Physics 16 – Spring 2010 – Problem Set 4

5.62. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each object . Constant speed means $a = 0$.

SET UP: The free-body diagrams are sketched in Figure 5.62. T_1 is the tension in the lower chain, T_2 is the tension in the upper chain and $T = F$ is the tension in the rope.

EXECUTE: The tension in the lower chain balances the weight and so is equal to w. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to w and the tension in the rope, which equals F, is $w/2$. Then, the downward force on the upper pulley due to the rope is also w, and so the upper chain exerts a force w on the upper pulley, and the tension in the upper chain is also w.

EVALUATE: The pulley combination allows the worker to lift a weight w by applying a force of only $w/2$.

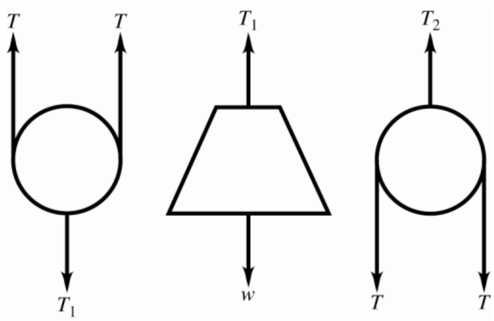
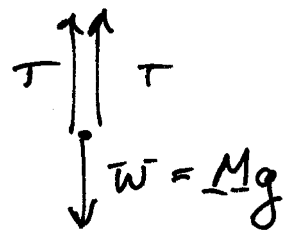


Figure 5.62

5.62 addendum:

FBD for weight + its pulley is (for tension $T = F$)

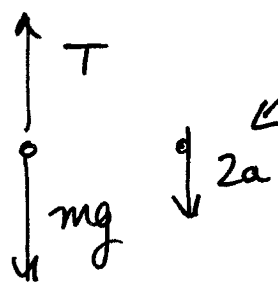


$$2T - Mg = Mg$$

$$\boxed{T = \frac{1}{2}M(a+g)}, \text{ or}$$

$$T = \frac{1}{2}W\left(1 + \frac{a}{g}\right)$$

If you supply this T with a hanging mass (m), remember that $T \neq mg$ b/c the mass m is <u>not</u> in static equilibrium:
FBD for hanging mass:



acceleration is <u>twice</u> the acc^n of W because of the free pulley.

<u>So</u>

$$T - mg = m(-2a)$$

$$T = m(g - 2a). \quad \text{Comparing to } T \text{ above:}$$

$$\frac{1}{2}M(a+g) = T = m(g - 2a)$$

$$\Rightarrow \boxed{m = \frac{\frac{1}{2}M(a+g)}{g - 2a}} \quad \left(\text{where } M = W/g\right).$$

This works for $a = 0$ (the book's question): $m(a=0) = \dfrac{\frac{1}{2}Mg}{g} = \frac{1}{2}M$ ✓.

5.65. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: Constant speed means $a = 0$. When the blocks are moving, the friction force is f_k and when they are at rest, the friction force is f_s.

EXECUTE: (a) The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g = (m_1 g \sin\alpha + \mu_k m_1 g \cos\alpha)$ and $m_2 = m_1(\sin\alpha + \mu_k \cos\alpha)$.

(b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2 g = (m_1 g \sin\alpha - \mu_k m_1 g \cos\alpha)$, or $m_2 = m_1(\sin\alpha - \mu_k \cos\alpha)$.

(c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha + \mu_s \cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha - \mu_s \cos\alpha)$.

EVALUATE: In parts (a) and (b) the friction force changes direction when the direction of the motion of m_1 changes. In part (c), for the largest m_2 the static friction force on m_1 is directed down the incline and for the smallest m_2 the static friction force on m_1 is directed up the incline.

5.66. IDENTIFY: The system is in equilibrium. Apply Newton's 1st law to block A, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

(a) SET UP: The free-body diagram for the hanging block is given in Figure 5.66a.

