Physics 16 Problem Set 6 Solutions

Y&F Problems

1.55. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle ϕ as $\phi = \arccos\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\frac{A_x B_x + A_y B_y}{AB}\right)$.

SET UP: Eq.(1.14) shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22$, $A = \sqrt{40}$, $B = \sqrt{13}$, and so $\phi = \arccos\left(\frac{-22}{\sqrt{40}\,\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60$, $A = \sqrt{34}$, $B = \sqrt{136}$, $\phi = \arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0$, $0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0$, $90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0$, $\phi = 90°$ and the two vectors are perpendicular.

1.93. **IDENTIFY:** Find the angle between specified pairs of vectors.

SET UP: Use $\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB}$

EXECUTE: **(a)** $\vec{A} = \hat{k}$ (along line *ab*)

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$ (along line *ad*)

$A = 1, \quad B = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\vec{A}\cdot\vec{B} = \hat{k}\cdot(\hat{i} + \hat{j} + \hat{k}) = 1$

So $\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = 1/\sqrt{3}; \quad \phi = 54.7°$

(b) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ (along line *ad*)

$\vec{B} = \hat{j} + \hat{k}$ (along line *ac*)

$A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}; \quad B = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A}\cdot\vec{B} = (\hat{i} + \hat{j} + \hat{k})\cdot(\hat{i} + \hat{j}) = 1 + 1 = 2$

So $\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{2}{\sqrt{3}\sqrt{2}} = \frac{2}{\sqrt{6}}; \quad \phi = 35.3°$

EVALUATE: Each angle is computed to be less than $90°$, in agreement with what is deduced from Fig. 1.43 in the textbook.

6.3. **IDENTIFY:** Each force can be used in the relation $W = F_{\parallel}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{net} = W_{worker} + W_{grav} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{worker} = f_k = \mu_k n$.

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{worker} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}} \cos\phi)s = [(74\text{ N})(\cos 0°)](4.5\text{ m}) = +333\text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k \cos\phi)s = [(74\text{ N})(\cos 180°)](4.5\text{ m}) = -333\text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0$ J.

(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 333\text{ J} = 0.0\text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.50. **IDENTIFY** and **SET UP:** Use Eq.(6.15) to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.

EXECUTE: Find the total mass that can be lifted:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{\text{av}}t}{gh}$$

$$P_{\text{av}} = (40\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 2.984\times10^4\text{ W}$$

$$m = \frac{P_{\text{av}}t}{gh} = \frac{(2.984\times10^4\text{ W})(16.0\text{ s})}{(9.80\text{ m/s}^2)(20.0\text{ m})} = 2.436\times10^3\text{ kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is $2.436\times10^3\text{ kg} - 600\text{ kg} = 1.836\times10^3\text{ kg}$. The number of passengers is $\dfrac{1.836\times10^3\text{ kg}}{65.0\text{ kg}} = 28.2$. 28 passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

6.63. **IDENTIFY:** The effective force constant is defined by $k_{\text{eff}} = F/x$, where F is the force applied to each end of the spring combination and x is the amount the spring combination is stretched.

SET UP: Consider a force F applied to each end of the combination. Then F_1 and F_2 are the forces applied to each spring and $F = F_1 + F_2$. Each spring stretches the same amount x.

EXECUTE: **(a)** $F = k_{\text{eff}}x$. $F = F_1 + F_2 = k_1x + k_2x$. Equating the two expressions for F gives $k_{\text{eff}} = k_1 + k_2$.

(b) The same procedure as in part (a) gives $k_{\text{eff}} = k_1 + k_2 + \cdots + k_N$.

EVALUATE: The effective force constant of the configuration is greater than any of the force constants of the individual springs. More force is required to stretch the parallel combination that is required to stretch each separate spring the same amount.

6.64. **IDENTIFY:** The effective force constant is defined by $k_{\text{eff}} = F/x$, where F is the force applied to each end of the spring combination and x is the amount the spring combination is stretched.

