# Regulating Algorithmic Pricing

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**Abstract**

Algorithms have led to more sophisticated pricing strategies adopted by firms. They can lower consumers' search costs through recommendations, track their search sequences, and monitor rival prices. Previous literature has studied the effects of tracking technology in facilitating tacit collusion, which is an anti-competitive concern for antitrust regulators. My thesis presents a countervailing argument that suggests the pro-competitive potential for strategic algorithmic price discrimination. I model sequential consumer search on a Hotelling line with differing search costs in a duopoly market. The model illustrates how the three algorithmic features jointly expand market overlap, create a credible threat of multiple searches, and lower Perfect Bayesian Nash Equilibrium prices for all consumers. It also shows that the payoff structure of firms' investment in algorithms resembles a Prisoner's Dilemma. My analysis provides regulatory insights on balancing the pro-competitive features and anti-competitive concerns of algorithmic pricing in antitrust enforcement.

**Keywords:** algorithm, price discrimination, antitrust, search cost

**JEL Classification:** C72, D43, D47, D82, K21, L11, L41, O33

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# 1 Introduction

Over the past decade, algorithmic technology, compounded by increasingly available consumer information, has led to more sophisticated pricing strategies adopted by firms. A pricing algorithm automates the retailer's pricing decision at each instance by dynamically mapping observed inputs about market characteristics to a desired set of price offering outputs. By analyzing vast amounts of processed data, obtained from sources such as cookies and data brokers, these algorithms can provide valuable insights into consumer preferences and the actions of competitors within the online market space. European Commission's 2020 Final report on the E-commerce Sector Inquiry finds that "a majority of retailers track the online prices of competitors and consumer profiles. Three fourths of them use software programs that autonomously infer the willingness to pay for consumers and observe prices of competitors."[1] Per US Department of Justice, most large online shopping sites in the US, including Amazon, Walmart, eBay, Bookings, Zillow, Walgreens, and Uber collect and process consumers' purchase data, browsing history, and demographic characteristics either in their own database or through third-party sources.[2] Even smaller retailers have been relying on independent algorithm providers such as Intelligence Node, which advertises its algorithm service as "having eyes on competitor movements at all times and ... automatically update your prices to maximize profit."[3]

In this thesis, I identify three salient features of modern pricing algorithms that seem to facilitate strategic pricing the most. Machine learning algorithms analyze consumer preferences and provide customized recommendations that reduce the time and effort required for the consumer to find relevant products or services. By narrowing down the range of choices and presenting options that are more likely to be of interest, algorithms help reduce

1. "Sector inquiry into e-commerce," https://competition-policy.ec.europa.eu/sectors/ict/sector-inquiry-e-commerce_en.

2. "Competition and Antirust Law Enforcement Act Proposes Wholesale Changes to U.S. Antitrust Law," The National Law Review, https://www.natlawreview.com/article/competition-and-antirust-law-enforcement-act-proposes-wholesale-changes-to-us.

3. Marco Bertini and Oded Koenigsberg,"The Pitfalls of Pricing Algorithms," Harvard Business Review.

consumers search cost. Second, algorithms track the consumers' sequence of search by using browser cookies or requiring the consumer to create an account and log in to a website or app. As the consumer navigates the website or app, the algorithm records each search query, click, and other actions taken by the user. Importantly, with access to cookies, the firms can detect if a consumer browses a competitor's website for a similar product and infer the consumer's search sequence. Third, algorithms monitor rival price offerings from earlier searches. They can do this by crawling rival websites or by subscribing to third-party data providers that collect and analyze pricing information. Through such monitoring, algorithms can arrive at pricing strategies responsive to actual rival prices.

My thesis responds to concerns from regulatory authorities and legal scholars over these features of pricing algorithms. In addition to privacy concerns, they worry that real-time tracking of consumers and rival firms' behavior may facilitate greater tacit collusion among firms, enabling far more effective coordination of prices even without an explicit agreement. Members of the Congress have voiced concern over the lack of regulatory tools to restrict algorithmic consumer harms. Senator Amy Klobuchar, while introducing her bill "Competition and Antitrust Law Enforcement Reform Act of 2021," stated that algorithms "only gets more common and more dangerous" over time.[4] Senator Sherrod Brown, Chair of the Senate Committee on Banking, Housing, and Urban Affairs, directly addressed to Lina Khan, Chair of the Federal Trade Commission (FTC), and urged the experts to review rent-setting optimization software like RealPage's YieldStar and AI Revenue Management.[5] A narrative of "inevitable threat" that an algorithm would surely produce to raise prices, lower outputs, and facilitate collusion corroborates with lawmakers' deep-seated anxiety on the current antitrust framework which, in their characterization, are "ill-equipped" to tackle technologically complex challenges.

4. S.225 - 117th Congress: Competition and Antitrust Law Enforcement Reform Act of 2021. https://www.congress.gov/bill/117th-congress/senate-bill/225.

5. "Brown Calls on FTC to Review Whether Rental Pricing Algorithms Violate the Law — U.S. Senator Sherrod Brown of Ohio," https://www.brown.senate.gov/newsroom/press/release/sherrod-brown-calls-on-ftc-to-review-whether-rental-pricing-algorithms-violate-the-law.

Meanwhile, the nascent but growing scholarship in law, computer science, and economics has expended their effort mainly on two aspects of algorithmic competition: documenting empirical evidence for algorithmic interactions and analyzing the technical specificities for which algorithmic collusion could be possible. Chen et al. (2016) documented that a significant fraction of sellers in a large online marketplace (Amazon US), where many different types of goods are traded, adopted algorithmic pricing in 2015. Calvano et al. (2018) recorded large-scale price dispersion when consumers of different demographic profiles accessed the item. Clark et al. (2021) empirically estimated the price changes as a result of algorithm adoption for German gasoline retailers. To analyze how algorithms may facilitate tacit or overt collusion, Asker et al. (2021) investigated how the design of machine learning protocols such as Q-learning could lead to competitive or supracompetitive price outcomes when competing in a simple Bertrand pricing game. Their paper informed Brown and MacKay (forthcoming in 2023) to model oligopolistic competition in which firms differ in pricing frequency and choose pricing algorithms that respond to rivals' prices. Despite the recent success in detecting the use of algorithms in retail spaces, scholars disagree on whether algorithms make collusion a more likely choice for firms in oligopoly. Analysis of algorithmic effect other than collusion is limited in academic literature.

I consider the joint effects of all three features and provide a potentially countervailing argument to the abovementioned concern. I argue that, in the same market environment where pricing algorithms may facilitate tacit collusion, pricing algorithms could also allow the firms to strategically price-discriminate in a manner that generates pro-competitive effects. I formally explore this argument using a duopolistic price competition model with search costs that differ across consumers (à la the Hotelling line). The consumers engage in sequential search, and the decision whether to search further is based on their search costs and observed or expected prices. The model is solved using the equilibrium concept of Perfect Bayesian Nash. The model is used to examine how firms would price given algorithms and why they might choose to invest in pricing algorithms.

I show that given certain market conditions, algorithms can fundamentally shift retailers' equilibrium pricing behavior and result in pro-consumer outcomes. While each feature may improve consumer welfare on their own, their combination magnifies the pro-competitive effect. Product recommendation contributes to expansion of trade, as consumers lower their search cost for either or both retailers. The reduction in search cost increases the demand elasticity, for consumers can switch shopping sites easily to compare prices across platforms. Search sequence detection and rival price monitoring could result in greater credible threat of price-matching: since the second-searched firm could precisely undercut the first price offering, the first-searched firm is incentivized to discourage their first searchers from searching twice by offering them a lower price. The algorithms enable credible threat of switching consumers and undercutting rivals, hence lower prices for both first and second searchers in the market.

I further demonstrate that the firms' algorithmic investment game may resemble a Prisoner's Dilemma. A firm enjoys the most algorithmic advantage when it invests in the algorithm, but its rival does not. This advantage dissipates when both firms have symmetric algorithm, and their competitive payoff may be lower than when neither has an algorithm. Given reasonable upfront investment cost, the firms treat having algorithms as a dominant strategy primarily because of the disadvantage of not having one when its rival does. Pricing algorithms sometimes benefit the consumers more than the firms.

