Rehypothecation and Banking Fragility

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Abstract

This paper investigates whether the practice of collateral rehypothecation in short-term money markets, specifically repo markets, contributes to elevated fragility in banking. I first develop an extension to an existing game-theoretic bank run model to include rehypothecation of collateral. The model illustrates how rehypothecation increases fragility of banks through increasing leverage, which makes it more difficult for banks to react to runs induced by sunspots. I then empirically test whether rehypothecation increases fragility by employing a structural VAR and plotting impulse responses of measures of fragility such as interbank credit spreads and unemployment. I found that rehypothecation shocks led to tighter credit conditions for over a year and increased unemployment for around two years. The structural VAR also confirmed an implication of the theoretical model that suggested rehypothecation would fall when there is elevated uncertainty. These results suggest that greater regulation and oversight on rehypothecation of collateral may be effective in maintaining banking stability and preventing bank runs.

JEL Classification: E44, G21, G23

Keywords: Banking Crisis, Bank Run, Financial Intermediaries, Shadow Bank

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Introduction

This thesis examines whether rehypothecation, the reuse of collateral already backing outstanding loans for new borrowing, can increase banking fragility. Rehypothecation is a common practice in short-term money markets and allows for greater liquidity, but also greater leverage in the banking system. I show both theoretically and empirically that rehypothecation of collateral can lead to elevated fragility and tighter conditions in short-term credit markets. My theoretical bank run model suggests that allowing for rehypothecation results in greater difficulty for borrowers to avoid bankruptcy in run scenarios. To support the finding of elevated fragility that rehypothecation introduces, I run a structural VAR model that found that rehypothecation shocks lead to elevated credit spreads and higher unemployment.

The 2007-2009 Great Financial Crisis (GFC) brought attention to the fragility of wholesale funding mechanisms of financial intermediaries. The reliance of leveraged financial institutions on collateralized short-term borrowing, such as repurchase agreements (repo), has become a key feature of the modern financial system. With over $3 trillion in repo markets transacted daily (Baklanova, Caglio, Cipriani, and Copeland 2016), repos allow institutions to manage cash balances, dealers to reduce funding costs by efficiently making markets for various securities, and institutional investors to leverage their market bets. This reliance presents a critical source of instability that manifested itself in 2008 and then again in March 2020 when the Covid-19 pandemic shook financial markets. In both instances, the Federal Reserve was forced to enter money markets and support short-term funding of banks.

A significant portion of the funding of financial institutions takes place in the shadow banking system, which "conduct maturity, credit, and liquidity transformation without explicit access to central bank liquidity or public sector credit guarantees"

(Pozsar, Adrian, Ashcraft, and Boesky 2010). While shadow banking improves market efficiency and provides liquidity in the form of money-like claims for institutions, it also contributes to systemic risk (Gertler, Kiyotaki, and Prestipino 2016). Today's traditional banking system was stabilized through deposit insurance and backing by the public sector, but the shadow banking system relies solely on the solvency of private parties. This risk materialized during the GFC as a system-wide bank run. A core sector of the shadow banking system is the repo market, in which financial institutions can engage in short-term collateralized agreements. Since the 1980s, large banks have become increasingly dependent on funding through repos, meaning that when repo funding dropped in 2008, a massive chunk of their funding was cut off, and they were forced to sell off their more illiquid assets for funds. The "bank run" in the repo market was a run by financial institutions on banks such as Lehman Brothers and Bear Stearns by pulling their repo funding and demanding higher collateralization for new loans (Gorton and Metrick 2009).

Further contributing to the severity of the crisis was the practice of rehypothecation, which is when collateral posted for a loan is reused for a separate loan. Rehypothecation leads to increased leverage in the system and added uncertainty regarding ownership of collateral, as upwards of 10 different institutions may have a claim on a single piece of collateral. Singh and Aitken (2010) found that rehypothecation was widespread in money markets leading up to the GFC. Data gathered on repledged collateral as a multiple of annual revenue from each major bank's 10-K annual filings from 2007 are plotted below in Figure 1. By the end of September 2008, the top repledger, Merrill Lynch had to be rescued from bankruptcy through acquisition. The second highest repledger, Lehman Brothers, ended up bankrupt while Bear Stearns, like Merrill Lynch, needed to be acquired to avoid bankruptcy.

Since 2008, new banking regulations, such as the Basel Leverage Ratio and the Global Systemically Important Bank (G-SIB) capital surcharge, require banks to hold capital against exposures associated with repos and have significantly boosted their

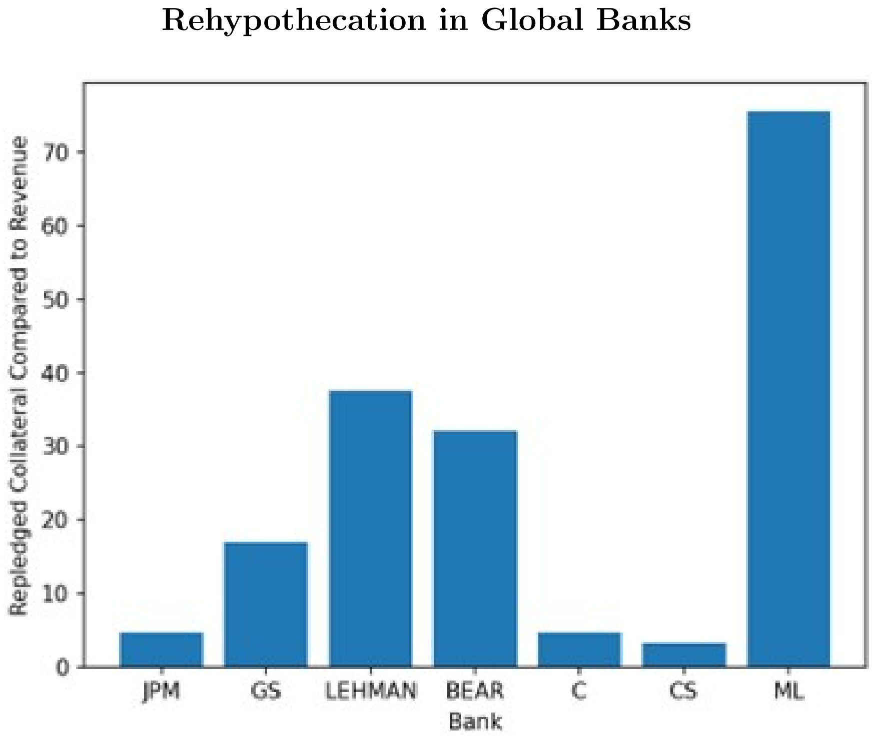

Figure 1: Repledged collateral as a ratio of yearly revenue is graphed for a sample of the largest investment banks in 2007. From left to right, the banks are JP Morgan, Goldman Sachs, Lehman Brothers, Bear Stearns, Citi, Credit Suisse, and Merrill Lynch. Data is collected from the banks' 10-K filings.

safety. However, events such as the repo rates spike in 2019 and the treasury market stresses in March 2020 illustrate how fragility is still present in the system. In both instances, the Fed was forced to step into the repo market and provide funding. In order to support the trillion-dollar repo markets, the Fed offered up to $1 trillion each week for short-term treasuries to stabilize the financial system.

I first develop an extension to an existing infinite horizon, game-theoretic model of short-term collateralized lending to include the possibility for rehypothecation. The model introduces sunspot shocks to steady-state equilibria to derive liquidity and collateral constraints that measure fragility. The model reveals how adding the ability to rehypothecate collateral increases overall utility in the steady state but increases the fragility of these lending markets by making it more difficult for borrowers to survive runs. It also provides insight into financial institutions' choice of whether or not to rehypothecate.

To empirically investigate the relationship between rehypothecation and runs on repos, I employ a structural vector autoregression (VAR) model. The results of the structural VAR model suggest that an increase in the level of rehypothecation is associated with a persistent increase in credit risk in the months following a shock to the level of rehypothecation. This finding supports the view that the practice of rehypothecation in financial markets, such as the repo market, can contribute to the destabilization of financial intermediaries. The structural VAR also suggests that a shock to uncertainty in which volatility rises and investors become more cautious leads to financial institutions cutting down their rehypothecation levels.