SET UP: Consider a force F applied to each end of the combination. The same force F is applied to each spring. Spring 1 stretches a distance x_1 and spring 2 stretches a distance x_2, where $x_1 = F/k_1$ and $x_2 = F/k_2$. The total distance the combination stretches is $x = x_1 + x_2$.

EXECUTE: **(a)** $x = x_1 + x_2$ gives $\dfrac{F}{k_{\text{eff}}} = \dfrac{F}{k_1} = \dfrac{F}{k_2}$ and $\dfrac{1}{k_{\text{eff}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.

(b) The same procedure as in part (a) gives $\dfrac{1}{k_{\text{eff}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \cdots + \dfrac{1}{k_N}$.

EVALUATE: For two springs the result in part (a) can be written as $k_{\text{eff}} = \dfrac{k_1 k_2}{k_1 + k_2}$. The effective force constant for the two springs in series is less than the force constant for each individual spring. It takes less force to stretch the combination an amount x than to stretch either separate spring an amount x.

6.77. **IDENTIFY** and **SET UP:** Use Eq.(6.6). Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.77.

EXECUTE:

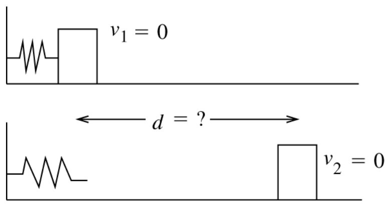

Figure 6.77

$W_{\text{tot}} = K_2 - K_1$

$K_1 = 0, \quad K_2 = 0$

$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250$ m (Spring force is in direction of motion of block so it does positive work.)

$W_{\text{fric}} = -\mu_k mgd$

Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$d = \dfrac{kx_1^2}{2\mu_k mg} = \dfrac{(250\text{ N/m})(0.250\text{ m})^2}{2(0.30)(2.50\text{ kg})(9.80\text{ m/s}^2)} = 1.1\text{ m}$, measured from the point where the block was released.

EVALUATE: The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero and the textbook starts and ends with zero kinetic energy.

6.82. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let h be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)gh and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))\,(9.80\text{ m/s}^2)\,(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\dfrac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

6.102. **IDENTIFY:** In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. Write W_0 in terms of the range R and speed v and in terms of the time of flight T and v.

SET UP: In both cases assume v is constant, so $W_0 = RF$ and $R = vT$.

EXECUTE: In terms of the range R and the constant speed v, $W_0 = RF = R\left(\alpha v^2 + \dfrac{\beta}{v^2}\right)$.

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \dfrac{\beta}{v}\right)$.

(a) Rather than solve for R as a function of v, differentiate the first of these relations with respect to v, setting $\dfrac{dW_0}{dv} = 0$ to obtain $\dfrac{dR}{dv}F + R\dfrac{dF}{dv} = 0$. For the maximum range, $\dfrac{dR}{dv} = 0$, so $\dfrac{dF}{dv} = 0$. Performing the differentiation, $\dfrac{dF}{dv} = 2\alpha v - 2\beta/v^3$ which is solved for

$$v=\left(\frac{\beta}{\alpha}\right)^{1/4}=\left(\frac{3.5\times10^5\ \mathrm{N\cdot m^2/s^2}}{0.30\ \mathrm{N\cdot s^2/m^2}}\right)^{1/4}=32.9\ \mathrm{m/s}=118\ \mathrm{km/h}.$$

This is certainly an easier way to solve the problem than solving for R(v) and differentiating to find the maximum, but if you're calculus is up to snuff there's no reason you can't differentiate R(v).

(b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv)=0$; performing the differentiation,

$$3\alpha v^2-\beta/v^2=0.\ \ v=\left(\frac{\beta}{3\alpha}\right)^{1/4}=\left(\frac{3.5\times10^5\ \mathrm{N\cdot m^2/s^2}}{3(0.30\ \mathrm{N\cdot s^2/m^2})}\right)^{1/4}=25\ \mathrm{m/s}=90\ \mathrm{km/h}.$$

EVALUATE: When $v=(\beta/\alpha)^{1/4}$, F_{air} has its minimum value $F_{\mathrm{air}}=2\sqrt{\alpha\beta}$. For this v, $R_1=(0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1=(0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v=(\beta/3\alpha)^{1/4}$, $F_{\mathrm{air}}=2.3\sqrt{\alpha\beta}$. For this v, $R_2=(0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2=(0.57)\alpha^{-1/4}\beta^{-3/4}$. $R_1>R_2$ and $T_2>T_1$, as they should be.

