Math 28 Spring 2008: Exam 1

Instructions: Each problem is scored out of 10 points for a total of 50 points. You may not use any outside materials(eg. notes or calculators). You have 50 minutes to complete this exam.

Problem 1. Let $A \subseteq \mathbb{R}$ be bounded above. Show that $\sup(A) \in \overline{A}$.

Problem 2. Let $(a_n) \to a$ and $(b_n) \to b$ be convergent sequences. Show directly that $(a_n - b_n) \to a - b$. (i.e. without using the Algebraic Limit theorem).

Problem 3.

(a) State the definition of a compact set and the characterization of compact sets by the Heine-Borel Theorem.

(b) Show that the union of finitely many compact sets is compact.

Problem 4. Prove the $\sum_{n=1}^{\infty} a_n$ is convergent if and only if for any $m \in \mathbb{N}$ $\sum_{n=1}^{\infty} a_{m+n}$ is convergent. Moreover, $\sum_{n=1}^{\infty} a_n$ converges to $a_1 + \cdots + a_m + \sum_{n=1}^{\infty} a_{n+m}$.

Problem 5.

(a) State the Monotone Converge Theorem.

(b) Define $a_1 = 1$ and $a_n = 3 - \frac{1}{a_{n-1}}$ for $n \geq 2$. Determine the convergence or divergence of the sequence (a_n).