Literature Review

In 2008, the "run on repo" came in the form of steep rises in repo haircuts, which dictate the amount an institution can borrow using a certain level of collateral. Haircuts were essentially 0 prior to the crisis, meaning that collateral could be traded at par with cash. However, in 2008, the average haircut for collateral rose to nearly 50%, effectively halving the repo funding available to banks (Gorton and Metrick 2009). The short-term collateralized lending system between banks became insolvent, the shadow banking equivalent of the inability of depositors to withdraw money from banks. The shadow banking run that took place in 2007-2009 bears similarities to traditional bank runs of the 19th and 20th centuries, which are modeled in theoretical literature pioneered by Diamond and Dybvig (1983). This part of the economic literature focuses on liquidity mismatches—the combination of short-term liabilities and illiquid long-term assets on the balance sheet—within banking intermediaries. The liquidity mismatch leads to possible scenarios where all depositors attempt to withdraw their deposits after ex-ante expectations that other depositors will withdraw, reducing bank liquidity and thereby causing bank fragility.

This paper contributes to a strand of literature building upon Diamond and

Dybvig's bank-run model as it applies to shadow banking and collateralized short-term funding. Moreira and Savov (2017) created a shadow banking model, but instead of focusing on bank runs in which depositors withdraw en masse, they found potential for collateral runs, in which investors demand additional collateral for existing loans. However, they did note that a bank run could arise in their model if the collateral is rehypothecated among investors. More recently, the Kuong (2021) bank run model of short-term lending markets demonstrates how risk-taking incentives of firms result in a coordination failure that leads to fire sales of collateral and a systematic run. They focus on bank runs brought about by anticipation of fire sales that lower collateral values, leading to the fire sales themselves. I primarily build my research on the Martin, Skeie, and Thadden (2014) model, which analyzes repo runs on individual firms as a result of uncertainty regarding the liquidity of the borrower. They study differences in the microstructures of the bilateral and tri-party repo markets and derive measures of fragility for the markets. In the bilateral repo market, borrowers and lenders deal directly with one another, and collateral is directly transferred. By contrast, the tri-party repo market has a third-party clearing bank that acts as a central counterparty and manages the transaction. Martin, Skeie, and Thadden (2014) find that the tri-party repo is more fragile due to how bilateral repos allow haircuts to adjust quickly, which enables borrowers to offer contracts with better terms to entice new lenders to shore up any liquidity shortfalls. However, this conclusion contradicts other researchers' empirical findings. Krishnamurthy, Nagel, and Orlov (2012) studied repo quantities in the tri-party market and concluded that runs were smaller in scale than previously thought. Gorton, Laarits, and Metrick (2018) employed data from emergency lending facilities set up by the Federal Reserve to conclude that the rise in bilateral haircuts has high explanatory power over emergency borrowing after the fall of Lehman, signaling a run in the bilateral market.

One explanation for the differing conclusion of the model in Martin, Skeie, and Thadden (2014) to empirical findings could be that they do not account for the practice of collateral rehypothecation prevalent in the bilateral repo, but not possible in tri-party repo due to how a third-party custodian holds the collateral. Following the GFC, there have been strands of both empirical and theoretical research done on rehypothecation. Singh and Aitken (2010), Singh (2011), and Singh (2012) discuss the scale of the practice and how the fall of Lehman led to reduced rehypothecation as financial firms feared their prime brokers becoming insolvent which would lead to them losing their collateral.

The Eren (2015), Infante (2015), and Muley (2016) models of rehypothecation all present it as a way for dealer banks to obtain liquidity through differences in haircuts that they offer and receive. My model merges this idea of rehypothecation with the Martin, Skeie, and Thadden (2014) model of repo bank runs in order to derive new constraints for the bilateral repo that factor in the possibility of rehypothecation.

There has been little work in the literature on employing structural VAR techniques in shadow banking and money markets. I run a structural VAR that models the impact of rehypothecation on tightening credit conditions within large banks and tests if rising uncertainty may lead to reduced rehypothecation by adding contemporaneous relationships and decomposing potentially correlated errors into orthogonal shocks. This methodology is similar to Valenti, Bastianin, and Manera (2022), who run a structural VAR that analyzes crude oil prices, in that I am also using weekly data and implementing contemporaneous restrictions.

Model

In this section, I describe a model of collateralized borrowing, focusing specifically on repos, in which rehypothecation of collateral is possible. My simple model demonstrates how collateral rehypothecation provides a source of liquidity while allowing for greater leverage and ultimately, greater instability in the system. It yields constraints under which runs may occur following sunspot shocks, which allows us to assess the impact of rehypothecation on shadow banking stability. Rehypothecation of collateral would only be permitted under the bilateral repo market, so the model specifically applies to that segment of the repo system.

The Environment

The model has an infinite time horizon and contains 3 types of agents: borrowers, cash investors, and third-party investors. In practice, various financial institutions assume multiple roles from the model, but for the purpose of grounding ideas, the borrowers should be thought of as leveraged hedge funds seeking to borrow in order to maximize profitable investment, the cash investors as dealer banks who seek to earn interest from lending to borrowers, and the third-party investors as money market funds whose priority is to preserve current funds.

Cash investors live for 3 periods and at each date t, there are an N number born. In their first period, "young" investors receive an endowment of 1 unit of goods that they can invest. These investors have a need for cash that is uncertain: they are either "patient" and do not need cash until their old age, or "impatient" and need cash immediately in their middle age. They do not know whether they are patient or impatient cash investors at the time they are born and find out only in their middle age. The information about their type and age is private. With probability α, an investor is impatient and when N is sufficiently large, the overall fraction of impatient agents is also α due to the law of large numbers. These cash investors have no investment projects of their own and must invest their good with the borrower agents to receive a return, r.

Borrowers receive no endowments but have access to investment projects. These securities are assumed to be relatively illiquid compared to other investments within shadow banking markets and cannot be liquidated instantaneously in the model. These investment projects by borrowers yield return $R_i > 1$ two periods after the investment is made and I assume that returns are riskless for simplicity. I also assume that the investments are sufficiently profitable: $\beta^2 R_i > 1$. Borrowers accept funds from the cash investors with the promise to pay interest rate r and offer the ability to request payment either after one or after two periods. These agreements bear similarities to the demand deposit contracts in traditional bank run literature following Diamond and Dybvig (1983).

I assume that the investments yield constant returns to scale up to an exogenous capacity constraint, which can be interpreted as firm size. More specifically, the capacity constraint for each individual firm is $\bar{I}_i$ and the equation below shows the returns when investing I^t at date t:

$$\begin{cases} R_i I^t \text{ if } I^t \leq \bar{I}_i \\ R_i \bar{I}_i \text{ if } I^t \geq \bar{I}_i \end{cases} \tag{1}$$

The total investment capacity of the borrowers, $\bar{I} = \sum_i \bar{I}_i$ is greater than N, the total available cash to be invested by the cash investors. Furthermore, no single borrower is pivotal and even if the largest borrower fails, cash investors continue to compete to invest with the borrowers.

The investment returns of the borrowers are not verifiable to cash investors, so collateral has the role of an enforcement mechanism in this model, consistent with its conventional role in the economic literature. Baklanova, Caglio, Cipriani, and Copeland (2019) studied collateral in the bilateral repo market and found that US treasury collateral is important as an enforcement mechanism.[1] I define k_i as the collateralization rate, the amount of collateral posted per unit of borrowing. In the steady state, the rate of collateralization is $\hat{k}_i$.

As in Martin, Skeie, and Thadden (2014), cash investor agents who end up having to keep the collateral, which occurs in cases where the borrower defaults or is otherwise unable to repurchase it, realize a return of $\gamma_i^t R_i$ such that $\gamma_i^t < 1$, reflecting asymmetries such as transaction costs, timing costs, or different skills in

[1] However, they find that this is not collateral's only role; it also acts as a screening mechanism against adverse selection

valuing assets.

