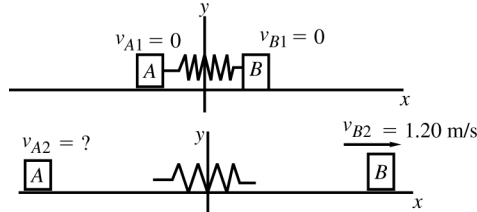**SET UP:** Initially both blocks are at rest.

Figure 8.20

EXECUTE: **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right) v_{B2x} = -\left(\frac{3.00 \text{ kg}}{1.00 \text{ kg}}\right)(+1.20 \text{ m/s}) = -3.60 \text{ m/s}$$

Block A has a final speed of 3.60 m/s, and moves off in the opposite direction to B.
(b) Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2 = \tfrac{1}{2}(1.00 \text{ kg})(3.60 \text{ m/s})^2 + \tfrac{1}{2}(3.00 \text{ kg})(1.20 \text{ m/s})^2 = 8.64 \text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64$ J

EVALUATE: The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

8.38. **IDENTIFY:** Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.
SET UP: Conservation of momentum applied to the collision between the bullet and the block: Let object A be the bullet and object B be the block. Let v_A be the speed of the bullet before the collision and let V be the speed of the block with the bullet inside just after the collision.
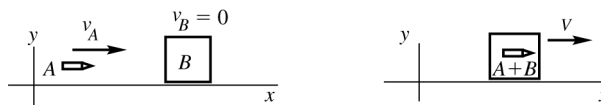

Figure 8.38a

P_x is constant gives $m_A v_A = (m_A + m_B)V$.
Conservation of energy applied to the motion of the block after the collision:
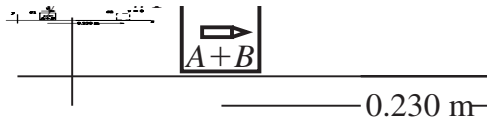
Figure 8.38b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80 \text{ m/s}^2)(0.230 \text{ m})} = 0.9495$ m/s

Use this in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00 \times 10^{-3} \text{ kg} + 1.20 \text{ kg}}{5.00 \times 10^{-3} \text{ kg}}\right)(0.9495 \text{ m/s}) = 229 \text{ m/s}$$

EVALUATE: When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

8.41. **IDENTIFY:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Since the collision is elastic, Eqs. 8.24 and 8.25 give the final velocity of each block after the collision.
SET UP: Let $+x$ be the direction of the initial motion of A.
EXECUTE: (a) Momentum conservation gives $(2.00 \text{ kg})(2.00 \text{ m/s}) = (12.0 \text{ kg})V$ and $V = 0.333$ m/s . Both blocks are moving at 0.333 m/s, in the direction of the initial motion of block A. Conservation of energy says the initial kinetic energy of A equals the total kinetic energy at maximum compression plus the potential energy U_b

stored in the bumpers: $\frac{1}{2}(2.00 \text{ kg})(2.00 \text{ m/s})^2 = U_b + \frac{1}{2}(12.0 \text{ kg})(0.333 \text{ m/s})^2$ and $U_b = 3.33$ J .

(b) $v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\frac{2.00 \text{ kg} - 10.0 \text{ kg}}{12.0 \text{ kg}}\right)(2.00 \text{ m/s}) = -1.33$ m/s . Block A is moving in the $-x$ direction at 1.33 m/s.

$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right)v_{A1x} = \frac{2(2.00 \text{ kg})}{12.0 \text{ kg}}(2.00 \text{ m/s}) = +0.667$ m/s . Block B is moving in the $+x$ direction at 0.667 m/s.
EVALUATE: When the spring is compressed the maximum amount the system must still be moving in order to conserve momentum.

8.68. **IDENTIFY:** The total momentum of the system is conserved and is equal to zero, since the pucks are released from rest.
SET UP: Each puck has the same mass m. Let $+x$ be east and $+y$ be north. Let object A be the puck that moves west. All three pucks have the same speed v.
EXECUTE: $P_{1x} = P_{2x}$ gives $0 = -mv + mv_{Bx} + mv_{Cx}$ and $v = v_{Bx} + v_{Cx}$. $P_{1y} = P_{2y}$ gives $0 = mv_{By} + mv_{Cy}$ and $v_{By} = -v_{Cy}$. Since $v_B = v_C$ and the y components are equal in magnitude, the x components must also be equal: $v_{Bx} = v_{Cx}$ and $v = v_{Bx} + v_{Cx}$ says $v_{Bx} = v_{Cx} = v/2$. If v_{By} is positive then v_{Cy} is negative.

