Physics 16 Problem Set 10 Solutions

Y&F Problems

10.20. **IDENTIFY:** Only gravity does work, so $W_{\text{other}} = 0$ and conservation of energy gives $K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$. $K_{\text{f}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$.

SET UP: Let $y_{\text{f}} = 0$, so $U_{\text{f}} = 0$ and $y_{\text{i}} = 0.750\text{ m}$. The hoop is released from rest so $K_{\text{i}} = 0$. $v_{\text{cm}} = R\omega$. For a hoop with an axis at its center, $I_{\text{cm}} = MR^2$.

EXECUTE: **(a)** Conservation of energy gives $U_{\text{i}} = K_{\text{f}}$. $K_{\text{f}} = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy$. $\omega = \frac{\sqrt{gy_{\text{i}}}}{R} = \frac{\sqrt{(9.80\text{ m/s}^2)(0.750\text{ m})}}{0.0800\text{ m}} = 33.9\text{ rad/s}$.

(b) $v = R\omega = (0.0800\text{ m})(33.9\text{ rad/s}) = 2.71\text{ m/s}$

EVALUATE: An object released from rest and falling in free-fall for 0.750 m attains a speed of $\sqrt{2g(0.750\text{ m})} = 3.83\text{ m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.42. **IDENTIFY:** Apply conservation of angular momentum to the diver.

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\text{ kg}\cdot\text{m}^2)/(18\text{ kg}\cdot\text{m}^2)$.

EXECUTE: If she had tucked, she would have made $(2\text{ rev})(3.6\text{ kg}\cdot\text{m}^2)/(18\text{ kg}\cdot\text{m}^2) = 0.40\text{ rev}$ in the last 1.0 s, so she would have made $(0.40\text{ rev})(1.5/1.0) = 0.60\text{ rev}$ in the total 1.5 s.

EVALUATE: Untucked she rotates slower and completes fewer revolutions.

10.90. **IDENTIFY:** Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$.

SET UP: For constant mass the moment of inertia is proportional to the square of the radius.

EXECUTE: $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = (R_0 + \Delta R)^2(\omega_0 + \Delta\omega) = R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega$, where the terms in $\Delta R\Delta\omega$ and $(\Delta\omega)^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives

$$\Delta R = -\frac{R_0}{2}\frac{\Delta\omega}{\omega_0} = -1.1\text{ cm}.$$

EVALUATE: $\Delta R/R_0$ and $\Delta\omega/\omega_0$ are each very small so the neglect of terms containing $\Delta R\Delta\omega$ or $(\Delta\omega)^2$ is an accurate simplifying approximation.

10.92. **IDENTIFY:** Angular momentum is conserved, since the tension in the string is in the radial direction and therefore produces no torque. Apply $\sum\vec{F} = m\vec{a}$ to the block, with $a = a_{\text{rad}} = v^2/r$.

SET UP: The block's angular momentum with respect to the hole is $L = mvr$.

EXECUTE: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\frac{v^2}{r}$. $T = mv^2\frac{1}{r} = \frac{m^2v^2}{m}\frac{r^2}{r^3} = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}$. The radius at which the string breaks is

$r^3 = \frac{L^2}{mT_{\text{max}}} = \frac{(mv_1r_1)^2}{mT_{\text{max}}} = \frac{((0.250\text{ kg})(4.00\text{ m/s})(0.800\text{ m}))^2}{(0.250\text{ kg})(30.0\text{ N})}$, from which $r = 0.440\text{ m}$.

EVALUATE: Just before the string breaks the speed of the rock is $(4.00\text{ m/s})\left(\dfrac{0.800\text{ m}}{0.440\text{ m}}\right) = 7.27\text{ m/s}$. We can verify that $v = 7.27\text{ m/s}$ and $r = 0.440\text{ m}$ do give $T = 30.0\text{ N}$.