Hypothesis Test/CI Example for the Difference in Two Proportions

Renting a living space is something almost everyone does at some point in their lives. How does the rental atmosphere change if you have AIDS and mention it? A study in 1999 investigated this question by having an adult male caller respond to 2 random samples of 80 advertisements for room for rent. For the first sample, he did not allude to having AIDS, and for the second, he alluded that he was in the hospital, about to be released and needed a place to live. For the first random sample (no AIDS reference), the caller was told a room was available in 61 calls. For the second random sample (AIDS reference), the caller was told a room was available in 32 calls. Is there sufficient evidence to conclude that a reference to receiving treatment for AIDS substantially decreases the likelihood of a room being stated as available? Use a significance level of .01, and be sure to discuss any possible issues with conditions necessary for carrying out the test. $\alpha = .01$

Hypotheses:

$\mathcal{H}_0: p_1 = p_2$

$\mathcal{H}_A: p_1 < p_2$

(equivalent to $p_1 - p_2 = 0$, $p_1 - p_2 < 0$)

p_1 = proportion of rooms available to rent with AIDs reference

p_2 = proportion of rooms available to rent without AIDs reference

Conditions/comments:

1. Need 2 $\perp$ groups. Yes, from what we can tell.
2. Randomization - Yes from what is described.
3. $n_1\hat{p}_1 = 32$ $\quad n_2\hat{p}_2 = 61$ $\quad$ All ≥ 10. ✓ yes.

   $n_1(1-\hat{p}_1) = 48$ $\quad n_2(1-\hat{p}_2) = 19$

   $\hat{p}_1 = .4$ $\quad \hat{p}_2 = .7625$ $\quad n_1 = n_2 = 80$

Test statistic and p-value computations:

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1+n_2} = \frac{32+61}{160} = \frac{93}{160} = .58125$$

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} = \frac{.4 - .7625}{\sqrt{\frac{.58125(.41875)(2)}{80}}} = \frac{-.3625}{.078006} = -4.6471 \approx -4.65$$

only b/c $n_1 = n_2$ here

p-value $= P(z < -4.65) \approx 0$ $\quad$ 0 to four decimal places

Reject $\mathcal{H}_0$

Conclusion:

We have significant evidence that the proportion of rooms available with an AIDs reference is lower than the proportion available with no reference.

Interpret your test statistic. $z = -4.65$

This tells us our observed difference in sample proportions (-.3625) is 4.65 null standard errors below 0.

This is a very unusual result if H_0 is true.

Find a 98% confidence interval for the difference in proportions. Is this equivalent to your hypothesis test above? Explain. (Conditions were checked above.)

98% CI $z^* = 2.326$

$$\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \Rightarrow .4 - .7625 \pm (2.326)\sqrt{\frac{.4(.6)}{80} + \frac{.7625(}{80}}$$

$$\Rightarrow -.3625 \pm 2.326\sqrt{.003 + .002264}$$

$$-.3625 \pm 2.326(.07255)$$

$$-.3625 \pm .1688 \Rightarrow (-.5313, -.1937)$$

98% is consistent with a one-sided test @ $\alpha = .01$

Interpret your confidence interval.

We are 98% confident the true difference in proportions of rooms available (AIDs wf - no AIDs wf) lies in (-.5313, -.1937).

Is your confidence interval consistent with your hypothesis test? Explain.

Yes. The entire CI is below 0 and we already determined 98% CI is equivalent to $\alpha = .01$ for a one-sided test.

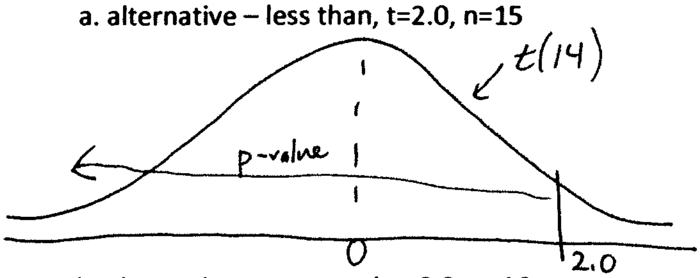
p-value Practice

Portion of Table T in the Appendix D: Probabilities to the RIGHT of t or 2-sided probabilities

Two-tail	.20	.10	.05	.02	.01
Df/ One-tail	.10	.05	.025	.01	.005
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921

For a-c below, draw the picture corresponding to the p-value and bracket it based on the table. Assume all are hypothesis tests about a single population mean, with alternative and n as specified. You can also determine whether or not to reject the null hypothesis using significance level .05.