My approach to locate algorithmic effect via search-based price discrimination is related to a rich reservoir of economics literature on strategic price discrimination. Scholarship on the impacts of price discrimination in competitive frameworks include Holmes, 1989, Tirole, 1994, Corts, 1998 and Armstrong and Vickers, 2001. Search costs may generate equilibrium price dispersion (Varian, 1980, Burdett, Judd, 1983, Stahl, 1989, Janssen, Moraga, 2004). Firms must possess some degree of market power and there must be factors limiting the potential for buyers' arbitrage. The structure of search sequence, combined with the algorithmic effect, allows the firm to price-discriminate. Without price discrimination, a retailer faces an essential tradeoff between quantity and price. A firm can forego second-searched sales in an expectation to gain maximum surplus for those who search for its products only. Alternatively, a firm can retain second-searched sales by offering a discount to all consumers, including those who would take the price at the first search. Price discrimination allows the firm to do both without necessarily decreasing overall consumer welfare. Ceteris peribus, consumers who do not incur the additional cost of searching twice would be charged a higher price, while those who do would be offered a discount given Nash conjectures. The amount of discount depends on the observed first price offering and the search cost at the second site. Rival price monitoring, the very technological feature that allows the firm to monitor each other while colluding according to Calvano (2018) and Asker (2021), may also motivate a firm to undercut each other's prices and improve consumer welfare.

My modeling result substantiates recent qualitative discussions by regulating authorities on possible antitrust mechanisms for algorithms. Although price discrimination is made illegal under the Sherman Antitrust Act. 15 U.S.C. §2, the Clayton Act, 15 U.S.C. §13, and by the Robinson-Patman Act, 15 U.S.C. §§13-13b, 21a, current U.S. antitrust doctrine applies a rule-of-reason, consumer-welfare-based test to determine its legality by weighing its anti-competitive effect against its pro-competitive effects. My model offers a guide by which antitrust authorities can examine such weighing. In some markets where collusion is less likely to be sustainable, some algorithms may operate to benefit consumers. My findings echo the intuition expressed by Terrell McSweeney, an FTC commissioner, who wrote, "As with algorithmic pricing generally, algorithmic price discrimination has the potential to provide consumer benefits, such as enabling companies to identify and offer discounts to targeted consumers who were previously priced out of certain markets."[6]

The rest of the paper discusses the three algorithmic effects in Chapter 2, sets up the model in Chapter 3, analyzes the model in Chapter 4, presents Perfect Bayesian Nash Equilibrium calibrations in Chapter 5, and concludes with possible extensions of the model.

6. "The Implications of Algorithmic Pricing for Coordinated Effects Analysis and Price Discrimination Markets in Antitrust Analysis," Federal Trade Commission, December 5, 2017.

# 2 Background on Pricing Algorithms

In this section, I discuss explicitly the above-mentioned three features of algorithms: search cost reduction, search sequence tracking, and rival price monitoring. I offer background information for each feature and elaborate on how their combined effects influence strategic competition among retailers to motivate my model.

## 2.1 Search Cost Reduction through Recommendations

Psychology studies have revealed that consumers are more willing to purchase a good if the product "came to them" at just the time they need it. In a survey covering 19 countries and territories, more than 95% of some 19,000 respondents stated having online shopping experience, more than half of them shopping online at least once a month.[7] Many studies have estimated consumers' online search cost for hotels, books, tablets, and so on (Blake, Nosko, and Tadelis 2016; Chen and Yao 2017; De los Santos 2008; De los Santos, Hortaçsu, and Wildenbeest 2012, 2017; Ghose, Ipeirotis, and Li 2018; Jiang et al. 2017; Moraga-González, Sándor, and Wildenbeest 2013; Moraga-González and Wildenbeest 2008). Most studies indicate that consumers' online per-item search cost is between $1 and $20.

Some products require higher search costs. Furniture and bedding products are typical examples of maximally differentiated product with many specifications– they require higher search cost than mass-produced, accessible, daily-use items such as toilet paper. With numerous filters, the consumers are able to locate their desired product without the need to record all product characteristics on their own. For products of higher research cost, online platforms offer the shopping platform as a service to consumers who, within several clicks, can apply the right filter and find the products they prefer. Wayfair designs their site such that consumers can locate their preferred dining table or mattress "within 6 clicks."[8]

7. Baojun Jiang and Tianxin Zou, "Consumer Search and Filtering on Online Retail Platforms," *Journal of Marketing Research* 57, no. 5 (2020): 900–916.

8. Babur De Los Santos, Ali Hortaçsu, and Matthijs R. Wildenbeest, "Search With Learning for Differentiated Products: Evidence from E-Commerce," *Journal of Business & Economic Statistics* 35, no. 4 (2017):

But algorithmic search cost reduction is most common when a retailer automates and personalizes product recommendations based on inferred consumer characteristics. Consumers who open the homepage of a shopping app receive an automated list of products based on their previous purchases, key words they search, and so on. Once a consumer visits the homepage of Amazon, she will immediately see a list of recommended products labeled as "Amazon selects." They may include items she browsed upon last visit, frequently purchased by people of the same ZIP code, or "flash discounts" on products purchased by consumers of similar profile nationally. For example, Townley (2017) found that those who recently purchase cosmetic products were often "recommended" a discount for feminine hygiene products.[9] Firms also aim to attract retailer-neutral consumers to their platform by using targeted advertisements. These consumers are more likely to click on ads that align closely with their desired products. By utilizing a sophisticated algorithm, firms can increase the number of retailer-neutral consumers who choose to search on their platform first.

## 2.2 Search Sequence Tracking

Online retailers can access the browsing history of every user who agrees with their Cookie policy. If their algorithm detects a competitor's domain in the searching history, it knows that a consumer has chosen to shop elsewhere before accessing its website. One of the more (in)famous examples involved Amazon, a case in which a user noticed that the price of a DVD offered to him dropped from \$26.24 to \$22.74 after he deleted cookies identifying him as a regular customer. Amazon attributed the price difference to random price tests, and eventually refunded all customers who had paid the higher price (Cavallo 2018). An alternative explanation is that the "regular consumer" Cookies helped identify the user as a consumer who likely searches the product primarily at Amazon. Relatedly, a regular user of Travelocity found that, if he browsed other ticket reservation platforms, Travelocity would

626–641, ISSN: 0735-0015, https://www.jstor.org/stable/44867754.

9. Christopher Townley, Eric Morrison, and Karen Yeung, *Big Data and Personalised Price Discrimination in EU Competition Law*, 3048688, Rochester, NY, October 6, 2017, https://doi.org/10.2139/ssrn.3048688, https://papers.ssrn.com/abstract=3048688.

more often offer him a discount on exactly the ticket he browsed (Townley et al., 2017). Subsequently, upon clearing her browsing history, the price rose by $59.99. (Townley et al. 2017). Pricing seems to depend on whether a potential consumer is identified as having searched elsewhere earlier.

However, research has revealed that only a small fraction of consumers pay attention to systematic price dispersion by online retailers (Kelley et al., 2022). 78% of surveyed consumers reported that they were not aware that their prices were given by an algorithm. Among those who were aware, almost 40% expressed no concern of being manipulated. Despite growing public discussions on the unfairness of algorithmic pricing, most consumers behave like price takers with idiosyncratic beliefs about the price a shopping site may offer. For example, over 50% of shoppers at Walmart believed that Walmart offered goods at a wholesale price, while market researches have shown otherwise. Researchers attribute this phenomenon as "the online cheap trap," arguing that consumers are not "skeptical enough" of algorithmic pricing schemes.[10] They do concede that while algorithms may not give the consumer the cheapest price in the market, they may save consumer significant search cost.

## 2.3 Rival Price Monitoring

Complementing search sequence tracking, the firm may also access log-in and shopping carts the consumer left behind from earlier searches at rivals. Each firm could observe the prices their rivals charged, enabling the firm to offer a price explicitly responsive to prices offered by rivals.

