Physics 16 Problem Set 12 Solutions

Y&F Problems

15.18. **IDENTIFY:** Apply $\sum F_y = 0$ to determine the tension at different points of the rope. $v = \sqrt{F/\mu}$.

SET UP: From Example 15.3, $m_{\text{samples}} = 20.0$ kg, $m_{\text{rope}} = 2.00$ kg and $\mu = 0.0250$ kg/m

EXECUTE: **(a)** The tension at the bottom of the rope is due to the weight of the load, and the speed is the same $88.5\,\text{m/s}$ as found in Example 15.3.

(b) The tension at the middle of the rope is $(21.0\ \text{kg})(9.80\,\text{m/s}^2) = 205.8$ N and the wave speed is $90.7\,\text{m/s}$.

(c) The tension at the top of the rope is $(22.0\ \text{kg})(9.80\ \text{m/s}^2) = 215.6\ \text{m/s}$ and the speed is 92.9 m/s. (See Challenge Problem (15.82) for the effects of varying tension on the time it takes to send signals.)

EVALUATE: The tension increases toward the top of the rope, so the wave speed increases from the bottom of the rope to the top of the rope.

15.41. **IDENTIFY:** Compare $y(x,\ t)$ given in the problem to Eq.(15.28). From the frequency and wavelength for the third harmonic find these values for the eighth harmonic.

(a) SET UP: The third harmonic standing wave pattern is sketched in Figure 15.41.

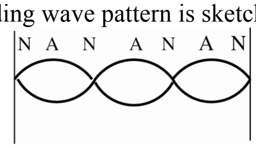

Figure 15.41

EXECUTE: **(b)** Eq. (15.28) gives the general equation for a standing wave on a string:

$y(x,\ t) = (A_{\text{SW}} \sin kx)\sin \omega t$

$A_{\text{SW}} = 2A$, so $A = A_{\text{SW}}/2 = (5.60\ \text{cm})/2 = 2.80\ \text{cm}$

(c) The sketch in part (a) shows that $L = 3(\lambda/2)$. $k = 2\pi/\lambda$, $\lambda = 2\pi/k$

Comparison of $y(x,\ t)$ given in the problem to Eq. (15.28) gives $k = 0.0340$ rad/cm. So,

$\lambda = 2\pi/(0.0340\ \text{rad/cm}) = 184.8\ \text{cm}$

$L = 3(\lambda/2) = 277\ \text{cm}$

(d) $\lambda = 185$ cm, from part (c)

$\omega = 50.0$ rad/s so $f = \omega/2\pi = 7.96$ Hz

period $T = 1/f = 0.126$ s

$v = f\lambda = 1470$ cm/s

(e) $v_y = dy/dt = \omega A_{\text{SW}} \sin kx \cos \omega t$

$v_{y,\ \text{max}} = \omega A_{\text{SW}} = (50.0\ \text{rad/s})(5.60\ \text{cm}) = 280\ \text{cm/s}$

(f) $f_3 = 7.96\ \text{Hz} = 3f_1$, so $f_1 = 2.65$ Hz is the fundamental

$f_8 = 8f_1 = 21.2$ Hz; $\omega_8 = 2\pi f_8 = 133$ rad/s

$\lambda = v/f = (1470\ \text{cm/s})/(21.2\ \text{Hz}) = 69.3\ \text{cm}$ and $k = 2\pi/\lambda = 0.0906$ rad/cm

$y(x,\ t) = (5.60\ \text{cm})\sin([0.0906\ \text{rad/cm}]x)\sin([133\ \text{rad/s}]t)$

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. In the 8th harmonic the frequency and wave number are larger than in the 3rd harmonic.

15.54. **IDENTIFY:** The maximum vertical acceleration must be at least g.

SET UP: $a_{\text{max}} = \omega^2 A$

EXECUTE: $g = \omega^2 A_{\text{min}}$ and thus $A_{\text{min}} = g/\omega^2$. Using $\omega = 2\pi f = 2\pi v/\lambda$ and $v = \sqrt{F/\mu}$, this becomes $A_{\text{min}} = \dfrac{g\lambda^2\mu}{4\pi^2 F}$.

