Fall, 2009 Science and Music

Name:_____

Science Lab #1

Standing Waves

In this experiment, you will set up standing waves on a string by mechanically driving one end of it. You will first observe the phenomenon of resonance. Though observations of resonance will only be qualitative in this lab, some experience of when resonance occurs will be important for the quantitative measurements you will be making. In the course of doing this lab, you will also come up with estimates of the uncertainties of measured quantities and see how those uncertainties affect your calculated values of wave velocity.

Properties of waves

The physical properties that you will measure, such as wavelength and frequency, are characteristics of standing waves. The wavelength and frequency of waves are related to the speed of wave propagation in the medium (the string, in our case):

$$v = \lambda f \tag{1}$$

where v is the speed of the wave, λ is its wavelength, and f its frequency. The speed of propagation of waves is determined by properties of the medium.

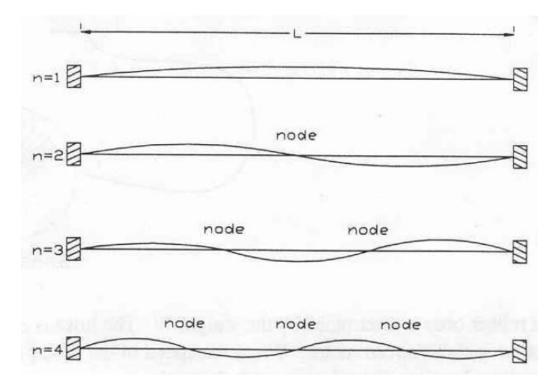


Figure 1: Standing waves on a string. n=1 corresponds to the fundamental mode; n>1 are harmonics.

When the string is held (more or less) fixed at the two ends, the standing wave patterns shown in Figure 1 are possible. The places where the wave displacement is zero are called the nodes and the places where the displacement is a maximum are called anti-nodes. One may think of a standing wave as the sum of two traveling waves, one traveling to the right and the other to the left. The distance between two successive nodes (or two successive anti-nodes) in a standing wave is half the wavelength of either of the traveling waves that comprise the standing wave. Clearly the fixed ends of the string are always nodes. With that constraint, the longest-wavelength standing wave corresponds to having one anti-node, n = 1 in the figure, and is variously called the "fundamental mode" or the "lowest note" of the string, the latter term coming from the fact that the frequency is lowest for these standing waves. The higher n "modes of vibration" correspond to harmonics of the fundamental, the overtones. The frequency for the nth resonant mode is given by

$$f_n = \frac{v}{\lambda_n} = nf_1 \tag{2}$$

Consider two points on the string, one that lies on a node and one that lies on an antinode. *Make a sketch of the displacement as a function of time for both of these points.*

If your sketches above represented real experimental data, explain how you would determine the frequency of the motion.

For a given mode (i.e. a given value of n), how does the motion depend on which point on the string you choose? That is, how would the motion be different if you were looking at a point that is between an antinode and a node?

In general, a string does not vibrate in only one of the basic patterns of standing waves shown in the figure. The excitation of a string (like that of a plucked guitar stringed) will produce a mixture of these different patterns with different amplitudes simultaneously. The mix of overtones, among other features, distinguishes a violin from a piano or other instrument playing the same "note", and lends music its richness. Understanding of the pure modes allows us to analyze even highly complicated mixtures of modes.

Typically, wave speed is the square root of an elastic property of the medium such as a tension or pressure divided by an inertial property such as mass density. In the case of waves on a string, the speed is given by

$$v = \sqrt{\frac{F}{\mu}} \,, \tag{3}$$

where F is the tension in the string, and μ is its mass per unit length. (We reserve the letter T for period, and hence don't use it for the tension in the string here).

Again, consider the motion of an antinode for a given mode (say n = 1). Using Eqs. 1 and 3, explain how the frequency of the point of the string at the antinode changes when you change either the tension or the mass density. Given what you know about the frequency of simple harmonic motion (our first lab), explain why these dependences make sense.