Rival price monitoring is a double-edged sword, as the same technology that may help the retailer undercut its rival may also help sustain collusion. Brown and MacKay (forthcoming, 2023) records sustained supracompetitive prices of firms using algorithm, suggesting the inevitability of algorithmic collusion. They argue that repeated interaction between two so-

10. Tom Blake, Chris Nosko, and Steven Tadelis, *Consumer Heterogeneity and Paid Search Effectiveness: A Large Scale Field Experiment*, 2444538, Rochester, NY, May 1, 2014, https://papers.ssrn.com/abstract=2444538.

phisticated algorithms may eventually result in a tacit agreement to not undercut their rival. Their finding does not necessarily contradict the pro-competitive side of price monitoring technology, as they assume consumers may observe all prices simultaneously in their model. In this thesis, sequential search is considered essential and shown to have key implications that differ from Brown and MacKay.

An online experiment by Vollero et al. (2022) documents the importance of this distinction. A consumer first searches on Amazon for a fridge, for which Amazon charged $327.79. Five minutes after she put the item into Amazon's shopping cart, Walmart's target advertisement on social media platform offered a "seasonal discount" which gave her $325.81.[11] Walmart essentially used "seasonal discount" as a cover for undercutting Amazon. In this case, even though Amazon attracts the consumer to their site first, subsequent selective discounting from Walmart poaches the consumer in the end.

## 2.4 Joint Features

To demonstrate how these three algorithmic features jointly influence firm's pricing strategy, consider a product being sold by two online retailers, Azamon (A) and Wamlart (W). A has invested more in developing a superior algorithm, while W has not. A and W independently decide their pricing strategy before making the product available for consumers.

Consumers in the market decide whether to search for the product. Every consumer incurs a distinct search cost for A and for W. Consumers can search at most twice: at both A and W. Loyal consumers of A, who have developed some idiosyncratic preference, incur a smaller search cost from searching at A, as A is better able to infer their preference and make precise personalized recommendations. Similar loyal consumers of W would have a smaller search cost at W. Holding all else equal, the more loyal a consumer is to A (W), the smaller search cost it incurs by searching at A (W), and the larger search cost it incurs by

11. Agostino Vollero, Domenico Sardanelli, and Alfonso Siano, "Exploring the role of the Amazon effect on customer expectations: An analysis of user-generated content in consumer electronics retailing," *Journal of Consumer Behaviour*, June 29, 2021, cb.1969, ISSN: 1472-0817, 1479-1838.

searching at W (A). Observing the price A (W) offers, the loyal A (W) consumer chooses among three options: give up buying the good; buy the good at the price A (W) offers; try searching at W (A). In order for the consumer to be willing to incur the additional, higher search cost at W (A), the consumer must expect the price at W (A) to be much lower than A (W)'s offering. A loyal A (W) consumer may be willing to search at A (W) but unlikely W (A) unless the expected price difference is substantial.

In contrast, consider those consumers who do not have a strong idiosyncratic preference for A or W. The search cost for both retailers may be sufficiently high, leading the consumer to consider not buying the product at all. With a superior algorithm that advertises for products the consumer may like, A shortens the time it takes for the indifferent consumer to locate her desired product. This is where the algorithm expands trade and benefits both the retailer and the consumer. But the reduction in search cost may also expand the set of consumers, not heavily loyal to A or W, who might consider searching at A and at W. Such twice-searching consumers potentially have multiple price offers to consider.

Suppose one such consumer second-searches at W after A. Her three options are: take W's second price offering; take A's first price offering with a small cost to return to A; exit the market with no purchase. With a sophisticated algorithm, A knows what this consumer is searching at A after having already searched at W. Moreover, the algorithm may reveal to A the price W offered to this consumer. This can facilitate A offering the consumer a competitive price, one she is likely to favor over W's.

Now assume that W has acquired an algorithm of similar quality. As a second-searched retailer, W can also match A's price and secure all second-searched sales. For any consumer searching twice, W and A would enter a Bertrand-like price competition. The sheer threat of being undercut from the second-searched firm and losing all second-searched sales would incentivize the first-searched firm to lower its price for first-searchers and deter them from searching twice. This leads to lower prices for all searchers.

# 3 Model

I present a sequential search model with two firms and a unit mass of consumers uniformly distributed on a Hotelling line from 0 to 1, which captures differences in search cost. The model demonstrates how the three effects of algorithm shape firm pricing strategies. The equilibrium results of the model offer an analytical characterization to the lessons narrated earlier.

## 3.1 Firms and Consumers

Two retailers $i \in \{A, W\}$ are located at opposite end of the Hotelling line. They compete to sell a maximum of one unit of homogeneous product per consumer. But differing search costs make shopping at the two retailers' products differentiated to consumers. I normalize marginal cost to zero for both firms. Retailer's objective is to maximize expected total profit across all potential consumers.

With an algorithm, firm $i$ can charge two sets of prices: $p_{i1}$ for the consumers who search $i$ first and $p_{i2}(p_{-i1})$ for the consumers who search $i$ second. Its rival with an algorithm can also charge two prices: $p_{-i1}$ and $p_{-i2}(p_{i1})$, analogous to $i$'s prices. Without an algorithm, the firm charges a uniform price of $p_{i,-i}$ to all consumers as it cannot distinguish search sequence.

Consumers of type $k$ are defined by their location on the Hotelling line: $\theta_k \in [0, 1]$. Each consumer incurs a distinct search cost for A and for W depending on their location on the line if they decide to search.

- The consumer's search cost for A is $s_{Ak} = \sigma_A \theta_k$
- The consumer's search cost for W is $s_{Wk} = \sigma_W (1 - \theta_k)$

Consumer with 0 search cost for A lives at $\theta_k = 0$; consumer with 0 search cost for W lives at $\theta_k = 1$. A consumers' search cost for A is strictly increasing in $\theta_k$ while her search cost for W is strictly decreasing in $\theta_k$ and increasing in $(1 - \theta_k)$.

Every consumer has the same exogenous value $v$ of obtaining the good. Their objective is to maximize their surplus, which also incorporates any search cost and purchase price incurred.

Consumers are naive expected price-takers. In other words, they do not conjecture how their actions impact firm's pricing strategies.[12] Before searching, consumer's expectation of the price firm $i$ will offer is modeled as a given fraction of her value: $\bar{p}_i = (1 - \gamma_i)v$. Her expected payoff of consuming the good (inclusive of purchase price but not search cost) is conveniently $v - \bar{p}_i = \gamma_i v$.

## 3.2 Sequential Search Game

The model consists of the following sequential stages.

- Stage 0: Firms simultaneously choose whether to invest in algorithm or not. Investment in algorithm requires a fixed capital cost $K$. Firms with algorithm are able to obtain a $\sigma_i$ that is smaller than the default; tell whether the firm is first-searched or second-searched by the consumer; track rival prices.

- Stage 1: Observing the outcome to Stage 0, firms simultaneously choose their pricing strategy.