EVALUATE: When the amplitude of the motion increases, the maximum acceleration of a point on the rope increases.

15.68. IDENTIFY: The time between positions 1 and 5 is equal to $T/2$. $v = f\lambda$. The velocity of points on the string is given by Eq.(15.9).

SET UP: Four flashes occur from position 1 to position 5, so the elapsed time is $4\left(\dfrac{60\text{ s}}{5000}\right) = 0.048\text{ s}$. The figure in the problem shows that $\lambda = L = 0.500\text{ m}$. At point P the amplitude of the standing wave is 1.5 cm.

EXECUTE: **(a)** $T/2 = 0.048\text{ s}$ and $T = 0.096\text{ s}$. $f = 1/T = 10.4\text{ Hz}$. $\lambda = 0.500\text{ m}$.

(b) The fundamental standing wave has nodes at each end and no nodes in between. This standing wave has one additional node. This is the 1st overtone and 2nd harmonic.

(c) $v = f\lambda = (10.4\text{ Hz})(0.500\text{ m}) = 5.20\text{ m/s}$.

(d) In position 1, point P is at its maximum displacement and its speed is zero. In position 3, point P is passing through its equilibrium position and its speed is $v_{\text{max}} = \omega A = 2\pi f A = 2\pi(10.4\text{ Hz})(0.015\text{ m}) = 0.980\text{ m/s}$.

(e) $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{FL}{m}}$ and $m = \dfrac{FL}{v^2} = \dfrac{(1.00\text{ N})(0.500\text{ m})}{(5.20\text{ m/s})^2} = 18.5\text{ g}$.

EVALUATE: The standing wave is produced by traveling waves moving in opposite directions. Each point on the string moves in SHM, and the amplitude of this motion varies with position along the string.

15.35. IDENTIFY: Evaluate $\partial^2 y/\partial x^2$ and $\partial^2 y/\partial t^2$ and see if Eq.(15.12) is satisfied for $v = \omega/k$.

SET UP: $\dfrac{\partial}{\partial x}\sin kx = k\cos kx$. $\dfrac{\partial}{\partial x}\cos kx = -k\sin kx$. $\dfrac{\partial}{\partial t}\sin\omega t = \omega\cos\omega t$. $\dfrac{\partial}{\partial t}\cos\omega t = -\omega\sin\omega t$

EXECUTE: **(a)** $\dfrac{\partial^2 y}{\partial x^2} = -k^2\left[A_{\text{sw}}\sin\omega t\right]\sin kx$, $\dfrac{\partial^2 y}{\partial t^2} = -\omega^2\left[A_{\text{sw}}\sin\omega t\right]\sin kx$, so for $y(x,t)$ to be a solution of Eq.(15.12), $-k^2 = \dfrac{-\omega^2}{v^2}$, and $v = \dfrac{\omega}{k}$.

(b) A standing wave is built up by the superposition of traveling waves, to which the relationship $v = \lambda/k$ applies.

EVALUATE: $y(x,t) = (A_{\text{SW}}\sin kx)\sin\omega t$ is a solution of the wave equation because it is a sum of solutions to the wave equation.

15.21. IDENTIFY: For a point source, $I = \dfrac{P}{4\pi r^2}$ and $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$.

SET UP: $1\ \mu\text{W} = 10^{-6}\text{ W}$

EXECUTE: **(a)** $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (30.0\text{ m})\sqrt{\dfrac{10.0\text{ W/m}^2}{1\times10^{-6}\text{ W/m}^2}} = 95\text{ km}$

(b) $\dfrac{I_2}{I_3} = \dfrac{r_3^2}{r_2^2}$, with $I_2 = 1.0\ \mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\dfrac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\ \mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\text{ W/m}^2)(4\pi)(30.0\text{ m})^2 = 1.1\times10^5\text{ W}$

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.