Physics 16 Problem Set 9 Solutions

Y&F Problems

9.6. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. $\omega_{\text{av-}z} = \dfrac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250\ \text{rad/s}) - (40.0\ \text{rad/s}^2)t - (4.50\ \text{rad/s}^3)t^2$. $\alpha_z = -(40.0\ \text{rad/s}^2) - (9.00\ \text{rad/s}^3)t$.

EXECUTE: **(a)** Setting $\omega_z = 0$ results in a quadratic in *t*. The only positive root is $t = 4.23\ \text{s}$.

(b) At $t = 4.23\ \text{s}$, $\alpha_z = -78.1\ \text{rad/s}^2$.

(c) At $t = 4.23\ \text{s}$, $\theta = 586\ \text{rad} = 93.3\ \text{rev}$.

(d) At $t = 0$, $\omega_z = 250\ \text{rad/s}$.

(e) $\omega_{\text{av-}z} = \dfrac{586\ \text{rad}}{4.23\ \text{s}} = 138\ \text{rad/s}.$

EVALUATE: Between $t = 0$ and $t = 4.23\ \text{s}$, ω_z decreases from 250 rad/s to zero. ω_z is not linear in *t*, so $\omega_{\text{av-}z}$ is not midway between the values of ω_z at the beginning and end of the interval.

9.15. **IDENTIFY:** Apply constant angular acceleration equations.
SET UP: Let the direction the flywheel is rotating be positive.
$\theta - \theta_0 = 200$ rev, $\omega_{0z} = 500\ \text{rev/min} = 8.333\ \text{rev/s}$, $t = 30.0\ \text{s}$.

EXECUTE: **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00\ \text{rev/s} = 300\ \text{rpm}$

(b) Use the information in part (a) to find α_z: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111\ \text{rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111\ \text{rev/s}^2$, $\omega_{0z} = 8.333\ \text{rev/s}$ in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0\ \text{s}$ and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\theta - \theta_0 = 312\ \text{rev}$.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.46. **IDENTIFY:** The work done on the cylinder equals its gain in kinetic energy.
SET UP: The work done on the cylinder is *PL*, where *L* is the length of the rope. $K_1 = 0$. $K_2 = \tfrac{1}{2}I\omega^2$.
$I = \tfrac{1}{2}mr^2 = \tfrac{1}{2}\left(\dfrac{w}{g}\right)r^2$.

EXECUTE: $PL = \dfrac{1}{2}\dfrac{w}{g}v^2$, or $P = \dfrac{1}{2}\dfrac{w}{g}\dfrac{v^2}{L} = \dfrac{(40.0\ \text{N})(6.00\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(5.00\ \text{m})} = 14.7\ \text{N}.$

EVALUATE: The linear speed *v* of the end of the rope equals the tangential speed of a point on the rim of the cylinder. When *K* is expressed in terms of *v*, the radius *r* of the cylinder doesn't appear.

9.47. **IDENTIFY** and **SET UP:** Combine Eqs.(9.17) and (9.15) to solve for *K*. Use Table 9.2 to get *I*.
EXECUTE: $K = \tfrac{1}{2}I\omega^2$

$a_{\text{rad}} = R\omega^2$, so $\omega = \sqrt{a_{\text{rad}}/R} = \sqrt{(3500\ \text{m/s}^2)/1.20\ \text{m}} = 54.0\ \text{rad/s}$

For a disk, $I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(70.0\ \text{kg})(1.20\ \text{m})^2 = 50.4\ \text{kg}\cdot\text{m}^2$

Thus $K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(50.4\ \text{kg}\cdot\text{m}^2)(54.0\ \text{rad/s})^2 = 7.35\times10^4\ \text{J}$

EVALUATE: The limit on a_{rad} limits ω which in turn limits *K*.

9.85. **IDENTIFY:** Apply conservation of energy to the system consisting of blocks *A* and *B* and the pulley.
SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.85.

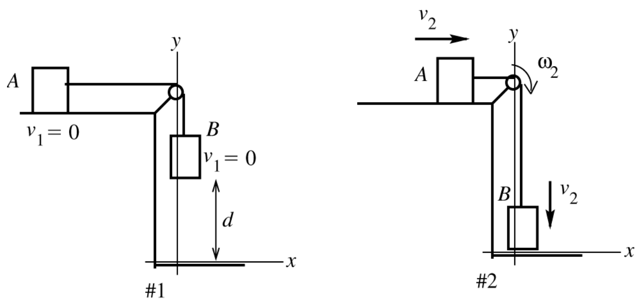

Figure 9.85

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block *B* after it has descended. The tension in the rope does positive work on block *A* and negative work of the same magnitude on block *B*, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block *B* and kinetic friction does work on block *A*. Therefore $W_{\text{other}} = W_f = -\mu_k m_A g d.$

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d;\ \ U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2} m_A v_2^2 + \frac{1}{2} m_B v_2^2 + \frac{1}{2} I \omega_2^2.$

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2 / R$ and

$K_2 = \frac{1}{2}(m_A + m_B) v_2^2 + \frac{1}{2} I (v_2 / R)^2 = \frac{1}{2}(m_A + m_B + I / R^2) v_2^2$

Putting all this into the work-energy relation gives

$m_B g d - \mu_k m_A g d = \frac{1}{2}(m_A + m_B + I / R^2) v_2^2$

$(m_A + m_B + I / R^2) v_2^2 = 2gd(m_B - \mu_k m_A)$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I / R^2}}$$

EVALUATE: If $m_B >> m_A$ and I / R^2, then $v_2 = \sqrt{2gd}$; block *B* falls freely. If *I* is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of *B* will be larger than the friction force on *A*. I / R^2 has units of mass and is in a sense the "effective mass" of the pulley.

9.89. IDENTIFY: $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.9.

SET UP: $\omega = \dfrac{v}{R_1}$ for part (b) and $\omega = \dfrac{v}{R_2}$ for part (c).

EXECUTE: **(a)** $I = \dfrac{1}{2} M_1 R_1^2 + \dfrac{1}{2} M_2 R_2^2 = \dfrac{1}{2}((0.80\text{ kg})(2.50 \times 10^{-2}\text{ m})^2 + (1.60\text{ kg})(5.00 \times 10^{-2}\text{ m})^2)$

$I = 2.25 \times 10^{-3}\text{ kg} \cdot \text{m}^2.$

(b) The method of Example 9.9 yields $v = \sqrt{\dfrac{2gh}{1 + (I / m R_1^2)}}$.

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1 + ((2.25 \times 10^{-3}\text{ kg} \cdot \text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}} = 3.40\text{ m/s}.$$

The same calculation, with R_2 instead of R_1 gives $v = 4.95\text{ m/s}$.

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega$, so when $R = R_2$ the factor that relates *v* to ω is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides with the block. The total kinetic energy is the same in both cases (equal to *mgh*), so when $R = R_2$ the kinetic energy and speed of the block are greater.