The Experiment

In this experiment, you will choose a value of *F*, and make measurements of the wavelengths of standing waves for at least five different resonant frequencies. For a given tension, your data will allow you to investigate Eq. 1 and compare your results to the value predicted by Eq. 3.

We will use a wave driver (essentially a modified audio speaker) to stimulate the string at various frequencies. The wave driver is electrically driven by a function generator. The function generator should already be on and the "Output" button should be lit, indicating that it is sending a sinusoidally varying voltage to the wave driver. (If it is not lit, press it once to activate the output.) You will only need to adjust the frequency of this driving voltage. Use the arrow keys and knob to set the frequency to 1.000 Hz. (Alternatively, you can also change frequency by using the numeric keypad.) You are now ready to begin the experiment. Slowly increase the frequency until you see standing waves on your string.

To produce a tension in the string, you will hang some weight off the end of the string. Your station includes a "hanger" and some masses that you can hang on it. Select some masses to give you a total hanging mass (including the hanger itself) between 200 and 800 grams. Place the masses securely on the hanger. *Measure the total mass with the scale*:

. Now, hang the mass off the end of your string.

The tension in the string is equal to the force of gravity pulling on the hanging mass. Recalling that the force of gravity is given by, $F_q = mg$ (where $g = 10 \text{ m/s}^2$), calculate the tension in the string:

The mass density of the string μ will be provided to you by the instructor. Use it and Eq. 3 to calculate the wave velocity for your string.

First, let's get a qualitative feel for the standing-wave resonances of the string. Slowly increase the frequency produced by the function generator. As you adjust the frequency, you will notice that, from time to time, the vibrations of the string suddenly grow in amplitude to reach a maximum when the string takes on one of the shapes depicted in the figure above, and then, as you increase the frequency further, the amplitude dies away just as suddenly. This is the phenomenon of resonance. When the driving frequency coincides with one of the natural frequencies of the medium (the string), the response of the medium increases sharply. Because this phenomenon is of central importance in many parts of physics and chemistry, spend a few moments just observing it qualitatively. Now, slowly raise and lower the driving frequency and observe as many standing wave resonances as you can. Pay attention to how "sharp" each resonance is. That is, is the build-up slow or sudden as you gently ramp up or down the frequency? Explain how this sharpness impacts how well you can determine the frequency of the resonance. (You might want to include a sketch of amplitude as a function of frequency to augment your answer.)

Now you can begin to study the resonances more quantitatively. Carefully adjust the drive frequency so that the string is vibrating in the fundamental mode (n=1). Note the frequency on the function generator. Repeat the frequency measurement at least twice, each time moving away from resonance and moving back. Share the task of judging when you have hit resonance equally with your lab partner. Use the range of frequencies as an estimate of experimental uncertainty. Record your value of f_1 and the corresponding

uncertainty in the table below. Now measure the distance between successive nodes. This will allow you to calculate the wavelength. Estimate how well you can locate the nodes, and hence the experimental uncertainty in the wavelength. Repeat the above steps for as many harmonics as you can. You should be able to measure to at least n = 5. Record your results in the table. The last two columns of the table allow you to check the accuracy of Eq. 2 as well as to calculate the wave velocity using Eq. 1.

n	$f_n \pm \Delta f_n$	$\lambda_n \pm \Delta \lambda_n$	nf ₁	<i>V ± ΔV</i>
1			XXXX	
2				
3				
4				
5				
6				
7				

Is Eq. 2 satisfied to within your experimental uncertainty?

Do the values of v you calculated in the last column all agree with each other? What can you say about how wave velocity depends on frequency?

Do these values for wave velocity agree with the one you calculated using Eq. 3?

 $^{^{1}}$ For n = 1, the location of only one node is well defined. The other end of the string is being driven and thus has no node. Notice, however, that as you move along the string in the direction of the driver, the amplitude of the vibration is decreasing. You can still estimate the wavelength for the fundamental mode by extrapolating to the point where the node would exist if the string continued without interruption. This point will often be very near the rod which the string is tied to, but its exact position is often difficult to pinpoint.