My model allows the cash investors to rehypothecate the collateral they receive from the borrower so that they can now borrow money and expand their potential consumption. Instead of just holding onto the collateral as an enforcement mechanism and it having no other uses for these agents, they can now rehypothecate it and gain access to a new source of funding. Within the bilateral repo market, rehypothecation is very common, with the average collateral security being used in around 4-5 different repo agreements (Michl and Park 2022), which makes my model a more accurate representation of the bilateral market than Martin, Skeie, and Thadden (2014), as they leave out the possibility for reusing collateral.

The new type of agent that I introduce to the model to enable rehypothecation is the third-party investor. These agents' sole priority is to store their endowed good as safely as possible; they represent institutions such as money market funds that have a "do not lose" mandate. The third-party investors live for three periods and seek to maximize their utility in their last period of life. They are endowed with 1 good, but have no investment opportunities and must invest with the cash investors—they have no access to the borrower. This assumption is rooted in two potential explanations connected to the real-world structure of the repo markets. First, institutional investors such as hedge funds are established prime brokerage clients of dealer banks that they borrow from, and there are additional time and transaction costs for these borrowers to seek different sources of funding, especially for large funds with high trading volumes. The borrower can only access their prime broker, which can then in turn rehypothecate the collateral to other institutions. Second, collateral obtained in the bilateral repo is often rehypothecated to other repo markets such as the tri-party repo market. Institutions lending in these other markets can differ from the types of institutions transacting purely in the bilateral market. For instance, tri-party market institutions such as money market funds, central banks, commercial banks, corporations, and asset managers would not transact directly with the types of institutions borrowing from the bilateral repo market.

Similar to the cash investor agents, at every time period t, there are an M number of third-party investors born such that there is an effectively infinite supply of third-party funds. M is sufficiently large enough that there will always be competition to lend to the cash investors. This assumption is rooted in the Pozsar (2014) paper that found that cash managers prioritizing safety have experienced a secular growth in scale. As a result of a systemic shortage in safe government-backed securities, institutions have turned to alternatives such as the repo market, which offers collateralized, short-term contracts that are their next-best option for safe storage of money.

In the model, if the third-party investors do not conduct repos and simply hold onto their endowments, they have an ϵ chance of losing their holdings such that $(1-\epsilon) < 1$. This means that they choose to repo with the cash investors despite receiving 0 interest because they can then consume their full endowment without the chance of losing anything in the steady state equilibrium. The contracts that the third-party and the cash investor agree to can either be for one period or two periods.

Just as borrowers have a capacity constraint that limits their profit, I assume that cash investors also have a capacity constraint that limits their investment to only their endowment of 1. This prevents them from borrowing without restriction from the third-party investors. The capacity constraint of the cash investors is described below and shows the single-period returns when investing n at date t.

$$\begin{cases} n^t r_i \text{ if } n^t \leq 1 \\ r_i \text{ if } n^t \geq 1 \end{cases} \tag{2}$$

Since cash investors are capped at investing up to their endowment, only the impatient cash investors have a purpose for rehypothecation in this setup. In the model without rehypothecation, the impatient cash investors withdraw their funds from the borrower and consume it immediately given that they only care about consumption in their middle age. With rehypothecation possible, instead of withdrawing funds, these impatient investors will opt to repo the collateral they are holding with the third-party investors for a one period contract and consume their newly borrowed money instead. I assume that the collateralization rate of cash investor and third-party investor contracts is equivalent to that of borrower and cash investor contracts, meaning the same piece of collateral can be used by the cash investor to borrow the same amount that the borrower borrowed. This way, impatient cash investors can consume 2 periods of interest rather than the single period of interest in the no rehypothecation model.

To better understand how rehypothecation allows for this elevated consumption, I describe in detail the actions each agent takes over the duration of a single generation of cash investors. At their young age, cash investors all invest their endowment with a borrower and receive $\hat{k_i}$ in collateral. The borrowers take the fresh funds and invest I_i, which will pay off in two periods. In the cash investors' middle age, they find out whether they are impatient or not, with the impatient cash investors now deciding to use the collateral received to borrow r^2 from the third-party investors and consuming that immediately. Both patient and impatient cash investors roll over the contract with the borrower for another period. The third-party investors are willing to lend the money out because in the steady state, they now have a safe way of storing their funds without the ϵ chance of losing it all. Finally, in the cash investors' old age, all cash investors in this generation now receive r^2 from the borrower. Patient investors consume the proceeds while impatient cash investors use the proceeds to repay the third-party investors that they borrowed from in the previous period. These third-party investors have another period to live before they consume so they find other cash investors to invest with. In effect, all cash investors now become patient in the eyes of the borrowers since the impatient investors now have the option to consume borrowed funds and continue rolling over their contract with the borrowers.

With effectively all investors acting patiently in the eyes of the borrowers, the borrowers' expected cash flow (profits) in equilibria with no bank runs is:

$$\pi_i^t = R_i I_i^{t-2} + b_i^t - r_{2i}^{t-2} b_i^{t-2} - I_i^t + C_i^{t-1} - C_i^t \tag{3}$$

Where $R_i I_i^{t-2}$ represents the return on an investment project two periods ago, b_i^t represents the borrowing from new investors, $-r_{2i}^{t-2} b_i^{t-2}$ represents the payment to all investors in their old age, $-I_i^t$ represents new investments made, and $C_i^{t-1} - C_i^t$ is the change in cash carried over from the previous period that needs to be accounted for.

At each date, the borrowers consume their profit, π_i^t. If $\pi_i^t < 0$ at any date, then the borrower is bankrupt. The borrowers' objective is to maximize $\sum_{\tau=t}^{\infty} \beta^{(\tau-t)} \pi_i^{\tau}$ where β is the discount rate.

Steady State Equilibria

In our model, a steady state equilibrium is a set of conditions and decisions by the agents such that none of the agents would prefer another state, given the observed behavior of all other agents.

Borrowers always want to invest up to their capacity constraint to maximize their profit each period. They must decide how to fund that investment, whether through their own funds (profit that they have made, $\pi_i > 0$) or by borrowing from cash investors. I derive a "dynamic participation constraint" which determines the interest rate that the borrower pays the cash investors such that the borrowers are indifferent between using borrowed funds or their own profits. A borrower's steady state expected discounted profit on a unit of borrowed money is $\beta^2(R_i - r^2)$ while their profit on a unit of their own money is $\beta^2 R_i - 1$. Since competition drives interest rates up to where the two rates of return are equal, the dynamic participation constraint is $\beta^2(R_i - r^2) = \beta^2 R_i - 1$, which solves to $\bar{r} = \frac{1}{\beta}$ where $\bar{r}$ is the market rate at equilibrium. With interest rates for cash investors at $\bar{r} = \frac{1}{\beta}$, borrowers are indifferent between investing their own funds and borrowing.

In the steady equilibria,

- all investors roll over their loans with their borrowers
- all impatient investors take out a loan with the third-party investors for middle period liquidity needs
- investment is maximal: $I_i = \bar{I}_i$
- borrowers hold no cash: $C_i = 0$
- all borrowers pay same interest rate $\bar{r} = \frac{1}{\beta}$
- all investor cash is lent
- all borrowers make positive profits equal to $R_i I_i^{t-2} + b_i^t - r_{2i}^{t-2} b_i^{t-2} - I_i^t$

We focus only on the interesting equilibria with positive profits, although it is important to note that there are possible zero-profit equilibria in which borrowers are excessively leveraged and would like to reduce borrowing but cannot because they need to repay past loans.[2]

Run Scenarios

In this model, runs are triggered by sunspots, which are extrinsic shocks that happen at random and cannot be anticipated by any agents. It could be interpreted as an arbitrary shift in expectations where investors in the repo market become fearful of their borrowers' stability and suddenly attempt to withdraw the contracts. To analyze the fragility of these markets, I derive liquidity and collateral constraints, which are conditions under which sunspots never lead to bankruptcy for any given borrower. If both the liquidity and collateral constraints are violated, then a run is possible. In the event of a run, borrowers cannot meet obligations with their own

[2] In the appendix, I show this more formally.

funds as their investment returns are still incoming, and they are unable to raise more funding from young investors through the posting of more collateral.

