Amherst College
Department of Economics
Economics 58
Fall 2009

Name________________

Second Hour Test

SOLUTIONS

There are three questions on this 80 minute examination. Each is of equal weight in grading.

<u>Question 1</u>. The state preference model of decision-making under uncertainty is a good tool for examining some issues in insurance contracts. This question illustrates this approach. Throughout the problem assume that there are only two states of the world: Good Times (g) and Bad Times (b). The probability of good times will be denoted by π. Since there are only two states of the world, the probability of bad times is given by $1-\pi$.

a. Suppose a person believes $\pi = 0.5$ and that he or she faces a potential loss of \$100 in bad times. Describe the transactions in the contingent contracts "\$1 in good times" and "\$1 in bad times" that would allow him or her to fully insure against this loss. Assume that the prices of these contingent contracts are fair – that is $p_g = 0.5,\ p_b = 0.5$. (Note: here "fully insure" means that the value of this person's wealth is the same regardless of whether good or bad times occur).

Hint: For this and all later parts it may be helpful to think first about how much wealth <u>in both</u> states must eventually be reduced to allow for the expected value of the loss.

Need to transfer \$50 from good times to bad times. This is accomplished by selling 50 good time (gt) contingent contracts at \$.50 each (yielding \$25) and using the \$25 to buy 50 bad times (bt) contingent contracts. The result of this trade is that

$$W_g = W_0 - 50 = W_b = W_0 - 100 + 50 = W_0 - 50.$$

b. Suppose now there are two types of people. Type-A people are low-risk for whom $\pi_A = 0.6$. People of type-C are high risk ($\pi_C = 0.4$). If contingent contracts are priced separately for type-A and -C people, what sorts of transactions will each type of person make in order to insure fully against a loss of \$100?

Type A needs to lose \$40 in both states. This is accomplished by selling 40 gt at 0.6 and using the resulting \$24 to buy 60 bt (@.40 each). Resulting wealth is

$$W_g = W_0 - 40 = W_b = W_0 - 100 + 60 = W_0 - 40.$$

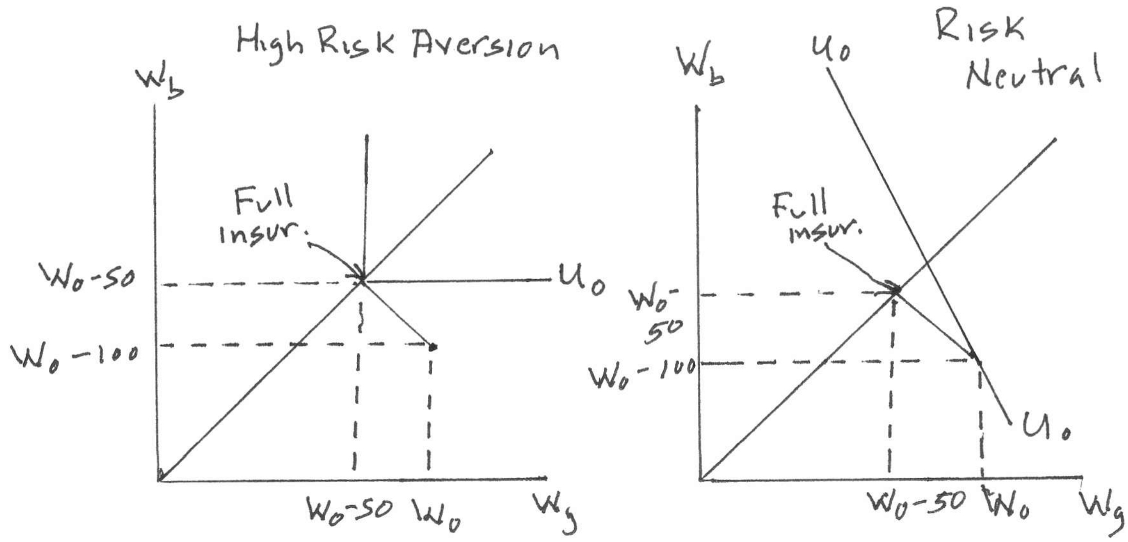
Type C needs to lose \$60 in both periods. This is accomplished by selling 60 gt @0.4 (yielding \$24) and using this to buy 40 bt @0.6. Resulting wealth is:

$$W_g = W_0 - 60 = W_b = W_0 - 100 + 40 = W_0 - 60.$$

c. Suppose now sellers of contingent contracts cannot tell whether a person is type A or type C. Show that type C people will be able to increase the expected value of their wealth by buying contingent contracts intended for type A persons.
Suppose C does the same transaction as in part b. Selling 60 gt @ 0.6 yields $36. Using that to buy bt @0.4 provides 90. Hence
$W_g = W_0 - 60 \quad W_b = W_0 - 100 + 90 = W_0 - 10$. **Expected wealth for type C person is**
$E(W) = 0.4W_g + 0.6W_b = W_0 - 24 - 6 = W_0 - 30$. **Which is much better than in part b.**
Another way to answer is to note that selling one gt allows the purchase of 1.5 bt and the expected value of this transaction is $0.4(-1) + 0.6(+1.5) = 0.5$. **Each such transaction increases expected wealth by $.50. (that is why selling 60 improves expected wealth by $40).**

d. Explain why sellers of contingent contracts will lose money by selling type-A contracts to type-C people. If type-A people and type-C people are equally numerous, what prices should sellers of contingent contracts charge so as to break even? Will this contract still be advantageous to type-C people? Explain.
Clearly the parties that take the other side of C's transactions are losing $.50 on each trade in expected value. No one would stay in that business. If these traders opt for a "pooled" pricing strategy they will charge $.50 for both gt and bt. These are still advantageous for type-C people. If they sell one gt they can buy one bt, but the expected value of this transaction is $0.4(-1) + 0.6(+1) = 0.20$.

e. Describe why the contract described in part d is "unfair" to type-A people. Use a state preference graph to show that type-A people may or may not fully insure against the $100 loss depending on their degree of risk aversion.
Buying gt and bt at these prices is unfair to type-A people. The expected value of a transaction is $0.6(-1) + 0.4(+1) = -0.20$. **But they may still make this transaction if they are risk averse enough. In the graphs below, indifference curves for a fully risk averse person show that fully insuring is still utility-maximizing. On the other hand a mildly risk-averse person will only go part way toward the certainty line.**



Question 2: Suppose a particular production process requires a firm to use 3 units of capital for each unit of labor. Suppose also that this process exhibits diminishing returns to scale with a scale factor of $s = 0.8$.

a. What is the production function for this firm?

The production function is $q = [Min(k, 3l)]^{0.8}$

b. Calculate the total cost function for this firm.

Unit costs here are $C(1, v, w) = v + w/3 = c$

Total costs are $kc = q^{1/0.8}c = q^{5/4}c$

c. Calculate the profit function for this firm (Hint: this is probably most easily accomplished using the $p = mc$ approach to profit maximization).

Set $p = mc$ $\quad p = \frac{5}{4}q^{1/4}c \quad q^{1/4} = \frac{4}{5}\frac{p}{c} \quad q = \left(\frac{4}{5}\right)^4 \left(\frac{p}{c}\right)^4$. **Note this is homogeneous of degree zero in p and c as it should be. To save notation, let $r = 4/5$. Profits are now:**

$\pi = pq - TC = r^4 p^5 c^{-4} - q^{5/4}c = r^4 p^5 c^{-4} - r^5 p^5 c^{-4} = bp^5 c^{-4}$ **where**

$b = r^4 - r^5 = r^4(1 - r) > 0$. **Note that this profit function is homogeneous of degree one in p and c, as it should be.**

d. Calculate this firm's supply function. Use this function to discuss which input price has a greater effect in shifting the supply curve. Explain your result intuitively.

$q = \partial\pi/\partial p = 5bp^4 c^{-4}$. **Is this the same supply relationship already derived in part c?**

An increase in v will reduce supply more than a similar increase in w because $\partial c/\partial v = 1 > \partial c/\partial w = 1/3$. **Note again that this relationship is homogeneous of degree zero in p and c.**

e. Calculate this firm's demand for labor function. Explain why this function is downward sloping despite the fact that the firm's inputs are used in fixed proportions.

$l = -\partial\pi/\partial w = 4bp^5 c^{-5} \cdot \partial c/\partial w = \frac{4}{3}bp^5 c^{-5}$. **Here clearly** $\partial l/\partial w == \frac{-20}{6}bp^5 c^{-6} < 0$.

This negative wage effect arises solely from the output effect of a wage change because there is no substitution effect with fixed proportions. Note that this firm's demand for labor is homogeneous of degree zero in p and c as it should be.