a. alternative – less than, t=2.0, n=15



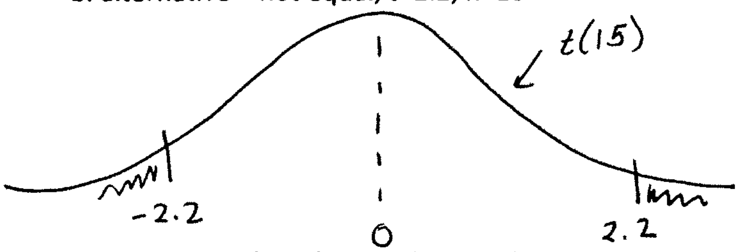
2 is btw 1.761 and 2.145

p-value is btw 1 − .05 and 1 − .025

i.e. btw .95 and .975 ⇒ Huge

Do Not Reject H_0

b. alternative – not equal, t=2.2, n=16

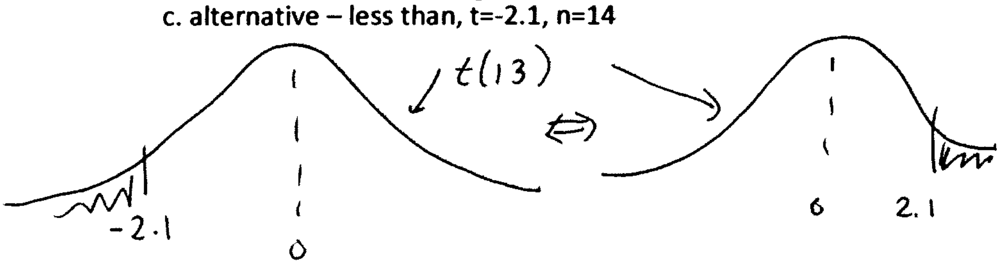


2.2 is btw 2.131 and 2.602

So p-value is btw .02 and .05

Reject H_0

c. alternative – less than, t=-2.1, n=14



2.1 is btw 1.771 and 2.16

very close to 2.160

p-value is btw .025 and .

Reject H_0

You can also find the critical values – t* for confidence intervals. Bottom row of the Table has the confidence levels (you should be able to figure these out yourselves also), and row above that is z*!

d. What would the t* be for a 90% confidence interval for a population mean when n=15?

90% ⇒ .10   2 sided   df = 14 $t^* = 1.761$

e. How about for a 98% CI for a population mean when n=14?

98% ⇒ .02   2-sided   df = 13 $t^* = 2.650$

Hypothesis Test Example for a Population Mean (Based on related question in Moore)

An environmentalist group collects a liter of water from each of 40 locations along a stream and measures the amount of dissolved oxygen in each specimen (recorded in milligrams). Perform a hypothesis test to determine if the stream has a mean oxygen content of less than 5 milligrams per liter. (Note the R output provided may not give the direction you want for the alternative, but you CAN still get the correct p-value with a modification.)

μ = population mean level of dissolved O_2 in a liter of water

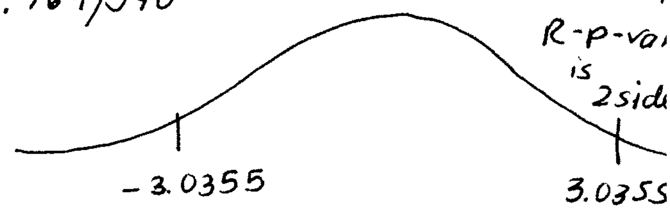
$H_0: \mu = 5$ $\quad H_A: \mu < 5$ $\quad n = 40$ $\quad \alpha =$ (discuss)

Conditions: 1. Randomization and $\perp$ observations → assume far apart and hence to assume representative sample $\perp$

2. Need population to be nearly normal or have a large n. n = 40 is probably large enough, but still check graphs. QQ plot, hist, boxplot look good → we see a unimodal symmetric bell-shaped dist with only 1 outlier.

Test stat: $t = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} = \dfrac{4.633 - 5}{.764/\sqrt{40}} = -3.0355$ Confirm via output

p-value: $\dfrac{.004263}{2} = .0021315$ R-p-value is 2sided



Decision: Reject H_0

Conclusion: We have significant evidence to conclude that the mean amount of dissolved O_2 in the stream is less than 5 mg/L.

Interpret your test statistic and p-value.

Test stat: Our test stat of -3.0355 means our observed $\bar{X} = 4.633$ is 3.0355 standard errors below the hypothesized value of 5.

p-value: Our p-value of .0021315 means if we repeated this study, we would attain a test stat of -3.0355 or lower in only .213 % of repetitions if the true mean amount of dissolved O_2 is really 5 mg/L.

Hypothesis Test/CI Example for a Population Mean

DDT concentrations in blood of 20 randomly selected people were collected (Devore/Peck) and are listed: 24,26,30,35,35,38,39,40,40,41,42,42,52,56,58,61,75,79,88,92.

Check all conditions and provide a 90% confidence interval for the concentration of DDT in the blood. Interpret the standard error, confidence interval and confidence level in the context of the problem. (The original population is not well described by the problem). Note the R output provided contains a 95% interval, but not a 90% interval.

Conditions and Interval Calculations:

1. Randomization and $\perp$. $\perp$ is reasonable and RS is stated.
2. Need a nearly normal population or large n. The n=20 is small, so look @ the graphs. The distribution is slightly right-skewed but there are no outliers.

CI

$\bar{x} \pm t^* \, s/\sqrt{n}$ $\quad t^*_{19} = 1.729$ for 90% $\quad s/\sqrt{n} = 4.49$

$49.65 \pm 1.729\left(\frac{20.095}{\sqrt{20}}\right)$

$49.65 \pm 7.769 \Rightarrow (41.881, 57.419)$

Interpretations:

Standard Error: Our standard error of 4.49 is the estimated average distance sample $\bar{x}$ values would be from the true mean μ, roughly in repeated sampling.

Confidence Interval: We are 90% confident that the true mean DDT concentration in the blood lies in (41.881, 57.419).

Confidence Level: If we repeated this experiment and generated many 90% CIs, we would expect 90% of the generated CIs to contain the true mean DDT concentration in the blood.