Extra Problem

1. In raising a mass m to a height h, the work done against gravity is $W=mgh$. If the total distance traveled is s (along the incline), then $h = s\sin(\theta)$, giving $W = mgs\sin(\theta)$. The work per meter per kg is then $W/ms = g\sin(\theta)$.
2. This is the function that is plotted in the figure. Since the angles are all pretty small, $\sin(\theta)$ looks pretty linear.
3. The difference between the walking curve (the lower curve) and the line $g\sin(\theta)$ is the physiological inefficiency.
    a. You can eyeball the point where the curves are closest together, or notice that the minimum gap occurs when their slopes are equal. (Displace the straight line upward, keeping the slope the same, to convince yourself of this).
    b. A grade of about –2° or –2.5° gives the smallest inefficiency.
    c. At –2.5°, the gap is about 1.5 J/kg.m.

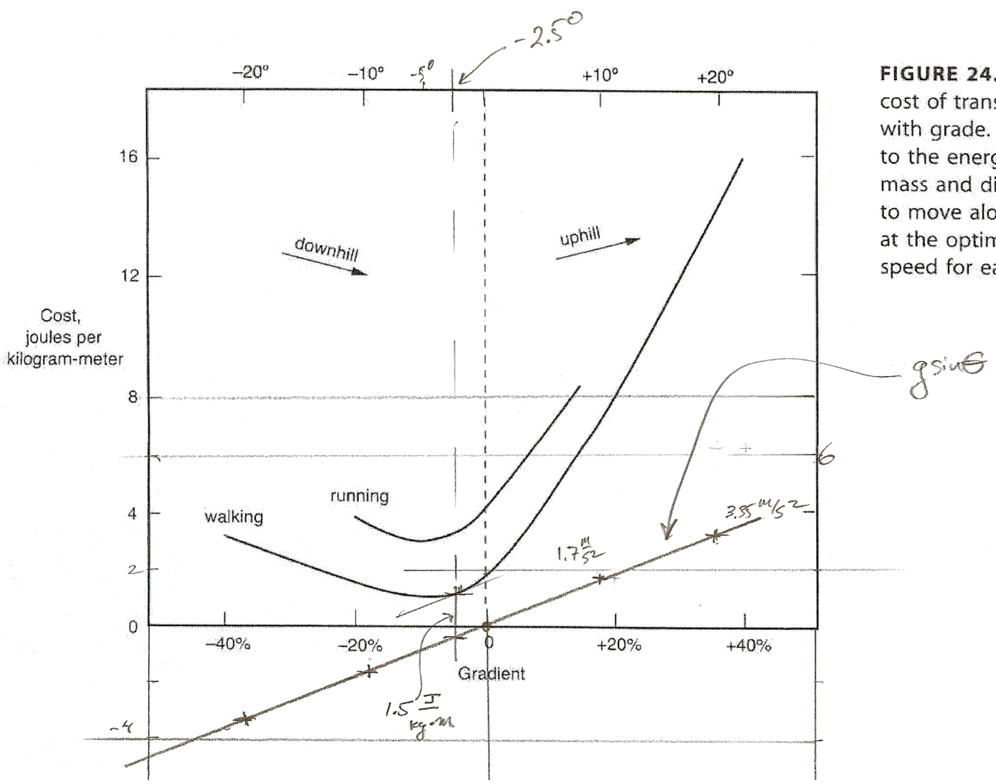

FIGURE 24.8 How our cost of transport varies with grade. The data refer to the energy relative to mass and distance needed to move along the grades at the optimal (cheapest) speed for each.