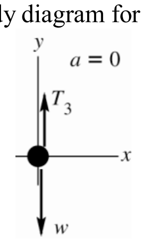


Figure 5.66a

EXECUTE:

$\sum F_y = ma_y$

$T_3 - w = 0$

$T_3 = 12.0 \text{ N}$

SET UP: The free-body diagram for the knot is given in Figure 5.66b.

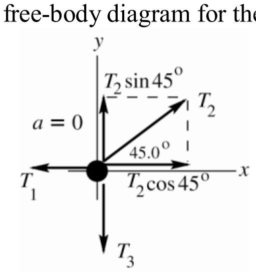


Figure 5.66b

EXECUTE:

$\sum F_y = ma_y$

$T_2 \sin 45.0° - T_3 = 0$

$T_2 = \dfrac{T_3}{\sin 45.0°} = \dfrac{12.0 \text{ N}}{\sin 45.0°}$

$T_2 = 17.0 \text{ N}$

$\sum F_x = ma_x$

$T_2 \cos 45.0° - T_1 = 0$

$T_1 = T_2 \cos 45.0° = 12.0 \text{ N}$

SET UP: The free-body diagram for block A is given in Figure 5.66c.

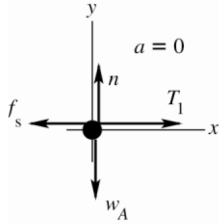


Figure 5.66c

EXECUTE:

$\sum F_x = ma_x$

$T_1 - f_s = 0$

$f_s = T_1 = 12.0 \text{ N}$

EVALUATE: Also can apply $\sum F_y = ma_y$ to this block:

$$n - w_A = 0$$
$$n = w_A = 60.0\ \text{N}$$

Then $\mu_s n = (0.25)(60.0\ \text{N}) = 15.0\ \text{N}$; this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n$; for this value of w the static friction force can hold the blocks in place.

(b) SET UP: We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0\ \text{N}$. Then $T_1 = f_s = 15.0\ \text{N}$.

EXECUTE: From the equations for the forces on the knot

$T_2 \cos 45.0° - T_1 = 0$ implies $T_2 = T_1 / \cos 45.0° = \dfrac{15.0\ \text{N}}{\cos 45.0°} = 21.2\ \text{N}$

$T_2 \sin 45.0° - T_3 = 0$ implies $T_3 = T_2 \sin 45.0° = (21.2\ \text{N}) \sin 45.0° = 15.0\ \text{N}$

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0\ \text{N}$.

EVALUATE: Compared to part (a), the friction is larger in part (b) by a factor of (15.0/12.0) and w is larger by this same ratio.

5.92. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: Use coordinates where $+x$ is directed down the incline.

EXECUTE: (a) Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; *i.e.*, the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\ \text{kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\ \text{kg})a$, or $11.11\ \text{N} - T = (4.00\ \text{kg})a$, and similarly for the larger, $15.44\ \text{N} + T = (8.00\ \text{kg})a$. Adding these two relations, $26.55\ \text{N} = (12.00\ \text{kg})a$, $a = 2.21\ \text{m/s}^2$.

(b) Substitution into either of the above relations gives $T = 2.27\ \text{N}$.

(c) The string will be slack. The 4.00-kg block will have $a = 2.78\ \text{m/s}^2$ and the 8.00-kg block will have $a = 1.93\ \text{m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

EVALUATE: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger μ_k.

5.94. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the ball, to m_1 and to m_2

SET UP: The free-body diagrams for the ball, m_1 and m_2 are given in Figures 5.94a-c. All three objects have the same magnitude of acceleration. In each case take the direction of $\vec{a}$ to be a positive coordinate direction.

EXECUTE: (a) $\sum F_y = ma_y$ applied to the ball gives $T\cos\theta = mg$. $\sum F_x = ma_x$ applied to the ball gives $T\sin\theta = ma$. Combining these two equations to eliminate T gives $\tan\theta = a/g$.