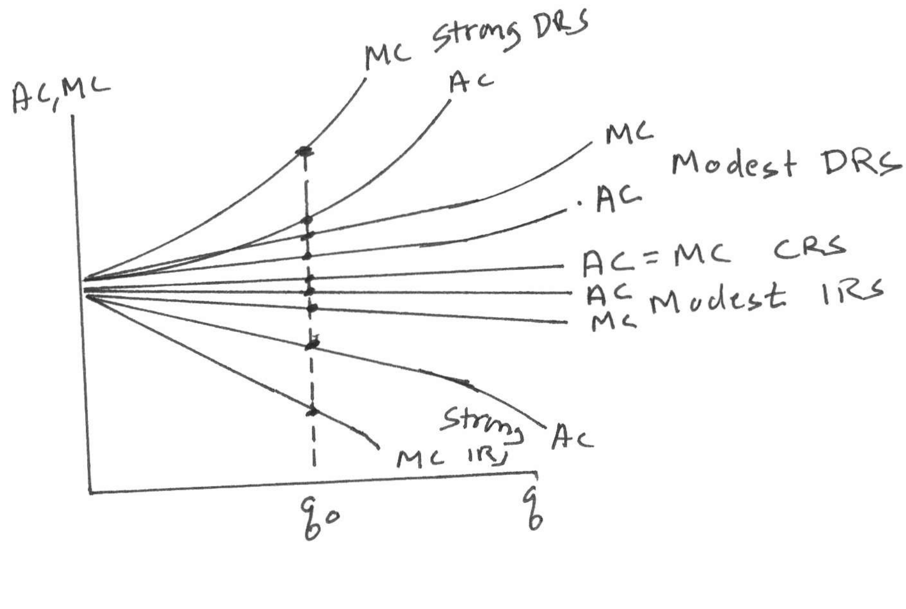
Question 3. The Christenson and Green paper is one of the most influential empirical studies in economics. Use this paper to address the following questions.

a. Why do you think the paper was so influential in the debate about deregulation of electricity generation? What question does it address? How is this question relevant to regulatory policy?

Whether deregulation of electricity generation will improve efficiency depends in part on whether the resulting market will be relatively competitive. That in turn depends on whether one (or a few) generating firms might dominate the market by exploiting economies of scale. Because C+G show that average cost curves are relatively flat across a broad range of output levels, it appeared that such problems would not materialize and that deregulation might proceed (but stay tuned for more on this).

b. A primary focus of the C+G paper is to estimate what the authors term "scale economies". This term is defined in equation 10 on page 661. Show that this definition can be re-written as $SCE = 1 - MC/AC$. Now use a standard cost curve diagram to devise an intuitive analysis of what SCE means.

$SGE = \dfrac{\partial \ln C}{\partial \ln q} = \dfrac{\partial C}{\partial q} \cdot \dfrac{q}{C} = \dfrac{MC}{AC}$. **In the graph below notice that at a given q this ratio does indeed indicate scale economies.**



c. What do the authors conclude about their scale economies measure? How is this conclusion reflected in Figure 3 on page 674?

This graph shows a composite average cost curve based on 1970 data. The fact that the curve is quite flat implies that there are constant returns to scale over a fairly broad range of output levels for generating plants.

d. The authors' "Model F" is in fact a Cobb-Douglas cost function. Using the notation from class, this function has the general form $C = Bq^{1/s}v^{\alpha/s}w^{\beta/s}$. Estimates of the parameters of the logarithmic form of this function are presented in the right column of Table 3 on page 665. What would you conclude about returns to scale from these results?
According to the authors is this result right or wrong?

For this Cobb-Douglas cost function: $\ln C = \ln B + \frac{1}{s}\ln q + \frac{\alpha}{s}\ln v + \frac{\beta}{s}\ln w$. **In Table 3, the coefficient of *lnq* (that is,** $\alpha_Y = 1/s$**) is given as 0.797. Hence** $s = \frac{1}{0.797} = 1.255$. **So this estimate suggests significant returns to scale. But the authors believe this is incorrect because the true cost function is not Cobb-Douglas.**

e. In Table 6 on page 667 the authors present their estimates of the own price elasticities of demand for electric utility inputs. Using the 1970 figures, discuss who you think would pay the majority of a "carbon tax" placed on the fuel used by these utilities. In your answer, keep in mind that electricity prices are regulated so as to provide a "fair return" to their investors. That is, price is set so that $P = AC + R$ where R is set so as to provide the promised fair rate of return to investors and is held constant over long periods.

Here the price elasticity of demand for fuel is estimated as -0.086. So demand for fuel is very inelastic. A tax on fuel would therefore be paid almost exclusively by demanders of fuel (i.e. electric generating companies). This would raise their average costs. And, because of the regulatory policy, these increases in AC would show through directly into prices for electricity. Ultimately the tax would be almost completely paid by electricity consumers