Physics 16 Problem Set 11 Solutions

Y&F Problems

13.27. **IDENTIFY:** Conservation of energy says $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ and Newton's second law says $-kx = ma_x$.

SET UP: Let $+x$ be to the right. Let the mass of the object be m.

EXECUTE: $k = -\frac{ma_x}{x} = -m\left(\frac{-8.40\text{ m/s}^2}{0.600\text{ m}}\right) = (14.0\text{ s}^{-2})m$.

$A = \sqrt{x^2 + (m/k)v^2} = \sqrt{(0.600\text{ m})^2 + \left(\frac{m}{[14.0\text{ s}^{-2}]m}\right)(2.20\text{ m/s})^2} = 0.840\text{ m}$. The object will therefore travel $0.840\text{ m} - 0.600\text{ m} = 0.240\text{ m}$ to the right before stopping at its maximum amplitude.

EVALUATE: The acceleration is not constant and we cannot use the constant acceleration kinematic equations.

13.41. **IDENTIFY:** $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = 0.25$ s.

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on L and g.

13.57. **IDENTIFY** and **SET UP:** Use Eq.(13.43) to calculate ω', and then $f' = \omega'/2\pi$.

(a) EXECUTE: $\omega' = \sqrt{(k/m) - (b^2/4m^2)} = \sqrt{\frac{2.50\text{ N/m}}{0.300\text{ kg}} - \frac{(0.900\text{ kg/s})^2}{4(0.300\text{ kg})^2}} = 2.47$ rad/s

$f' = \omega'/2\pi = (2.47\text{ rad/s})/2\pi = 0.393$ Hz

(b) IDENTIFY and **SET UP:** The condition for critical damping is $b = 2\sqrt{km}$ (Eq.13.44)

EXECUTE: $b = 2\sqrt{(2.50\text{ N/m})(0.300\text{ kg})} = 1.73$ kg/s

EVALUATE: The value of b in part (a) is less than the critical damping value found in part (b). With no damping, the frequency is $f = 0.459$ Hz; the damping reduces the oscillation frequency.

13.64. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$. The period changes when the mass changes.

SET UP: M is the mass of the empty car and the mass of the loaded car is $M + 250$ kg.

EXECUTE: The period of the empty car is $T_E = 2\pi\sqrt{\frac{M}{k}}$. The period of the loaded car is

$T_L = 2\pi\sqrt{\frac{M + 250\text{ kg}}{k}}$. $k = \frac{(250\text{ kg})(9.80\text{ m/s}^2)}{4.00\times10^{-2}\text{ m}} = 6.125\times10^4$ N/m

$M = \left(\frac{T_L}{2\pi}\right)^2 k - 250\text{ kg} = \left(\frac{1.08\text{ s}}{2\pi}\right)^2 (6.125\times10^4\text{ N/m}) - 250\text{ kg} = 1.56\times10^3\text{ kg}$.

$T_E = 2\pi\sqrt{\frac{1.56\times10^3\text{ kg}}{6.125\times10^4\text{ N/m}}} = 1.00\text{ s}$.

EVALUATE: When the mass decreases, the period decreases.

13.68. IDENTIFY: In SHM, $a_{\text{max}} = \dfrac{k}{m_{\text{tot}}}A$. Apply $\sum \vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\sum F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{\text{max}} = \left(\dfrac{k}{M+m}\right)A$. Set $a_{\text{max}} = a$ and solve for A: $\mu_s g = \left(\dfrac{k}{M+m}\right)A$ and $A = \dfrac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.