The liquidity constraint is the condition for which borrowers have enough funds to cover obligations under a run scenario. In order to derive the liquidity constraint, first notice that a run would have no consequence if:

$$(R_i - 1)\bar{I}_i \geq (\bar{r}^2 + \bar{r})b_i \tag{4}$$

because this means the borrower can repay obligations to all investors, $(\bar{r}^2+\bar{r})b_i$, while maintaining full investment. In scenarios where the borrower is able to survive, but cannot maintain full investment, $\bar{I}_i$, then they will need to reduce their investment until they are able to meet their obligations. They would continue to seek to maximize investment by investing all available cash. A surviving borrower's cash position, and therefore investment, I_0, is:

$$I_0 = R_i\bar{I}_i - (\bar{r}^2 + \bar{r})b_i \tag{5}$$

If $I_0 \geq 0$ but (4) doesn't hold, the borrower is able to survive a run but cannot maintain full investment. As long as $I_0 \geq 0$, the borrower has enough funds to meet obligations; it is the liquidity constraint. $I_0 \geq 0$ implies that $R_i\bar{I}_i \geq (\bar{r}^2 + \bar{r})b_i$, which leads to the formal liquidity constraint below:[3]

$$\beta^2 R_i\bar{I}_i \geq (1 + \beta)b_i \tag{6}$$

If a borrower satisfies this liquidity constraint, then a run is not possible because their lenders have no strict incentives to run; the borrower will have enough liquidity to cover any withdrawals. This liquidity constraint is just as the Martin, Skeie, and Thadden (2014) model, but the percentage of impatient investors, α, is 0; every

[3] I show the full derivation in the appendix.

investor is patient in the eyes of the borrowers. This is more binding than that of Martin et al's model, $\beta^2 R_i \bar{I}_i \geq (1-\alpha+\beta)b_i$, because borrowers need to pay impatient investors for the full two periods; they are more leveraged and have more outstanding borrowing at any given period. The liquidity constraint has now become harder to satisfy, signaling increased fragility.

I also derive a collateral constraint, which is the condition under which cash investors do not run because they have sufficient collateral protecting them in the event of their borrower going bankrupt. While the liquidity constraint found the conditions where borrowers have the funds to fend off a run, the collateral constraint captures situations where the borrower lacks funds to pay off running investors, i.e., $I_0 < 0$, but they can offer better terms to new borrowers to shore up missing funds and survive bankruptcy. As Martin, Skeie, and Thadden (2014) noted, a key feature of the bilateral market is the ability to quickly change collateralization rate margins. To find the amount of collateral that a borrower can offer new investors to satisfy any shortfall in cash and avoid bankruptcy in a run, I first find the shortfall of cash, m_i:[4]

$$m_i = (\bar{r} + \bar{r}^2)b_i - R_i\bar{I}_i \tag{7}$$

At the time of pledging collateral, the borrower has $\bar{I}_i$ in investments that they can send to investors as collateral. To try to draw in fresh funds, the maximum collateralization rate, $\bar{k_i}$, that borrowers can offer in this run scenario would be:

$$\bar{k_i} = \frac{\bar{I}_i}{m_i} \tag{8}$$

Borrowers offer as much collateral as they are able to for the newly-born, young cash investors to draw in enough fresh funds to avoid bankruptcy. I now consider a young cash investor's payoffs to examine their decision-making process when considering

[4] m_i is equivalent to $-I_0$.

whether or not to accept the borrower's offering of $\bar{k_i}$ in collateral for their investment. The cash investor's payoffs are outlined below and show what they expect to receive in the following period.

		Other Cash Investors	
		Invest	Don't
Young Investor	Invest	$\hat{r_i}$	$\gamma_i^t R_i \bar{k_i}$
	Don't	$\bar{r}$	$\bar{r}$

(9)

The bottom row of the table shows the returns that the young investor would receive if they decided to invest with a different borrower, $\bar{r}$, which is the common market return. The top row is the returns they would receive by investing in that particular borrower, which depends on the actions of other cash investors. If other investors in that borrower invest, then they would all receive the interest offered by that borrower, $\hat{r_i}$. If the cash investor invests but others run, then they would keep the collateral and receive the reduced return that the investor can obtain from the collateral, $\gamma_i^t R_i \bar{k_i}$. These are young investors, so no rehypothecation has taken place yet. The run outcome (don't,don't) is only not a strict equilibrium if:

$$\gamma_i^t R_i \bar{k_i} \geq \bar{r} \tag{10}$$

because the offered collateralization rate, $\bar{k_i}$ is high enough such that even if the cash investor must consume the collateral, they end up having at least as much as if they had just invested with another borrower and received $\bar{r}$. Plugging (7) into (8) into (10) yields the collateral constraint below because this is when young cash investors decide to invest with a borrower upon being offered the best terms that the borrower is able to offer:[5]

$$\beta^2 R_i \bar{I}_i \geq \frac{1+\beta}{1+\gamma_i^t \beta}(b_i) \tag{11}$$

A run occurs if both the liquidity constraint, (6) and collateral constraint, (11) are violated. Martin et al's original tri-party collateral constraint is $\beta^2 R_i \bar{I}_i \geq \frac{(1-\alpha+\beta)}{\gamma_i^t(1+\beta)} b_i$ while their bilateral collateral constraint is $\beta^2 R_i \bar{I}_i \geq \frac{(1-\alpha+\beta)}{1+\gamma_i^t \beta} b_i$. Upon adding rehypothecation, both the liquidity constraint and collateral constraint have become more difficult to satisfy for the bilateral repo market, signaling increased fragility in the system. Whereas their model has the tri-party collateral constraint strictly more fragile than that of the bilateral, upon allowing rehypothecation, the bilateral collateral constraint is now more fragile than the tri-party collateral when $\frac{1+\beta}{1+\gamma_i^t \beta} > \frac{1-\alpha+\beta}{\gamma_i^t + \gamma_i^t \beta}$. This simplifies to the following equation that describes conditions for when the bilateral constraint with rehypothecation have become harder to satisfy than that of the tri-party.

$$\alpha > \frac{1+\beta-\gamma_i^t-\gamma_i^t \beta}{1+\gamma_i^t \beta} \tag{12}$$

In sum, after rehypothecation is allowed, both the liquidity and collateral constraints in the bilateral repo market have become harder to meet, and in certain conditions, are now more difficult to meet than the Martin, Skeie, and Thadden (2014) model's tri-party collateral constraint. These results shift Martin et al's model results to better match other empirical papers' findings that the bilateral repo is more fragile.

Extended Model With Greater Cash Investor Capacity

In the previously described model, we set the capacity constraint for the cash investors at their endowment, meaning that only the impatient investors had any purpose for rehypothecation and these investors all engaged in one period repos with the third-party investors. Here, we set the capacity constraint to be c, such that c is greater than the endowment, i.e., $c > 1$. With greater investment capacity, all cash

[5] The full derivation is found in the appendix.

investors are now incentivized to rehypothecate to maximize their investments with the borrowers. Whereas before rehypothecation fulfilled the sole purpose of providing liquidity for impatient investors, rehypothecation can now provide increased leverage for the cash investors as well. Furthermore, since cash investors are rehypothecating in their young age, they now engage in two-period contracts with the third-party rather than one-period contracts that the impatient investors engaged in.

