Physics 16 Problem Set 11 Solutions

Y&F Problems

13.15. **IDENTIFY:** Apply $T = 2\pi\sqrt{\frac{m}{k}}$. Use the information about the empty chair to calculate *k*.

SET UP: When $m = 42.5\text{ kg}$, $T = 1.30\text{ s}$.

EXECUTE: Empty chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(42.5\text{ kg})}{(1.30\text{ s})^2} = 993\text{ N/m}$

With person in chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $m = \frac{T^2 k}{4\pi^2} = \frac{(2.54\text{ s})^2(993\text{ N/m})}{4\pi^2} = 162\text{ kg}$ and

$m_{\text{person}} = 162\text{ kg} - 42.5\text{ kg} = 120\text{ kg}$.

EVALUATE: For the same spring, when the mass increases, the period increases.

13.27. **IDENTIFY:** Conservation of energy says $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ and Newton's second law says $-kx = ma_x$.

SET UP: Let +*x* be to the right. Let the mass of the object be *m*.

EXECUTE: $k = -\frac{ma_x}{x} = -m\left(\frac{-8.40\text{ m/s}^2}{0.600\text{ m}}\right) = (14.0\text{ s}^{-2})m$.

$A = \sqrt{x^2 + (m/k)v^2} = \sqrt{(0.600\text{ m})^2 + \left(\frac{m}{[14.0\text{ s}^{-2}]m}\right)(2.20\text{ m/s})^2} = 0.840\text{ m}$. The object will therefore travel $0.840\text{ m} - 0.600\text{ m} = 0.240\text{ m}$ to the right before stopping at its maximum amplitude.

EVALUATE: The acceleration is not constant and we cannot use the constant acceleration kinematic equations.

13.41. **IDENTIFY:** $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = 0.25\text{ s}$.

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on *L* and *g*.

13.57. **IDENTIFY and SET UP:** Use Eq.(13.43) to calculate ω', and then $f' = \omega'/2\pi$.

(a) EXECUTE: $\omega' = \sqrt{(k/m) - (b^2/4m^2)} = \sqrt{\frac{2.50\text{ N/m}}{0.300\text{ kg}} - \frac{(0.900\text{ kg/s})^2}{4(0.300\text{ kg})^2}} = 2.47\text{ rad/s}$

$f' = \omega'/2\pi = (2.47\text{ rad/s})/2\pi = 0.393\text{ Hz}$

(b) IDENTIFY and SET UP: The condition for critical damping is $b = 2\sqrt{km}$ (Eq.13.44)

EXECUTE: $b = 2\sqrt{(2.50\text{ N/m})(0.300\text{ kg})} = 1.73\text{ kg/s}$

EVALUATE: The value of *b* in part (a) is less than the critical damping value found in part (b). With no damping, the frequency is $f = 0.459\text{ Hz}$; the damping reduces the oscillation frequency.

13.64. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$. The period changes when the mass changes.

SET UP: *M* is the mass of the empty car and the mass of the loaded car is $M + 250\text{ kg}$.

EXECUTE: The period of the empty car is $T_E = 2\pi\sqrt{\frac{M}{k}}$. The period of the loaded car is

$$T_L = 2\pi\sqrt{\frac{M + 250\text{ kg}}{k}}.\quad k = \frac{(250\text{ kg})(9.80\text{ m/s}^2)}{4.00\times10^{-2}\text{ m}} = 6.125\times10^4\text{ N/m}$$

$$M = \left(\frac{T_L}{2\pi}\right)^2 k - 250\text{ kg} = \left(\frac{1.08\text{ s}}{2\pi}\right)^2(6.125\times10^4\text{ N/m}) - 250\text{ kg} = 1.56\times10^3\text{ kg}.$$

$$T_E = 2\pi\sqrt{\frac{1.56\times10^3\text{ kg}}{6.125\times10^4\text{ N/m}}} = 1.00\text{ s}.$$

EVALUATE: When the mass decreases, the period decreases.

13.68. **IDENTIFY:** In SHM, $a_{max} = \frac{k}{m_{tot}}A$. Apply $\sum\vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\sum F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{max} = \left(\frac{k}{M+m}\right)A$. Set $a_{max} = a$ and solve for A: $\mu_s g = \left(\frac{k}{M+m}\right)A$ and $A = \frac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.

13.89. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2 g h_0 = \frac{1}{2}m_2 v^2$ and

$$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s}.$$

Momentum conservation during collision: $m_2 v = (m_2 + m_3)V$ and

$$V = \frac{m_2 v}{m_2 + m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s}.$$

Energy conservation during upward swing: $Mgh_f = \frac{1}{2}MV^2$ and

$$h_f = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm}.$$

Figure 13.89 shows how the maximum angular displacement is calculated from h_f. $\cos\theta = \frac{48.4\text{ cm}}{50.0\text{ cm}}$ and $\theta = 14.5°$. $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{0.500\text{ m}}} = 0.705\text{ Hz}$.

EVALUATE: $14.5° = 0.253\text{ rad}$. $\sin(0.253\text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and Eq.(13.34) is accurate.

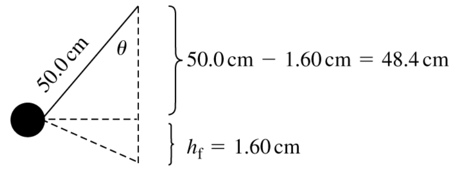

Figure 13.89