Physics 16 – Spring 2010 – Problem Set 5

5.104. **IDENTIFY:** The block has acceleration $a_{rad} = v^2/r$, directed to the left in the figure in the problem. Apply $\sum \vec{F} = m\vec{a}$ to the block.

SET UP: The block moves in a horizontal circle of radius $r = \sqrt{(1.25\text{ m})^2 - (1.00\text{ m})^2} = 0.75\text{ m}$. Each string makes an angle θ with the vertical. $\cos\theta = \dfrac{1.00\text{ m}}{1.25\text{ m}}$, so $\theta = 36.9°$. The free-body diagram for the block is given in Figure 5.104. Let $+x$ be to the left and let $+y$ be upward.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $T_u\cos\theta - T_l\cos\theta - mg = 0$.

$$T_l = T_u - \frac{mg}{\cos\theta} = 80.0\text{ N} - \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 31.0\text{ N}.$$

(b) $\sum F_x = ma_x$ gives $(T_u + T_l)\sin\theta = m\dfrac{v^2}{r}$.

$$v = \sqrt{\frac{r(T_u + T_l)\sin\theta}{m}} = \sqrt{\frac{(0.75\text{ m})(80.0\text{ N} + 31.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 3.53\text{ m/s}.$$ The number of revolutions per second is $\dfrac{v}{2\pi r} = \dfrac{3.53\text{ m/s}}{2\pi(0.75\text{ m})} = 0.749\text{ rev/s} = 44.9\text{ rev/min}$.

(c) If $T_l \to 0$, $T_u\cos\theta = mg$ and $T_u = \dfrac{mg}{\cos\theta} = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 49.0\text{ N}$. $T_u\sin\theta = m\dfrac{v^2}{r}$.

$$v = \sqrt{\frac{rT_u\sin\theta}{m}} = \sqrt{\frac{(0.75\text{ m})(49.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 2.35\text{ m/s}.$$ The number of revolutions per minute is

$$(44.9\text{ rev/min})\left(\frac{2.35\text{ m/s}}{3.53\text{ m/s}}\right) = 29.9\text{ rev/min}$$

EVALUATE: The tension in the upper string must be greater than the tension in the lower string so that together they produce an upward component of force that balances the weight of the block.

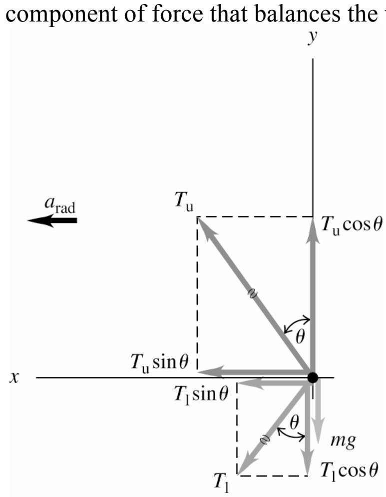

Figure 5.104

5.115. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the bead. Also use Eq.(5.16) to relate a_{rad} to the period of rotation T.

SET UP: The bead and hoop are sketched in Figure 5.115a.

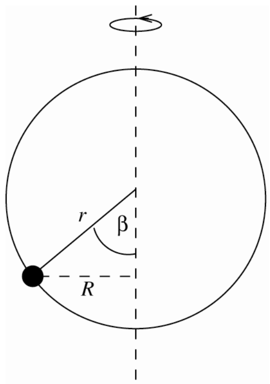

Figure 5.115a

The bead moves in a circle of radius $R = r\sin\beta$.

The normal force exerted on the bead by the hoop is radially inward.

The free-body diagram for the bead is sketched in Figure 5.115b.

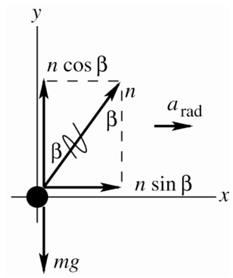

Figure 5.115b

EXECUTE:

$\sum F_y = ma_y$

$n\cos\beta - mg = 0$

$n = mg/\cos\beta$

$\sum F_x = ma_x$

$n\sin\beta = ma_{rad}$

Combine these two equations to eliminate n:

$$\left(\frac{mg}{\cos\beta}\right)\sin\beta = ma_{rad}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{a_{rad}}{g}$$

$a_{rad} = v^2/R$ and $v = 2\pi R/T$, so $a_{rad} = 4\pi^2 R/T^2$, where T is the time for one revolution.

$R = r\sin\beta$, so $a_{rad} = \dfrac{4\pi^2 r\sin\beta}{T^2}$

Use this in the above equation: $\dfrac{\sin\beta}{\cos\beta} = \dfrac{4\pi^2 r\sin\beta}{T^2 g}$

This equation is satisfied by $\sin\beta = 0$, so $\beta = 0$, or by

$\dfrac{1}{\cos\beta} = \dfrac{4\pi^2 r}{T^2 g}$, which gives $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$

(a) 4.00 rev/s implies $T = (1/4.00)\text{ s} = 0.250\text{ s}$

Then $\cos\beta = \dfrac{(0.250\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})}$ and $\beta = 81.1°$.

(b) This would mean $\beta = 90°$. But $\cos 90° = 0$, so this requires $T \to 0$. So β approaches $90°$ as the hoop rotates very fast, but $\beta = 90°$ is not possible.

(c) 1.00 rev/s implies $T = 1.00\text{ s}$

The $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$ equation then says $\cos\beta = \dfrac{(1.00\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})} = 2.48$, which is not possible. The only way to have the $\sum \vec{F} = m\vec{a}$ equations satisfied is for $\sin\beta = 0$. This means $\beta = 0$; the bead sits at the bottom of the hoop.

EVALUATE: $\beta \to 90°$ as $T \to 0$ (hoop moves faster). The largest value T can have is given by $T^2 g/(4\pi^2 r) = 1$ so $T = 2\pi\sqrt{r/g} = 0.635$ s. This corresponds to a rotation rate of $(1/0.635)$ rev/s $= 1.58$ rev/s. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

5.118. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the car. It has acceleration $\vec{a}_{\text{rad}}$, directed toward the center of the circular path.

SET UP: The analysis is the same as in Example 5.24.

EXECUTE: **(a)** $F_A = m\left(g + \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 + \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

(b) $F_B = m\left(g - \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 - \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N.}$, where the minus sign indicates that the track pushes *down* on the car. The magnitude of this force is 30.4 N.

EVALUATE: $|F_A| > |F_B|$. $|F_A| - 2mg$.