With this change, all cash investors will rehypothecate collateral immediately upon receiving it in their first period and use their newly borrowed funds to engage in new contracts with the borrowers up to their capacity constraint, c. The patient cash investors behave similarly as before, investing c while young through rehypothecation, rolling over the loan in their middle age, then consuming cr_i^2 when old. Impatient cash investors will also rehypothecate and invest c while young. Then in their middle age, they will borrow cr_i^2 from third-party investors through additional rehypothecation and consume that immediately. In their old age, once they receive two periods of interest on their investment, i.e., cr_i^2 from the borrower, they will repay the third-party investors. The borrowers' updated expected profits are now:

$$\pi_i^t = R_i I_i^{t-2} + b_i^t - r_i^{t-2} b_i^{t-2} - I_i^t + d_i^t - r_i^{t-2} d_i^{t-2} + C_i^{t-1} - C_i^t \tag{13}$$

This is the same as before with the addition of new borrowing from all young cash investors with rehypothecated collateral, d_i where $d_i = N(c-1)$, and the repayment of that contract two periods later. In the steady state equilibrium, cash investors can now increase their investments, more third-party investors are now able to keep their money with the cash investors and avoid the possibility of losing it, and borrowers are indifferent given the dynamic participation constraint. Overall, there is greater utility in the system with all agents being at least indifferent if not better off in a steady state equilibrium with increased leverage.

I now derive new liquidity and collateral constraints in run scenarios given this change. It is now more difficult to have a run with no consequence:

$$(R_i - 1)\bar{I}_i \geq (\bar{r}^2 + \bar{r})b_i + (\bar{r}^2 + \bar{r})d_i \tag{14}$$

Borrowers have to pay an additional $(\bar{r}^2 + \bar{r})d_i$ in run scenarios because they are more leveraged due to the increased funding all young cash investors provide as a result of their own leveraging up. Reflecting this, the new investment, I_0, of borrowers is:

$$I_0 = R_i\bar{I}_i - (\bar{r}^2 + \bar{r})b_i - (\bar{r}^2 + \bar{r})d_i \tag{15}$$

As before, if $I_0 \geq 0$ but (14) doesn't hold, then the borrower needs to adjust their investments to survive a run. $I_0 \geq 0$ is the liquidity constraint and occurs when

$$\beta^2 R_i\bar{I}_i \geq (1 + \beta)b_i + (1 + \beta)d_i \tag{16}$$

This liquidity constraint is now both more binding than that of Martin et al's model and more binding than my base model where only impatient investors rehypothecate.

To derive the new collateral constraint, the shortfall of cash for borrowers who don't satisfy the liquidity constraint in a run is:

$$m_i = (\bar{r} + \bar{r}^2)b_i + (\bar{r}^2 + \bar{r})d_i - R_i\bar{I}_i \tag{17}$$

As before, the max value of collateral per unit borrowed that these borrowers are able to offer new investors would be:

$$\bar{k}_i = \frac{\bar{I}_i}{m_i} \tag{18}$$

Using the payoff table (9), the same as the base model, cash investors decide to invest if:

$$\gamma_i^t R_i\bar{k}_i \geq \bar{r} \tag{19}$$

Plugging (17) into (18) into (19) yields the new collateral constraint,

$$\beta^2 R_i \bar{I}_i \geq \frac{1+\beta}{1+\gamma_i^t \beta}(b_i + d_i) \tag{20}$$

Allowing a greater capacity constraint for cash investors and having rehypothecation be optimal for both patient and impatient investors leads to even tighter liquidity and collateral constraints. Note that the larger the capacity constraint c, is, the larger d_i is, making it more difficult to satisfy the liquidity and collateral constraints and increasing fragility.

In this extension, another source of fragility comes from the possibility of the third-party investor running on the cash investors since they are now engaged in two-period contracts rather than just one as before. If we assume that the third-party investors, like the cash investors, realize a reduced return from consuming the collateral as compared to the borrowers: $\gamma_i R_i \hat{k}_i$ such that consuming the collateral is less valuable than the expected consumption from just holding cash and risking ϵ: $\gamma_i R_i \hat{k}_i < (1-\epsilon)$, then the third-party investors may be incentivized to run in sunspots. A run in this case would be defined as a sunspot shock causing the third-party investors to be fearful that they will have to keep the collateral, so they close their contract with the cash investors and request repayment in their middle, rather than their late, period of life. In that scenario, the cash investors would have to pull funds from the borrowers in order to fulfill repayment. In effect, the third-party investors running forces the cash investors to run as well, increasing the potential sunspots that lead to systematic failures.

Also important to note, while cash investors would typically rehypothecate the collateral when young, in runs, the cash investors would make the decision to not rehypothecate the collateral they receive because in the event that the borrower cannot repay, the cash investor ends up with nothing as they are no longer holding the collateral. When runs occur in this extension, rehypothecation conducted falls.

The primary conclusion that can be drawn from the model described is that rehypothecation of collateral can increase fragility by making it more difficult for borrowers to survive runs. An additional observation coming from the extended model is that when there is increased uncertainty during sunspot shocks, rehypothecation falls as financial institutions may opt to hold their collateral rather than rehypothecate it.

Empirical Analysis

In this section, I empirically test conclusions of my model through structural VARs. Specifically, I test the finding that rehypothecation can be a source of fragility by analyzing the impact of rehypothecation shocks on credit conditions, which can be an indicator of fragility. I also empirically test the finding that uncertainty shocks can reduce rehypothecation activity. I use a measure of the level of rehypothecation and the CBOE Volatility Index (VIX), which represents the market's expectations of volatility over the next 30 days, to capture the causal effects of the sunspot shock from the theoretical model on real-world measures of fragility. I estimate the impacts of these shocks by setting short-run restrictions on the variables, which will allow us to construct relevant impulse responses and interpret causal effects.

Data

The sample for my initial analysis comprises weekly data for a measure of rehypothecation using a method I describe below, the TED spread—the interest rate spread between a 3-month LIBOR and a 3-month Treasury Bill, and the VIX from July 4, 2001 to March 27, 2013.

I measure rehypothecation with the ratio of repos to reverse repos conducted by primary dealers.[6] This is the same data source for measuring rehypothecation as Infante, Press, and Saravay (2020), who argue that primary dealers are representative

[6] The list of primary dealers during the 2008 Financial Crisis can be found in the Appendix.

of the overall financial system because they are at the core of many financial markets, often as market makers. I choose to measure rehypothecation through a ratio of repo to reverse repo because in the theoretical model, there is increased rehypothecation if the cash investor decides to reuse their collateral by initiating another repo contract, meaning that when there are increased repos relative to the funds lent out, which are termed reverse repos, then there is higher rehypothecation. In the real world, much of the rehypothecation is conducted between the primary dealers themselves but eventually, the collateral is passed outside of primary dealers to institutional cash investors through repos. When the ratio of repos to reverse repos for primary dealers is higher, this indicates that dealers are receiving more funding through repos, which would mean greater rehypothecation as much of the collateral held by dealers come from reverse repo arrangements. Although rehypothecation between dealers are not captured in this measure, increased repo funding from non-dealer counterparties would likely mean there is more inter-dealer repo funding, in which rehypothecation would be used more frequently. I plot my rehypothecation variable below in Figure 2, which shows rising rehypothecation levels in the years leading up to the GFC, signaling increases in leverage hand-in-hand with the greater economy. When the GFC began, my measure of rehypothecation dropped sharply, which is consistent with findings in Singh and Aitken (2010).

I use the TED spread as a proxy for credit conditions in interbank lending for my structural VAR. My theoretical model shows how rehypothecation can increase fragility in short-term lending between financial institutions, so I expect increases in the TED spread following increases in rehypothecation.