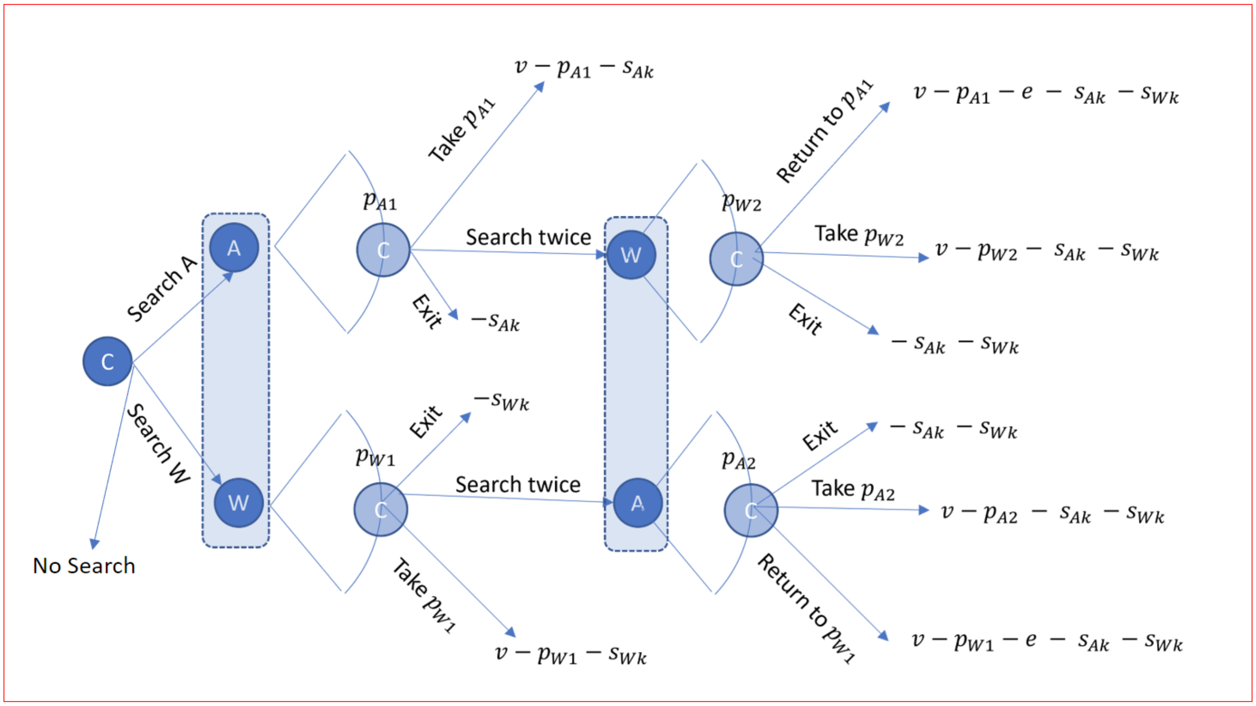
- Stage 2: Consumer chooses whether to search at all, and, if so, which retailer to search first. Searching at retailer $i$ for consumer $k$ costs $s_{ik}$.

- Stage 3: For those consumers who had chosen to search, choose whether to search again at the rival firm with an additional search cost; if not, choose whether to accept the first search firm's offer or exit the market without purchase.

- Stage 4: For those consumers who had chosen to search twice, choose among three options: accept the first offer with an additional small cost to return $e > 0$, accept the

12. This assumption simplifies the calculation of consumers' reservation values. It is not unrealistic to assume that consumers are naive price takers. See conclusion for extensions.

second offer, or exit the market without purchase.




Above is a visual representation of the sequential search game from Stage 2 to Stage 4.

## 3.3 Consumer Payoffs

The firms decide pricing strategy based on the consumers' (expected) payoffs at each stage.

- Stage 2:
  - Payoff from not searching: 0
  - Expected payoff from buying from first search at retailer $i$: $v - \bar{p}_i - s_{ik}$
  - Expected payoff from buying from first search at $-i$: $v - \bar{p}_{-i} - s_{-ik}$
- Stage 3: Assume the consumer first searches at $i$,
  - Payoff from exiting the market after searching once: $-s_{ik}$
  - Payoff from buying from the first-searched i: $v - p_{i1} - s_{ik}$
  - Expected payoff from searching twice and buying from second search: $v - \bar{p}_{-i2} - s_{ik} - s_{-ik}$

- Stage 4:
  - Payoff from searching twice and not buying: $-(s_{ik} + s_{-ik})$
  - Payoff from searching twice and buying from second retailer: $v - p_{-i2} - (s_{ik} + s_{-ik})$
  - Payoff from searching twice and buying from first retailer: $v - p_{i1} - e - (s_{ik} + s_{-ik})$

## 3.4 Algorithmic Effects

An algorithm confers three advantages to a firm.

- Search Cost Reduction. Through targeted advertising and automatic recommendations, a firm with algorithms can lower the perceived effort it takes for a consumer to shop her desired product. This is reflected in a smaller $\sigma_i$. Since $s_{Ak} = \sigma_A \theta_k$, $s_{Wk} = \sigma_W(1 - \theta_k)$, as $\sigma_i$ decreases, $s_{ik}$ decreases, too. A smaller $\sigma_i$ makes consumers more likely to be willing to search at $i$.

- Search Sequence Detection. By tracking Cookies, a firm with algorithm learns about a consumer's browsing history. If the consumer has visited rival's sites before on a similar product, the firm knows that it is being second-searched. Otherwise, the firm is first-searched. This distinction allows the firm to price-discriminate effectively between first-searchers and second-searchers and strategize with two sets of contingencies (first-searched case and second-searched case). Without such algorithm, the firm faces one contingency, charging uniform price for all.

- Rival Price Monitoring. Similarly, a firm with algorithm can access a second-searcher's browsing history, including the historical price its rival offered. This feature allows the firm to offer the second search consumer a price responsive to the earlier price after the consumer received.

# 4 Analysis

I adopt a Perfect Bayesian Equilibrium framework and use backward induction to analyze four possible pricing game scenarios: both firms have algorithm (but possibly asymmetric demand), denoted as (Alg, Alg); A has algorithm but W does not, denoted as (Alg, Non); W has algorithm but A does not, denoted as (Non, Alg); and neither have algorithm, denoted as (Non, Non).

## 4.1 Disjoint Markets or Overlapped Markets

For a sequential search model, it is critical to test if a market allows some consumers to benefit from second search. Otherwise, it turns into a model with two regional monopolies. The lack of contested market space eliminates strategic interaction between the two firms.

Consider the consumer located at $\theta_k^*$ who is indifferent between searching at A or W first, but possibly choosing to not search at all. Location $\theta_k^*$ is defined by:

$$v - \bar{p}_A - s_{Ak} = v - \bar{p}_W - s_{Wk}$$

$$\Rightarrow (1-\gamma_W)v - (1-\gamma_A)v = \sigma_A\theta_k^* - \sigma_W(1-\theta_k^*)$$

$$\theta_k^* = \frac{(\gamma_A - \gamma_W)v + \sigma_W}{\sigma_A + \sigma_W}$$

As the expected payoff from searching at A (W) strictly increases (decreases) with respect to $\theta_k$, markets for A and W overlap only when this indifferent consumer receives a strictly positive payoff from searching at A:

$$v - \bar{p}_A - \sigma_A\theta_k^* > 0$$

$$\Rightarrow \gamma_A v > \sigma_A \frac{(\gamma_A - \gamma_W)v + \sigma_W}{\sigma_A + \sigma_W}$$

$$\Rightarrow v > \frac{1}{\gamma_A} \frac{\sigma_A}{\sigma_A + \sigma_W}(\gamma_A - \gamma_W)v + \frac{1}{\gamma_A} \frac{\sigma_A \sigma_W}{\sigma_A + \sigma_W}$$

We call this threshold $\underline{v}$. Note: This can be explained as the middle person's surplus from buying A's products. If $\gamma_A = \gamma_W$, $\sigma_A = \sigma_W$, then $\gamma_A v > \frac{1}{2}\sigma_A$, the indifferent consumer is at the middle of the Hotelling line. For $\gamma_A \neq \gamma_W$ and $\sigma_A \neq \sigma_W$, the term $\frac{\sigma_A}{\sigma_A + \sigma_W}$ corrects for asymmetric $\sigma$, and $(\gamma_W - \gamma_A)$ corrects for asymmetric $\gamma$.

If the consumer's value exceeds $\underline{v}$, then it is potentially beneficial for some consumers to search twice. We say a market is overlapped when the indifferent consumer faces a positive expected payoff from searching at A or W.