The angle θ that puck B makes with the x axis is given by $\cos\theta = \dfrac{v/2}{v}$ and $\theta = 60°$. One puck moves

in a direction $60°$ north of east and the other puck moves in a direction $60°$ south of east.

8.69.   **IDENTIFY:**   The x and y components of the momentum of the system are conserved.
Set Up: After the collision the combined object with mass $m_{tot} = 0.100$ kg moves with velocity $\vec{v}_2$.
Solve for v_{Cx} and v_{Cy}.
EXECUTE: (a) $P_{1x} = P_{2x}$ gives $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{tot} v_{2x}$.

$$v_{Cx} = -\frac{m_A v_{Ax} + m_B v_{Bx} - m_{tot} v_{2x}}{m_C}$$

$$v_{Cx} = -\frac{(0.020 \text{ kg})(-1.50 \text{ m/s}) + (0.030 \text{ kg})(-0.50 \text{ m/s})\cos 60° - (0.100 \text{ kg})(0.50 \text{ m/s})}{0.050 \text{ kg}}.$$

$$v_{Cx} = 1.75 \text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{Ay} + m_B v_{By} + m_C v_{Cy} = m_{tot} v_{2y}$.

$$v_{Cy} = -\frac{m_A v_{Ay} + m_B v_{By} - m_{tot} v_{2y}}{m_C} = -\frac{(0.030 \text{ kg})(-0.50 \text{ m/s})\sin 60°}{0.050 \text{ kg}} = +0.260 \text{ m/s}.$$

(b) $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 1.77$ m/s . $\Delta K = K_2 - K_1$.

$\Delta K = \frac{1}{2}(0.100 \text{ kg})(0.50 \text{ m/s})^2 - [\frac{1}{2}(0.020 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2}(0.030)(0.50 \text{ m/s})^2 + \frac{1}{2}(0.050 \text{ kg})(1.77 \text{ m/s})^2]$
$\Delta K = -0.092$ J .
EVALUATE: Since there is no horizontal external force the vector momentum of the system is
conserved. The forces the spheres exert on each other do negative work during the collision and this
reduces the kinetic energy of the system.

8.76.   **IDENTIFY:**   The horizontal components of momentum of the system of bullet plus stone are conserved.
The collision is elastic if $K_1 = K_2$.
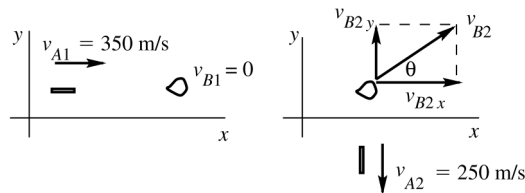**SET UP:**   Let A be the bullet and B be the stone.
(a)

Figure 8.76

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.
$m_A v_{A1} = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A}{m_B}\right) v_{A1} = \left(\frac{6.00 \times 10^{-3} \text{ kg}}{0.100 \text{ kg}}\right)(350 \text{ m/s}) = 21.0 \text{ m/s}$$

P_y is conserved so $m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$.
$0 = -m_A v_{A2} + m_B v_{B2y}$.

$$v_{B2y} = \left(\frac{m_A}{m_B}\right) v_{A2} = \left(\frac{6.00 \times 10^{-3} \text{ kg}}{0.100 \text{ kg}}\right)(250 \text{ m/s}) = 15.0 \text{ m/s}.$$

$$v_{B2} = \sqrt{v_{B2x}^2 + v_{B2y}^2} = \sqrt{(21.0 \text{ m/s})^2 + (15.0 \text{ m/s})^2} = 25.8 \text{ m/s}.$$

$$\tan\theta = \frac{v_{B2y}}{v_{B2x}} = \frac{15.0 \text{ m/s}}{21.0 \text{ m/s}} = 0.7143; \quad \theta = 35.5° \text{ (defined in the sketch).}$$

(b) To answer this question compare K_1 and K_2 for the system:

$$K_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}\left(6.00\times10^{-3} \text{ kg}\right)\left(350 \text{ m/s}\right)^2 = 368 \text{ J}.$$

$$K_2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2 = \tfrac{1}{2}\left(6.00\times10^{-3} \text{ kg}\right)\left(250 \text{ m/s}\right)^2 + \tfrac{1}{2}\left(0.100 \text{ kg}\right)\left(25.8 \text{ m/s}\right)^2 = 221 \text{ J}.$$

$$\Delta K = K_2 - K_1 = 221 \text{ J} - 368 \text{ J} = -147 \text{ J}.$$

EVALUATE: The kinetic energy of the system decreases by 147 J as a result of the collision; the collision is *not* elastic. Momentum is conserved because $\sum F_{\text{ext},x} = 0$ and $\sum F_{\text{ext},y} = 0$. But there are internal forces between the bullet and the stone. These forces do negative work that reduces K.