(b) $\sum F_x = ma_x$ applied to m_2 gives $T = m_2 a$. $\sum F_y = ma_y$ applied to m_1 gives $m_1 g - T = m_1 a$.

Combining these two equations gives $a = \left(\dfrac{m_1}{m_1 + m_2}\right) g$. Then $\tan\theta = \dfrac{m_1}{m_1 + m_2} = \dfrac{250\ \text{kg}}{1500\ \text{kg}}$ and $\theta = 9.46°$.

(c) As m_1 becomes much larger than m_2, $a \to g$ and $\tan\theta \to 1$, so $\theta \to 45°$.

EVALUATE: The device requires that the ball is at rest relative to the platform; any motion swinging back and forth must be damped out. When $m_1 << m_2$ the system still accelerates, but with small a and $\theta \to 0°$.

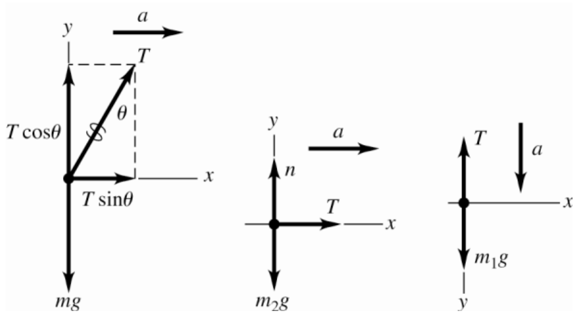


Figure 5.94a–c

5.99. IDENTIFY and SET UP: The monkey and bananas have the same mass and the tension in the rope has the same upward value at the bananas and at the monkey. Therefore, the monkey and bananas will have the same net force and hence the same acceleration, in both magnitude and direction.
EXECUTE: (a) For the monkey to move up, $T > mg$. The bananas also move up.
(b) The bananas and monkey move with the same acceleration and the distance between them remains constant.
(c) Both the monkey and bananas are in free fall. They have the same initial velocity and as they fall the distance between them doesn't change.
(d) The bananas will slow down at the same rate as the monkey. If the monkey comes to a stop, so will the bananas.
EVALUATE: None of these actions bring the monkey any closer to the bananas.

5.120. (included because it is referenced in the solution to 5.121. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the block and to the wedge.
SET UP: For both parts, take the x-direction to be horizontal and positive to the right, and the y-direction to be vertical and positive upward. The normal force between the block and the wedge is n; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is A, and the components of acceleration of the block are a_x and a_y.
EXECUTE: (a) The equations of motion are then $MA = -n\sin\alpha$, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$. Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y. An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is $\frac{a_y}{a_x - A} = -\tan\alpha$. At this point, algebra is unavoidable. A possible approach is to eliminate a_x by noting that $a_x = -\frac{M}{m}A$, using this in the kinematic constraint to eliminate a_y and then eliminating n. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+m)\tan\alpha}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

See how nasty the algebra is in this one? Aren't you glad I didn't ask you to do this one on the problem set?

(b) When $M >> m, A \to 0$, as expected (the large block won't move). Also,

$$a_x \to \frac{g}{\tan\ \alpha + (1/\tan\alpha)} = g\frac{\tan\ \alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha$$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha$.

(c) The trajectory is a spiral.

EVALUATE: If $m >> M$, our general results give $a_x = 0$ and $a_y = -g$. The massive block accelerates straight downward, as if it were in free-fall.

5.121. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: From Problem 5.120, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$ for the block. $a_y = 0$ gives $a_x = g\tan\alpha$.

EXECUTE: If the block is not to move vertically, both the block and the wedge have this horizontal acceleration and the applied force must be $F = (M + m)a = (M + m)g\tan\alpha$.

EVALUATE: $F \to 0$ as $\alpha \to 0$ and $F \to \infty$ as $\alpha \to \infty$.