**Anatomy of an Overlapped Market**

When the markets are overlapped, the four potential consumer groups are

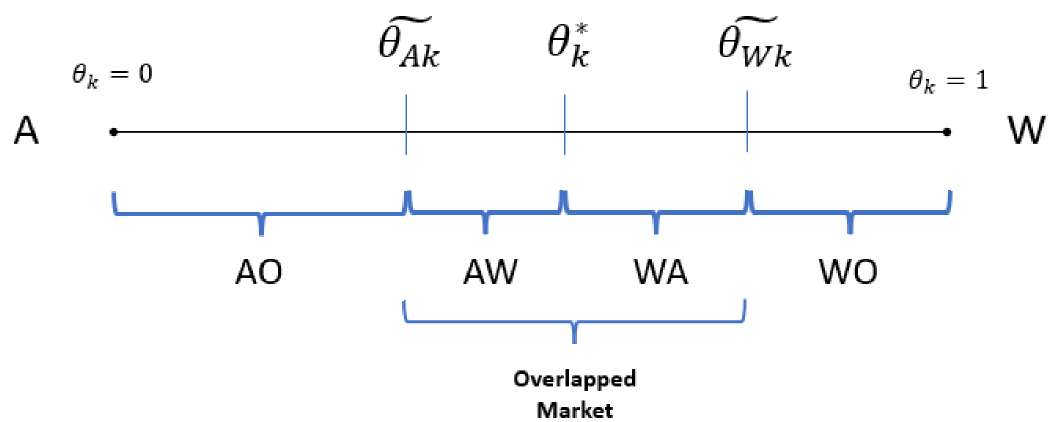
- Search A only (AO): $\theta_k \in [\ 0\ ,\ \tilde{\theta}_{Ak}\ ]$
- Search A then W (AW): $\theta_k \in [\ \tilde{\theta}_{Ak}\ ,\ \theta_k^*\ ]$
- Search W then A (WA): $\theta_k \in [\ \theta_k^*\ ,\ \tilde{\theta}_{Wk}\ ]$
- Search W only (WO): $\theta_k \in [\ \tilde{\theta}_{Wk}\ ,\ 1\ ]$

I will use the following terms to characterize market demand and firms' pricing strategies. I will calculate each term explicitly in scenario analyses:

- $\theta_k^*$ is the location of the consumer indifferent between searching A or W
- $\tilde{\theta}_{Ak}$ is the location of the consumer indifferent as to searching W after having already searched A given first price $p_A$
- $\tilde{\theta}_{Wk}$ is the location of the consumer indifferent as to searching A after having already searched W given first price $p_W$

- $\tilde{p}_{A1}$ is the highest A price at which all first search A consumers only search A
- $\tilde{p}_{W1}$ is the highest W price at which all first search W consumers only search W
- $p^*_{A1}$ is the price that maximizes A's profits from just AO consumers
- $p^*_{W1}$ is the price that maximizes W's profits from just WO consumers

Consumers of each search group are located in an interval over the Hotelling line. Assuming a uniformly distributed mass of consumers, we can represent quantity demanded by calculating the metrical distance from one end of the interval to another. For example, the demand for AO is ($\tilde{\theta}_{Ak} - 0 = \tilde{\theta}_{Ak}$), while the demand for AW is ($\theta^*_k - \tilde{\theta}_{Ak}$). Later in my analysis, I will explicitly refer to the four groups of consumers by their acronym. To visualize the demand, consider the following Hotelling line illustration.




**Strategy for Disjoint Markets**

Each firm's strategy when the markets are disjoint should be consistent across three scenarios as the two firms do not interact strategically. With none of their searchers willing to search at the rival firm, they essentially operate as monopolies. They should charge $p_i = v - \epsilon, \epsilon > 0$. Note that search costs are sunk. They are too exorbitant for consumers to search at the other firm. As such, consumers would accept any offer as long as the first price offering is below their value: $p < v$. Moreover, their initial search decision is based on their expected price rather than actual price, which empirical evidence has documented to be driven primarily by idiosyncratic belief. Hereafter, we focus on the situation when markets are overlapped.

## 4.2 (Alg, Alg)

In this scenario, both retailers have algorithms. With overlapped markets, some consumers may choose to search for the rival product after observing the price offered by the first-searched firm. Their desire to search twice orients from a cost-benefit analysis of the marginal benefit and marginal cost of second search. Therefore, a firm faces two sets of contingencies: being first-searched and being second-searched. The firm chooses a price it offers all first-searched consumers and a price response to rival's first price offered to all second searchers.

If firm $i$ is second-searched, it could observe the first-searched firm's price offering $p_{-i1}$ and decide what price to offer as a best response. Firm $i$ will undercut the price as long as the price lies below $v$ and above the marginal cost, which I normalize to 0.

- If $p_{-i1} > v$, then its best response is to offer a price slightly below $v$, just like a monopoly $p_{i2} = v - \epsilon, \epsilon > 0$;
- If $0 < p_{-i1} < v$, then its best response is to match the rival price $p_{i2} = p_{-i1}$. I have assumed that in the case of a tie in first-searched and second-searched prices, a consumer would always take the second-searched price, because it is more convenient for them to stay at the current retailer and finish the transaction.
- If $p_{-i1} = 0$, Bertrand's Paradox, charge $p_{i2} \geq 0$.

If a firm is first-searched, then it has to account for the fact that consumers might opt to search at the rival firm, too.

**A's Strategy as first-searched**

First observe that the set of consumers who search at A first with overlapped markets are located at the interval $[0, \theta_k^*]$. Consider the highest price $p_{A1}$ such that all consumers in this set will opt not to search at the rival firm. This can be solved by finding the $p_{A1}$ at which the marginal indifferent consumer $\theta_k^*$ is indifferent to accepting $p_{A1}$ and searching twice at W after A.

$$v - \tilde{p}_{A1} - s_{Ak}^* = v - \bar{p}_W - s_{Ak}^* - s_{Wk}^*$$

$$\Rightarrow \tilde{p}_{A1} = \bar{p}_W + \sigma_W(1 - \theta_k^*) = (1 - \gamma_W)v + \sigma_W(1 - \theta_k^*)$$

For any price $p_{A1} > \tilde{p}_{A1}$, some consumer would search twice and take the second price whenever the second price matches the first, which is the best response for W, for reasons analogous to those shown for A.

A faces the tradeoff between charging a higher per-unit price to lower quantity sold, similar to a monopolist's problem. Suppose that A charges a price slightly higher than $\tilde{p}_{A1}, p_{A1} = \tilde{p}_{A1} + \delta, \delta > 0$. More consumers would choose to search twice and take W's price– a loss of quantity demanded for A. In other words, an increase in the first price A offers expands AW and shrinks AO. More specifically, the consumer indifferent between searching A only and continuing to search W is located at

$$\underbrace{\tilde{p}_{A1} + \delta - \bar{p}_W}_{\text{MB of 2nd search}} = \underbrace{\sigma_W(1 - \theta_k)}_{\text{MC of 2nd search}}$$

Inserting $\tilde{p}_{A1} = \bar{p}_W + \sigma_W(1 - \theta_k^*)$, we get the range of consumers given the price increase, $\tilde{\theta}_{Ak}$:

$$\tilde{\theta}_{Ak} = \theta_k^* - \frac{\delta}{\sigma_W}$$

To derive the demand for the first search firm in (Alg, Alg), start with recognizing that the person at point 0 of the type $\theta_k = 0$ is indifferent between accepting A's offer and searching at W when $p_{A1}$ is

$$v - p_{A1} - \sigma_A(0) = v - \bar{p}_W - \sigma_A(0) - \sigma_W(1-0)$$

$$\Rightarrow p_{A1} = \bar{p}_W + \sigma_W$$

which is the highest price A could offer without having everyone also searching at W. Thus, A chooses some price $p_{A1} \in [\tilde{p}_{A1}, \bar{p}_W + \sigma_W]$.

Note that $\bar{p}_W + \sigma_W$ can be rewritten as a function of $\tilde{p}_{A1}$. Recall that $\tilde{p}_{A1} = \bar{p}_W + \sigma_W(1-\theta_k^*)$:

$$\bar{p}_W + \sigma_W = \bar{p}_W + \sigma_W(1-\theta_k^*) + \sigma_W\theta_k^* = \tilde{p}_{A1} + \sigma_W\theta_k^*$$

The inverse demand A faces as the first-searched retailer for $p_{A1}$ is:

$$p_{A1} = (\tilde{p}_{A1} + \sigma_W\theta_k^*) - \sigma_W\tilde{\theta}_{Ak}$$

$$\Rightarrow \tilde{\theta}_{Ak} = \theta_k^* - \frac{p_{A1} - \tilde{p}_{A1}}{\sigma_W}$$

Thus we have the quantity demanded given A's price offering.