The VIX is the measure of volatility in financial markets implied by stock index options and is used as a proxy for uncertainty in my model, as stock market volatility is found to be correlated with cross-sectional measures of uncertainty (Bloom 2009). Analyzing uncertainty will allow us to test the finding from the theoretical model that rising uncertainty lowers rehypothecation. The sunspot shocks can be compared to

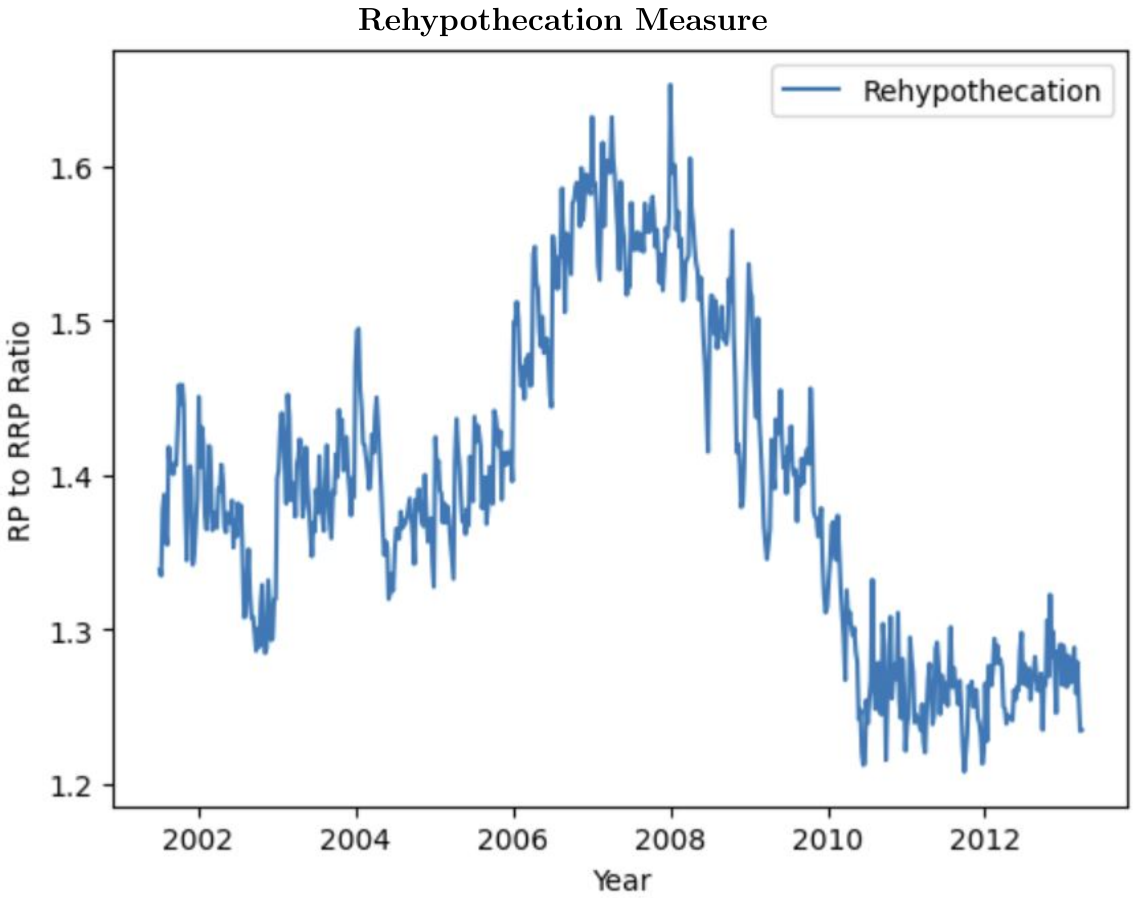

Figure 2: Ratio of repo agreements to reverse repo agreements by primary dealers is graphed over the time period of the sample.

uncertainty shocks because the suddenly elevated uncertainty by investors regarding their investments is what leads them to consider whether or not they should run. During these periods of uncertainty, the investors will stop rehypothecating collateral in order to avoid the possibility that their borrower fails, leaving them with no repayment on their investment or collateral to salvage.[7]

Structural VAR Model

In order to perform the desired causal analysis, the shocks imposed on the structural VAR model must first be orthogonal, i.e., the shocks should be serially uncorrelated

[7]The TED spread and VIX data are obtained from the Federal Reserve Economic Data (FRED) managed by the St. Louis Fed. I gathered data on primary dealers' collateral dealings from the Federal Reserve Bank of New York's primary dealer statistics.

with one another and second, have economic meaning, i.e., the shocks can be interpreted in a real-world setting and are an important driver of macroeconomic conditions.

The structural VARs that I employ fulfill the first condition of uncorrelated shocks by implementing short-term, contemporaneous restrictions. They also fulfill the second condition of economic meaning because rehypothecation is a significant source of leverage in the financial system and helps satisfy the massive demand for collateral. A potential rehypothecation shock can be interpreted as a sudden increase in demand for cash by a large financial institution, resulting in a need to borrow more money than usual. This could be due to various factors such as unexpected losses or a liquidity crisis.

My structural VAR can be represented by the following equation:

$$A_0 y_t = \sum_{j=1}^{12} B_j y_{t-j} + \epsilon_t \tag{21}$$

where A_0 is the $n \times n$ matrix of contemporaneous relationships that will allow us to conduct a Cholesky decomposition to obtain the uncorrelated shocks, the column y_t is a 3×1 column vector of the variables of interest ordered *VIX-TED-Rehypothecation*, B_j are the matrices of structural coefficients of the lags, and ϵ_t is the vector of serially uncorrelated shocks. The model incorporates 12 lagged values, about 3 months, which is consistent with the number of lags employed by other papers running structural VARs on weekly data such as in Valenti, Bastianin, and Manera (2022).

The restrictions follow a recursive scheme. Using the ordering of variables stated above, I assume that the VIX can impact the TED spread and the level of rehypothecation immediately, but the TED spread and rehypothecation have no impact on the VIX contemporaneously. I order rehypothecation last because primary dealers observe the TED and VIX and can then immediately adjust their rehypothecation levels within a week while rehypothecation most likely will impact the TED and VIX only later. I ordered the VIX before TED because an uncertainty shock could lead to immediate changes in interest rate spreads as investors adjust to the new volatility.

These relations allow us to decompose the errors to allow us to identify the structural shocks of interest:

$$\begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u^{VIX} \\ u^{TED} \\ u^{Rehypothecation} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon^{VIX} \\ \epsilon^{TED} \\ \epsilon^{Rehypothecation} \end{bmatrix} \tag{22}$$

Here, a 0 indicates a restriction placed while a_{ij} specifies the contemporaneous response of the variable corresponding to i to a shock of a variable corresponding to j.

Results

I plot the estimated impulse response functions in the initial structural VAR in Figure 3 below. I use 1-standard error bands corresponding to 68% confidence intervals per Sims and Zha (1999), who argue that one standard error bands may convey more information about the shape of the responses than two standard error bands.

Figure 3 shows that a positive rehypothecation shock leads to an initial fall in the TED spread for a few weeks but then a prolonged increase for at least 4 more years, suggesting that higher rehypothecation may indeed lead to weaker credit conditions for financial institutions. The short-term dip is likely a result of anticipatory movements in which the primary dealers have their own economic models that forecast credit conditions, which impacts their level of rehypothecation, which in turn affects credit conditions with a few weeks' lag. I address these anticipatory movements in the following section. The longer-term tightening of credit following rehypothecation is consistent with the results of the theoretical model as it indicates that rehypothecation elevates fragility in short-term lending markets.

Figure 3 impulse responses also show a strong impact of a VIX shock on rehypothecation. The plot shows a highly significant drop in rehypothecation for years following elevated