A's problem is to choose a $p_{A1}$ that maximizes the expected profit:

$$\begin{aligned}\max_{p_{A1}} \Pi_{A1} &= p_{A1}\tilde{\theta}_{Ak} \text{ such that } p_{A1} \in [\tilde{p}_{A1}, \bar{p}_W + \sigma_W] \\ &= p_{A1}(\theta_k^* - \frac{p_{A1} - \tilde{p}_{A1}}{\sigma_W}) \\ &= p_{A1}(\frac{\sigma_W\theta_k^* + \tilde{p}_{A1}}{\sigma_W} - \frac{p_{A1}}{\sigma_W})\end{aligned}$$

with FOC:

$$\frac{\sigma_W\theta_k^* + \tilde{p}_{A1}}{\sigma_W} - \frac{2p'_{A1}}{\sigma_W} = 0 \text{ as interior solution}$$

$$\Rightarrow p'_{A1} = \frac{\sigma_W \theta_k^* + \tilde{p}_{A1}}{2}$$

Therefore, A chooses to offer the maximum between $p'_{A1}$ and $\tilde{p}_{A1}$. Call this maximum $p^*_{A1}$. If $\tilde{p}_{A1} > \sigma_W \theta_k^*$, choose $\tilde{p}_{A1}$. Otherwise, choose $p'_{A1}$.[13]

**W's Strategy as first searched**

Using analogous arguments as with A, consider the best-case scenario for W: there exists a $\tilde{p}_{W1}$ such that the marginal consumer is indifferent between taking W's price and continuing to search A.

$$v - \tilde{p}_{W1} - s^*_{Wk} = v - \bar{p}_A - s^*_{Ak} - s^*_{Wk}$$

For any price $p_{W1} > \tilde{p}_{W1}$, some consumer would search twice and take the second price whenever the second price matches the first, which is the best response for A.

W faces the tradeoff between charging a higher per-unit price to lower quantity sold, similar to a monopolist's problem. Suppose that W charges a price slightly higher than $p_{W1} = \tilde{p}_{W1} + \delta, \delta > 0$. Then more consumers would choose to search twice and take A's price instead – a loss of quantity demanded for W. WA expands, and WO shrinks. More specifically, the consumer indifferent between searching W only and continuing to search A is located at

$$\underbrace{\tilde{p}_{W1} + \delta - \bar{p}_A}_{\text{MB of 2nd search}} = \underbrace{\sigma_A \theta_k}_{\text{MC of 2nd search}}$$

The range of consumers given the price increase, $\tilde{\theta}_{Wk}$, is:

$$\tilde{\theta}_{Wk} = \frac{\delta}{\sigma_A} + \theta_k^*$$

13. Note: If $\tilde{p}_{A1} = \sigma_W \theta_k^*$, then $p'_{A1} = \tilde{p}_{A1}$.

The highest price W can charge when the consumer indifferent between accepting W's offer and searching at A when $p_{W1}$ is:

$$p_{W1} = \bar{p}_A + \sigma_A$$

W chooses some price $p_{W1} \in [\tilde{p}_{W1}, \bar{p}_{A1} + \sigma_A]$. The demand function when W charges a price above $\tilde{p}_{W1}$ and below $\bar{p}_A + \sigma_A$ is:

$$\tilde{\theta}_k = \theta_k^* + \frac{p_{W1} - \tilde{p}_{W1}}{\sigma_A}$$

W's problem is to choose a $p_{W1}$ that maximizes the expected profit:

$$\max_{p_{W1}} \Pi_{W1} = p_{W1}(1 - \tilde{\theta}_{Wk}) \text{ such that } p_{W1} \in [\tilde{p}_{W1}, \bar{p}_{A1} + \sigma_A]$$

with FOC:

$$p'_{W1} = \frac{\sigma_A(1 - \theta_k^*) + \tilde{p}_{W1}}{2}$$

W chooses to offer the maximum between $p'_{W1}$ and $\tilde{p}_{W1}$. Call this maximum $p^*_{W1}$. If $\tilde{p}_{W1} < \sigma_A(1 - \theta_k^*)$, choose $\tilde{p}_{W1}$. Otherwise, choose $p'_{W1}$.

## 4.3 (Alg, Non)

In this scenario, A is equipped with an algorithm, but W is not. A faces two sets of contingencies but W only faces one.

**A's Strategy**

Since A has the algorithm as in the previous case, it faces the same sets of contingencies. Recall that $\tilde{p}_{A1}$ is the highest A price at which all first search A consumers are AO and none are AW.

When $p_W \leq \tilde{p}_{A1}$, A's profit maximization problem is analogous to the one from (Alg, Alg). A would never offer a price lower than $\tilde{p}_{A1}$, in which case no AW consumers exist, making $p_W$ irrelevant. But A may choose to charge a price higher than $\tilde{p}_{A1}$, in which case it loses all AW consumers but extracts more surplus from AO consumers. Thus, we need to compare the expected profit A receives when A charges $p^*_{A1}$ (monopolist's price) versus $p_W - \eta$ (undercut price). When A charges $p^*_{A1}$, it only retains the AO. When A charges $p_W - \eta$, it retains all AO+AW consumers.[14]

If $p_W > \tilde{p}_{A1}$, $p^*_{A1} > p_W > \tilde{p}_{A1}$, then we have

$$\underbrace{(p_W - \eta)\theta^*_k}_{\text{Profit for undercut price}} > \underbrace{p^*_{A1}\tilde{\theta}_{Ak}}_{\text{Profit for monopolist price}}$$

$$\Rightarrow p_W > p^*_{A1}\frac{\tilde{\theta}_{Ak}}{\theta^*_k} = p^*_{A1}[\frac{\theta^*_k - \frac{p^*_{A1} - \tilde{p}_{A1}}{\sigma_W}}{\theta^*_k}]$$

Therefore, A's profit-maximizing price is still $p^*_{A1} = \max\{\tilde{p}_{A1}, p'_{A1}\}$.[15]

When $p_W > \tilde{p}_{A1}$, then A can charge a price higher than $\tilde{p}_{A1}$ and still retain all of AO+AW consumers. A knows that with the algorithm, it can charge a price slightly lower than W's first price offering and retain AW consumers. This makes A's response price $p_W - \eta$, small $\eta > 0$. Alternatively, A could willingly forego some sales in AW and extract a higher surplus from the remaining consumers. This makes A's response price $p^*_{A1}$.

A wants to charge $p^*_{A1}$ if $p_W < p^*_{A1}\frac{\tilde{\theta}_K}{\theta^*_k}$, $p_W - \eta$ otherwise. By construction, we know that $\tilde{p}_{A1} \leq p^*_{A1}$, as the latter is the maximum between the former and the interior solution. So,

14. Note: AO depends on $p_A$. As $p^*_{A1} \geq p_W$, the AO for $p^*_{A1}$ will be smaller than the AO for $p_W - \eta$.

15. Note that the actual values of $(\tilde{p}_{A1}, p^*_{A1})$ differs between (Alg, Alg) and (Alg, Non) as $\sigma_W$ differs in value across those two algorithm scenarios. The way to calculate these values remains the same.

if $p^*_{A1} > p_W > \tilde{p}_{A1}$, A still wants to charge $p^*_{A1}$. But if $p_W > p^*_{A1} \geq \tilde{p}_{A1}$, then A wants to charge $p_W - \eta$ to undercut.

Overall, A's best response function is

$$\mathrm{BR}_A(p_W) \quad = \quad \begin{cases} p^*_{A1} & \text{if } p_W \leq p^*_{A1}\frac{\tilde{\theta}_{Ak}}{\theta^*_k} \\ p_W - \eta & \text{if } p_W > p^*_{A1}\frac{\tilde{\theta}_{Ak}}{\theta^*_k} \end{cases}$$

**W's Strategy**

Without an algorithm, W can only offer a uniform price $p_W$ to all consumers. Regardless of what price W offers for WA, it will always be undercut by A for any $p_W \geq 0$. Therefore, to maximize expected profit, W has to focus on WO and AW consumers.