Impulse Responses of VIX, TED, Rehypothecation

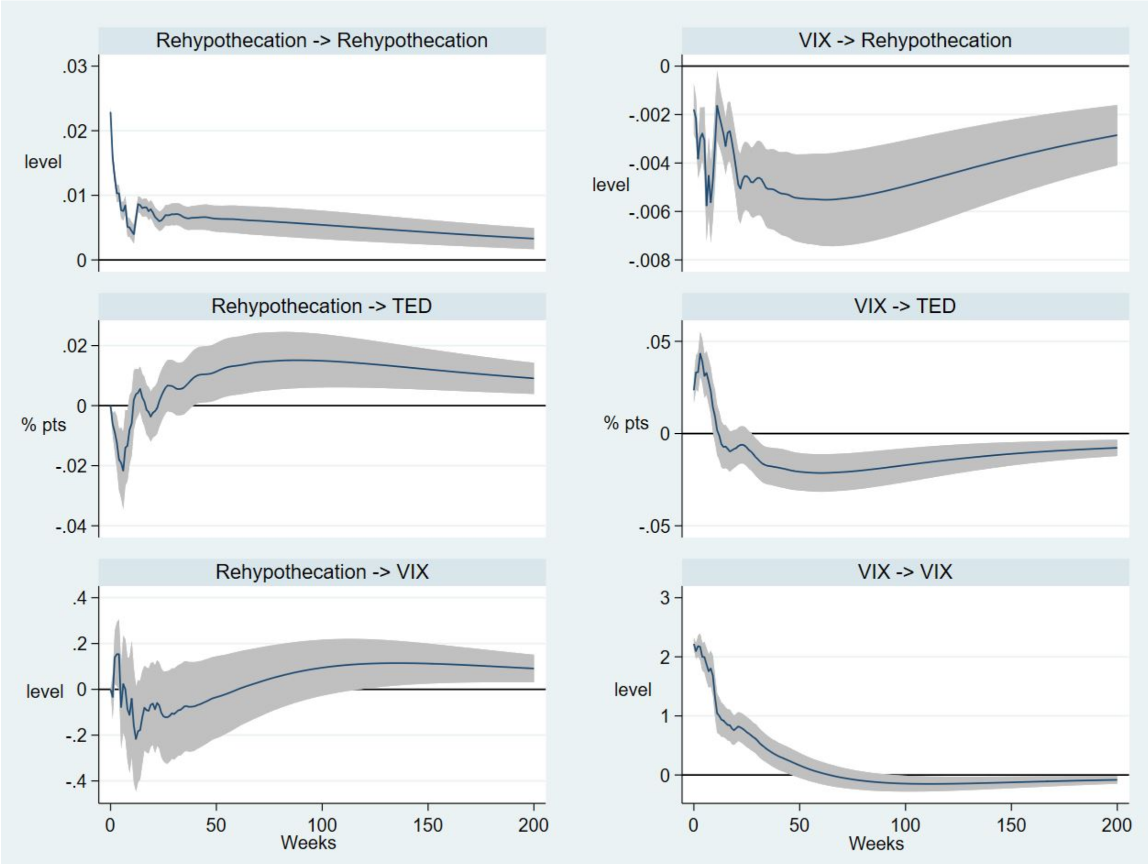

Figure 3: Impulse responses of rehypothecation, TED, and VIX to a one standard deviation rehypothecation shock are graphed on the left column while responses of these variables to a one standard deviation VIX shock are graphed on the right. These responses are plotted for 200 weeks following the shocks.

uncertainty. This is consistent with the model because in run scenarios where the cash investors become fearful that their loans will not be repaid, they will stop rehypothecating their collateral as they would be left with nothing if their loans go unpaid and they have pledged the collateral to another party.

Extended Empirical Analysis

Additional Variables

In order to rectify anticipatory movements and observe additional impacts of rehypothecation on potential recessions, I run a more comprehensive structural VAR with two additional variables: stock values and unemployment claims. By controlling for stock market values, we capture market expectations of future economic conditions reflected in current stock prices since stocks are forward-looking. The variable $Stocks$ is the log-transformed values of the Wilshire 5000 Total Market Index. I use the Wilshire 5000 index over alternatives such as the S&P 500 because the Wilshire 5000 is more comprehensive and is found to be the best index for a US stock market proxy (French 2017).

We are also interested in analyzing causal impacts that rehypothecation may have on recessions because the increased money market fragility from rehypothecation that our theoretical model predicts should negatively impact lending activity by large banks, which is a core driver of the real economy. We use the variable $Unemp$, which measures unemployment claims weekly, to analyze the recessionary impacts of rehypothecation levels.

We keep the recursive short-term restrictions from the initial structural VAR and add the two new variables in the order *(VIX, TED, Unemp, Stocks, Rehypothecation).* I order unemployment and stock values before rehypothecation because they are also economic indicators that primary dealers can observe as they decide their rehypothecation levels. The stock value variable in particular controls for any anticipatory movements for rehypothecation as well. Unemployment is ordered before stocks because stocks can instantly adjust to new unemployment values while unemployment takes time to adjust to changes in the stock market.

I again decompose the errors to allow us to identify the structural shocks of interest:

$$
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{bmatrix} \begin{bmatrix} u^{VIX} \\ u^{TED(\%)} \\ u^{Unemp} \\ u^{Stocks} \\ u^{Rehypothecation} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon^{VIX} \\ \epsilon^{TED(\%)} \\ \epsilon^{Unemp} \\ \epsilon^{Stocks} \\ \epsilon^{Rehypothecation} \end{bmatrix} \quad (23)
$$

Results

From the extended structural VAR's impulse responses in Figure 4, we see that with stock market values controlled for to remove anticipatory movements, the negative effect that rehypothecation had on the TED spread for the first few weeks following a shock has diminished a bit, but still exists. The causal impact of higher rehypothecation on weaker credit conditions has become less persistent, lasting for just under 2 years.

We also observe a positive impact of rehypothecation on unemployment beginning around 2.5 years after a shock, suggesting that rehypothecation may contribute to recessions.

As in the initial structural VAR above, a rise in the VIX lowers rehypothecation levels. However, the new impulse response now finds that this effect subsides around 3 years after the shock, which makes more sense since rehypothecation should recover and return to normal after the uncertainty has worn off and dealers feel safe enough to rehypothecate collateral once again.

In sum, the empirical data of impulse responses confirm findings in the model that rehypothecation increases banking fragility, which we observe with the tightening credit conditions. They also show how with elevated uncertainty, rehypothecation by primary dealers falls. Lastly, the positive effect rehypothecation seems to have on unemployment claims in the extended VAR further supports the greater risk of bank