If W considers $p_W > p^*_{A1}$, W knows that it will gain no sale from AW consumers, as its price is too high. Any profit W can expect to gain will come from WO consumers. W's maximization problem is thus

$$\max_{p_W} \Pi_W = \max_{p_W} p_W(1 - \tilde{\theta}_{Wk}) \text{ such that } p_W > p^*_{A1}\frac{\tilde{\theta}_{Wk}}{\theta^*_k}$$

We have studied a similar problem in (Alg, Alg). W's best response is to charge $p^*_W$.

If W considers $p_W \leq p^*_{A1}\frac{\tilde{\theta}_{Wk}}{\theta^*_k}$ instead, W may get some AW consumer sales in addition to WO sales. More concretely, its expected profit maximization problem is

$$\max_{p_W} \Pi_W = \underbrace{p_W(1 - \tilde{\theta}_{Wk})}_{\text{WO sales}} + \underbrace{p_W(\theta^*_k - \tilde{\theta}_{Ak})}_{\text{AW sales}} \text{ such that } p_W \leq p^*_{A1}\frac{\tilde{\theta}_{Wk}}{\theta^*_k}$$

Note that $p^*_W$ is the optimal price W can charge for the WO sales only. Therefore, W chooses between maximizing sales for WO at the expense of losing sales in AW or lowering the price to retain both WO and AW sales. But then we have the two following cases:

If $p_W^* \geq p_{A1}^* \frac{\tilde{\theta}_{Wk}}{\theta_k^*}$, then W needs to compare profits from WO consumers at $p_W^*$ versus profits from WO consumers plus all AW consumers at a price $p_{A1}^*$. Note that W would never charge a price below $p_{A1}^*$ as it can get all AW consumers with $p_w = p_{A1}^*$. Because A's best reponses depends on whether $p_W \geq p_{A1}^*$ and not by how much lower, the size of AW consumers will not increase even if $p_w$ were lower. Similarly, if W charges a price above $p_W^*$, it loses all AW consumers anyways, so its best strategy is still to maximize the profit from WO consumers.

If $p_W^* < p_{A1}^* \frac{\tilde{\theta}_{Wk}}{\theta_k^*}$, W faces two options: charging $p_W^*$ or charging slightly higher than $p_W^*$ but just below $p_{A1}^*$. If it charges $p_W^*$, it maximizes profits from WO consumers. If it charges $p_W^* + \delta$, where $0 < \delta < p_{A1}^* - p_W^*$, it does not maximize profit from WO consumers but possibly retains more sales from AW consumers. To characterize this tradeoff, I set up the maximization function as

$$\max_\delta \Pi_W(p_W^* + \delta) = \max_\delta (p_W^* + \delta)[\underbrace{(1 - \theta_k^* - \frac{\delta}{\sigma_A})}_{\text{shrunk WO}} + \underbrace{(\theta_k^* - \tilde{\theta}_{Ak})}_{\text{retained AW}}]$$

$$\text{such that } 0 < \delta < p_{A1}^* - p_W^* = \tilde{p}_W + (1 - \theta_k^*)\sigma_A - p_W^*$$

with FOC:

$$\delta' = \frac{\sigma_A(1 - \tilde{\theta}_{Ak}) - p_W^*}{2}$$

Suppose such value exists in the defined range. When $p_W^* < p_{A1}^*$, W compares the expected profit when $p_W = p_W^*$ versus when $p_W = p_W^* + \frac{\sigma_A(1-\tilde{\theta}_{Ak})-p_W^*}{2} = \frac{p_W^*}{2} + \frac{\sigma_A(1-\tilde{\theta}_{Ak})}{2}$. We know W will choose the latter because, had $p_W^*$ become the optimized value, $\delta'$ would be 0 or negative.

Overall, W's best response function is

$$\text{BR}_W(p_A) \;=\; \begin{cases} p^*_{A1} & \text{if } p_W < p^*_{A1} \leq p^*_W \\ \max\{p^*_W, \frac{p^*_W}{2} + \frac{\sigma_A(1-\tilde{\theta}_{Ak})}{2}\} & \text{if } p^*_W \leq p_W < p^*_{A1} \\ p^*_w & \text{if } p_W > p^*_{A1} \end{cases}$$

## 4.4 (Non, Alg)

This scenario flips the asymmetry in algorithm investment from the previous scenario: A does not have the algorithm but W does. As such, the best responses for A and W are derived based on analogous reasoning as in (Alg, Non).

### A's Strategy

Without an algorithm, A can only offer $p_A$ to all consumers, as it does not distinguish first-searchers from second-searchers. Regardless of what price A offers for AW, it will always be undercut by W for any $p_A \geq 0$. Therefore, to maximize expected profit, A has to focus on AO and WA consumers.

If A offers $p_A > p^*_{W1}$, A's best response is to charge $p^*_A$.

If A offers $p_A \leq p^*_{W1}$ instead, its expected profit maximization problem is

$$\max_{p_A} \Pi_A = \underbrace{p_A\tilde{\theta}_{Ak}}_{\text{AO sales}} + \underbrace{p_A(\tilde{\theta}_{Wk} - \theta^*_k)}_{\text{WA sales}} \text{ such that } p_A \leq p^*_{W1}$$

Note that $p^*_A$ is the optimal price A can charge for the AO sales only. Therefore, A chooses between maximizing sales for AO at the expense of losing sales in WA or lowering the price to retain both AO and WA sales. But then we have the two following cases:

If $p^*_A \geq p^*_{W1}$, then A needs to compare profits from AO consumers at $p^*_A$ versus profits from

AO consumers plus all WA consumers at a price $p^*_{W1}$.

If $p^*_A < p^*_{W1}$, A faces two options: charging $p^*_A$ or charging slightly higher than $p^*_A$ but just below $p^*_{W1}$. It maximizes expected profit as

$$\max_\delta \Pi_A(p^*_A+\delta) = \max_\delta (p^*_A+\delta)[\underbrace{(\theta^*_k - \frac{\delta}{\sigma_W})}_{\text{shrunk AO}} + \underbrace{(\tilde{\theta}_{Wk} - \theta^*_k)}_{\text{retained WA}}]$$

$$\text{such that } 0 < \delta < p^*_{W1} - p^*_A = \bar{p}_W + \sigma_W - p^*_W$$

with FOC:

$$\delta' = \frac{-p^*_A + \sigma_W \tilde{\theta}_{Wk}}{2}$$

Overall, A's best response function is

$$\text{BR}_A(p_W) \quad = \quad \begin{cases} p^*_{W1} & \text{if } p_A < p^*_{W1} \leq p^*_A \\ \max\{p^*_A, \frac{p^*_A+\sigma_W\tilde{\theta}_{Wk}}{2}\} & \text{if } p^*_A \leq p_A < p^*_{W1} \\ p^*_A & \text{if } p_A > p^*_{W1} \end{cases}$$

**W's Strategy**

Since W has the algorithm it faces two sets of contingencies. Recall that $\tilde{p}_{W1}$ is the highest W price at which all first search W consumers are WO and none are WA.

When $p_A \leq \tilde{p}_{W1}$, W's profit maximization problem is analogous to the one from (Alg, Alg). If $p_A > \tilde{p}_{W1}$, $p^*_{W1} > p_A > \tilde{p}_{W1}$, then we have

$$\underbrace{(p_A - \eta)(1-\theta^*_k)}_{\text{Profit for monopolist price}} \quad > \quad \underbrace{p^*_{W1}(1-\tilde{\theta}_{Wk})}_{\text{Profit for undercut price}}$$

$$\Rightarrow p_A > p^*_{W1}\frac{(1-\tilde{\theta}_{Wk})}{(1-\theta^*_k)} = p^*_{W1}[\frac{\theta^*_k + \frac{p'_{W1}-\tilde{p}_{W1}}{\sigma_A}}{1-\theta^*_k}]$$

Overall, W's best response function is

$$\text{BR}_W(p_A) \quad = \quad \begin{cases} p^*_{W1} & \text{if } p_A \leq p^*_{W1}\frac{1-\tilde{\theta}_{Wk}}{(1-\theta^*_k)} \\ p_A - \eta & \text{if } p_A > p^*_{W1}\frac{1-\tilde{\theta}_{Wk}}{1-\theta^*_k} \end{cases}$$

## 4.5 (Non, Non)

Both A and W will charge the same price to all consumers as neither can distinguish whether some consumer is searching them first or second. So, the pricing strategy for each retailer reduces to a single uniform price choice: $p_A$ and $p_W$ respectively.