Impulse Responses of VIX, TED, Unemployment, Stocks, Rehypothecation

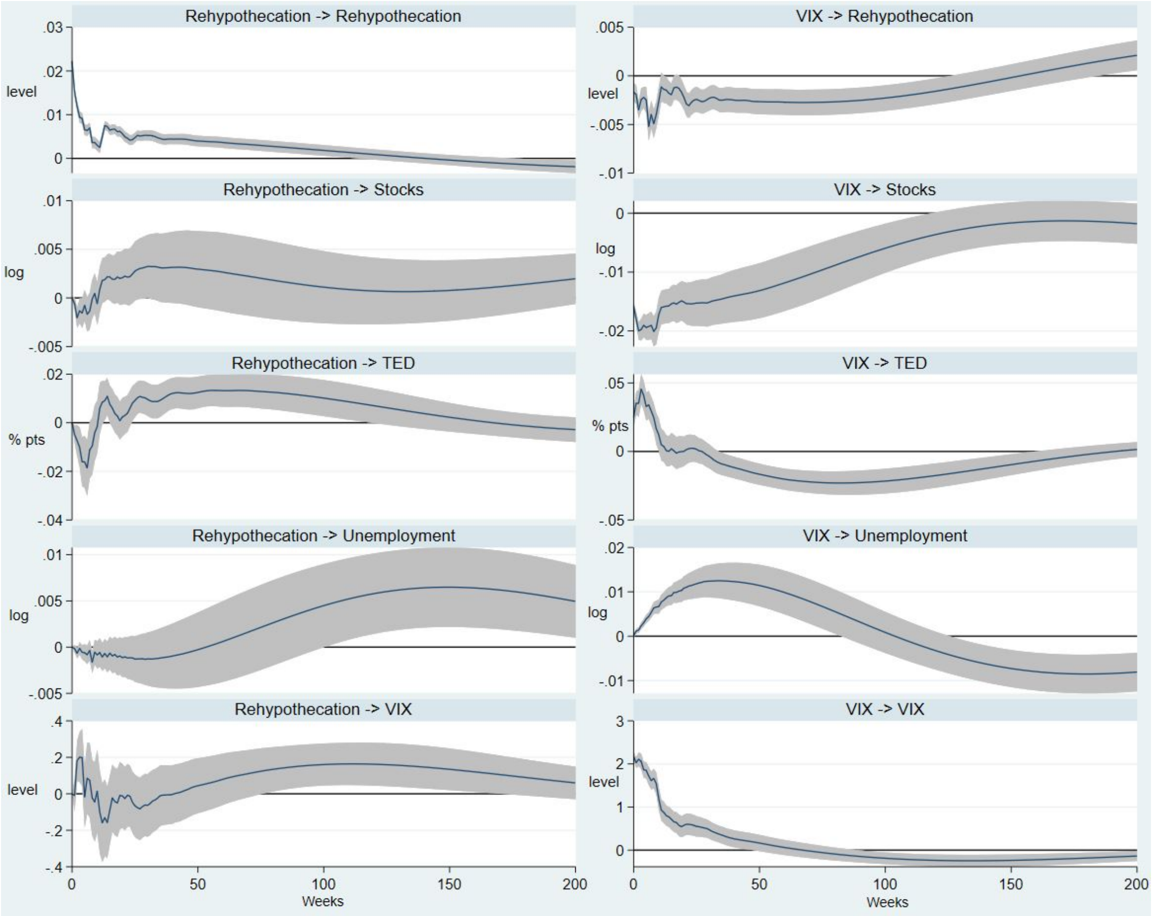

Figure 4: Impulse responses of rehypothecation, stocks, TED, unemployment, and VIX to a one standard deviation rehypothecation shock are graphed on the left column while responses of these variables to a one standard deviation VIX shock are graphed on the right. Again, responses are plotted for 200 weeks following the shocks.

runs that the model suggests.

Conclusion

The goal of this thesis is to analyze the impact that rehypothecation of collateral may have on banking stability. While existing literature has modeled bank runs in short-term interbank lending and rehypothecation of repo collateral separately, I incorporate rehypothecation into an existing theoretical model of repo runs and demonstrate how that increases fragility. While Martin, Skeie, and Thadden (2014) find that the tri-party repo market is more fragile than the bilateral market, I show that with rehypothecation considered, the bilateral market becomes more fragile, a conclusion that better fits existing empirical literature.

I also employ structural VARs to empirically support the model's finding that rehypothecation is a driver of fragility. From the results of impulse response analysis, I find that rehypothecation shocks lead to tighter credit conditions about a year after the shock and higher unemployment about 2.5 years after a shock. The impulse responses also confirm an implication of the theoretical model that rehypothecation levels fall when there is suddenly elevated uncertainty among bank dealers.

The rehypothecation and uncertainty shocks utilized in the structural VAR are all derived from the same sunspot shock in the theoretical model. While I find that with rehypothecation involved that there are additional possibilities for sunspots through the third-party investors, sunspots are still a zero-probability event in the model. Further research could incorporate stochasticity into the theoretical model to better understand uncertainty shocks.

While repo markets are the primary application of this research, the results can be generalized to other secured lending markets in which rehypothecation occurs. For instance, another potential application of my research could be the runs on crypto exchanges in 2022, notably FTX's collapse. The crypto exchange was found to be rehypothecating collateral used to back loans to their clients, which contributed to the eventual bank run once the value of their collateral dropped.

The results of this thesis suggest that greater oversight and constraints on rehypothecation of collateral may help prevent future banking crises. In the United States, Rule 15c3–3, known as the Consumer Protection Rule, limits a broker-dealer from rehypothecating more than 140% of their customer's debit balance. However, in the United Kingdom, there are no restrictions on rehypothecation, which is a key reason why many hedge funds, particularly fixed-income and macro funds, have opted for funding in Europe rather than the United States (Singh and Aitken 2010).

MF Global, a global financial broker, exploited this difference in supervision to rehypothecate client funds as collateral for repo funding in Europe. However, in 2011 they faced a repo run that led to a liquidity crisis and ultimately, bankruptcy. Tightening international discrepancies in the legality of rehypothecation, increasing supervision, and extending regulations to new markets such as cryptocurrencies are policies that may improve banking stability and reduce off-balance sheet agreements that elevate risk.

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Appendix

Zero-Profit Equilibria Description

In zero-profit equilibria, only some borrowers are active, they borrow more than they can invest ($b_i > I_i$), and investment is not optimal ($I_i < \bar{I}_i$). The active borrowers here would prefer to reduce their borrowing, but are forced to keep it higher than investment in order to repay previous loans. Therefore, these borrowers have choice but to borrow more to prevent bankruptcy, leading to zero profits.

Lemma: If $\pi_i = 0$, then $b_i > I_i$

Proof: $b_i > 0$ because if $b_i = 0$ then $\pi_i > 0$ by the profit function (3). Since there is borrowing and I assumed that borrowers are at least as well off by borrowing than using their own funds, the amount gained from investing one unit of borrowed funds must be at least as great as the payoff: $\beta^2 R_i \geq \beta^2 r^2$. This is equivalent to $\beta^2(R_i - r^2) \geq 0$ or $(R_i - r^2) \geq 0$.

Suppose that $b_i \leq I_i$. Then

$$\pi_i = (R_i - 1)I_i - (r^2 - 1)b_i$$
$$\pi_i \geq (R_i - r^2)I_i$$

Since $r = \frac{1}{\beta}$ and by assumption, $\beta^2 R_i > 1$, then $\pi_i > 0$ which is a contradiction.

Liquidity Constraint Derivation

The liquidity constraint covers scenarios where borrowers have the capacity to survive runs, which is when they have funds to invest more than 0, $I_0 \geq 0$. From the model section above, when $I_0 \geq 0$, then

$$R_i \bar{I}_i \geq (\bar{r}^2 + \bar{r})b_i$$
$$R_i \bar{I}_i \geq (1/\beta^2 + 1/\beta)b_i$$
$$\beta^2 R_i \bar{I}_i \geq (1 + \beta)b_i$$

Therefore, $\beta^2 R_i \bar{I}_i \geq (1 + \beta)b_i$ is the liquidity constraint for borrowers.

Collateral Constraint Derivation

I start with equation (10) and then plug in equations (7) and (8):

$$\gamma_i^t R_i \bar{k}_i \geq \bar{r}$$

$$\gamma_i^t R_i (\frac{\bar{I}_i}{m_i}) \geq \bar{r}$$

$$\gamma_i^t R_i (\frac{\bar{I}_i}{(\bar{r}+\bar{r}^2)b_i - R_i \bar{I}_i}) \geq \bar{r}$$

$$\gamma_i^t R_i \bar{I}_i \geq \bar{r}((\bar{r}+\bar{r}^2)b_i - R_i \bar{I}_i)$$

$$\gamma_i^t R_i \bar{I}_i \geq \frac{1}{\beta}((\frac{1}{\beta}+\frac{1}{\beta^2})b_i - R_i \bar{I}_i)$$

$$\beta\gamma_i^t R_i \bar{I}_i \geq (\frac{1}{\beta}+\frac{1}{\beta^2})b_i - R_i \bar{I}_i$$

$$(1+\beta\gamma_i) R_i \bar{I}_i \geq (\frac{1}{\beta}+\frac{1}{\beta^2})b_i$$

$$\beta^2(1+\beta\gamma_i) R_i \bar{I}_i \geq (\beta+1)b_i$$

$$\beta^2 R_i \bar{I}_i \geq \frac{1+\beta}{1+\gamma_i^t\beta}(b_i)$$

Therefore, $\beta^2 R_i \bar{I}_i \geq \frac{1+\beta}{1+\gamma_i^t\beta}(b_i)$ is the collateral constraint for borrowers.

List of Primary Dealers For Repo Data (As of July 15, 2008)

BNP Paribas Securities Corp.

Banc of America Securities LLC

Barclays Capital Inc.

Bear, Stearns & Co., Inc.

Cantor Fitzgerald & Co.

Citigroup Global Markets Inc.

Credit Suisse Securities (USA) LLC

Daiwa Securities America Inc.

Deutsche Bank Securities Inc.

Dresdner Kleinwort Securities LLC

Goldman, Sachs & Co.

Greenwich Capital Markets, Inc.

HSBC Securities (USA) Inc.

J. P. Morgan Securities Inc.

Lehman Brothers Inc.

Merrill Lynch Government Securities Inc.

Mizuho Securities USA Inc.

Morgan Stanley & Co. Incorporated

UBS Securities LLC.