### A's Strategy

A faces three potential sets of consumers: AO, AW, and WA. The profit it receives from each set of consumers depends on $p_A$ relative to $p_W$.[16]

$$\Pi_A \quad = \quad \begin{cases} p_A \left( \underbrace{\tilde{\theta}_{Ak}}_{\text{AO}} + \underbrace{(\theta^*_k - \tilde{\theta}_{Ak})}_{\text{AW}} + \underbrace{(\tilde{\theta}_{Wk} - \theta^*_k)}_{\text{WA}} \right) & \text{if } p_A < p_W \\ p_A \left( \underbrace{\tilde{\theta}_{Ak}}_{\text{AO}} + \underbrace{(0)}_{\text{AW}} + \underbrace{(\tilde{\theta}_{Wk} - \theta^*_k)}_{\text{WA}} \right) & \text{if } p_A = p_W \\ p_A \left( \underbrace{\tilde{\theta}_{Ak}}_{\text{AO}} + \underbrace{(0)}_{\text{AW}} + \underbrace{(0)}_{\text{WA}} \right) & \text{if } p_A > p_W \end{cases}$$

Suppose the conjectured $p_W = p^*_{A1}$. Charging $p_A > p_w = p^*_{A1}$ is weakly dominated by charging $p_A = p_W = p^*_{A1}$ as profits from AO consumers are maximized at $p_A = p^*_{A1}$ by construction. If A were to consider charging $p_A < p_W = p^*_{A1}$, then

- Profits from AO consumers are maximized at $p_A = p^*_{A1}$
- Profits from AW consumers are maximized at $p_A = p_W - \eta = p^*_{A1} - \eta$ and drops to zero if any higher

16. Note that $\tilde{\theta}_{Ak}$ differs across the three scenarios as the underlying $p_A$ differs.

- Profits from WA consumers are maximized at $p_A = p_W = p^*_{A1}$ and drops to zero if any higher

Therefore, as long as there are some AW consumers (i.e. $p^*_{A1} > \tilde{p}_{A1}$), the only $p_A < p_W = p^*_{A1}$ that A would consider is $p_A = p_W - \eta = p^*_{A1} - \eta$. But if $p^*_{A1} = \tilde{p}_{A1}$ and hence no WA consumers, A will prefer to charge $p_A = p^*_{A1}$.

Whether A wants to charge $p_A = p^*_{A1} - \eta$ or $p_A = p^*_{A1}$ depends on whether there exist AW consumers. If none, A's best response is to charge $p^*_{A1}$. If some, A should charge $p_A = p^*_{A1} - \eta$ as there exists some small $\eta > 0$ such that profits from AW more than offset profits lost from AO and WA from charging $\eta$ lower than the otherwise optimal $p^*_{A1}$.

Now suppose the conjectured $p_W > p^*_{A1}$. Any price $p_A < p_W$ gives A all the sales from AO, AW, and WA consumers. If A were to charge $p_A = p_W$, A gets the sales from AO and WA only but loses sales to AW consumers. But if there are no AW consumers at $p_A = p_W$, A would be better off charging $p_A = p_W$. This is not possible, as $p_w > p^*_{A1} \geq \tilde{p}_{A1}$. Therefore, at $p_A = p_W > p^*_{A1} \geq \tilde{p}_{A1}$, there are some AW consumers. Therefore, A should charge $p_A = p_w - \eta$ as there exists some small positive $\eta$ such that profits from AW gained from reducing price by $\eta$ swamps the slightly higher profits from AO and WA consumers stemming from charging $\eta$ higher price.

Now suppose the conjectured $p_W < p^*_{A1}$. There is a tension between profit gained from AO and profit from AW and WA as profit from AO is maximized at $p_A = p^*_{A1}$ and profits from AW and WA require $p_A \leq p_W$ for WA and $p_A < p_w$ for AW in order to be positive. As profit from AO declines monotonically as $p_A$ falls from $p^*_{A1}$ and profit from AW declines monotonically as $p_A$ falls from $p_W - \eta$ and profit from WA declines monotonically as $p_A$ falls from $p_W$, A should be choosing $p_A$ from among $\{p^*_{A1}, p_w, p_w - \eta\}$. If $p_W \leq \tilde{p}_{A1}$ then charging $p_A = p_W$ dominates charging $p_A = p_W - \eta$ as at $p_A = p_W \leq \tilde{p}_{A1}$ there are no AW consumers;

neither are there AW consumers for the even lower $p_A = p_W - \eta$.

If $p_W = \tilde{p}_{A1} = p^*_{A1}$ then $p_A = p_W = p^*_{A1}$ as A gets sales from AO and WA at the best price possible for those two consumers and there are no AW consumers. If $p_W < \tilde{p}_{A1} \leq p^*_{A1}$ then compare

$$p^*_{A1} \cdot \underbrace{\tilde{\theta}_{Ak}}_{\text{evaluated at } p_A = p^*_{A1}}$$

to

$$p_W \cdot \underbrace{\tilde{\theta}_{Wk}}_{\text{evaluated at } p_W}$$

A will choose whichever of the two $p_A$ has higher profits above

$$p_A = \begin{cases} p^*_{A1} & \text{if } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} \geq p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \\ p_w & \text{if } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} < p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \end{cases}$$

If $p_W > \tilde{p}_{A1}$ then there are some AW consumers even when $p_A = p_W > \tilde{p}_{A1}$ (or the even lower $p_A = p_W - \eta$). Therefore, charging $p_A = p_W - \eta$ for a sufficiently small positive $\eta$ dominates charging $p_A = p_W$.

- If $p_W \geq p^*_{A1} > \tilde{p}_{A1}$, then A will charge $p_A = p_W - \eta$ as A gets all three sets of consumers at that high price (and any higher would lead to losing AW consumers of which there will be some given $p_W - \eta > \tilde{p}_{A1}$. Note that A is fine with $p_A = p_W - \eta > p^*_{A1}$ as it captures all AW consumers even at that higher price

- if $p_W > p^*_{A1} \geq \tilde{p}_{A1}$, then A will charge $p_A = p_W - \eta$ for similar reasons as above

- If $p^*_{A1} > p_W > \tilde{p}_{A1}$, then compare

$$p^*_{A1} \cdot \underbrace{\tilde{\theta}_{Ak}}_{\text{evaluated at } p_A = p^*_{A1}}$$

to

$$(p_W - \eta) \cdot \underbrace{\tilde{\theta}_{Wk}}_{\text{evaluated at } p_W}$$

A will choose whichever of the two $p_A$ has higher profits above

$$p_A = \begin{cases} p^*_{A1} & \text{if } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} \geq p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \\ p_w - \eta & \text{if } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} < p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \end{cases}$$

Altogether

$$BR_A(p_w) = \begin{cases} p^*_{A1} & \text{if } p_W = p^*_{A1} = \tilde{p}_{A1} \\ p^*_{A1} - \eta & \text{if } p_W = p^*_{A1} > \tilde{p}_{A1} \\ p_W - \eta & \text{if } p_W > p^*_{A1} \geq \tilde{p}_{A1} \\ p^*_{A1} & \text{if } p_W < \tilde{p}_{A1} \leq p^*_{A1} \text{ and } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} \geq p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \\ p_W & \text{if } p_W < \tilde{p}_{A1} \leq p^*_{A1} \text{ and } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} < p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \\ p^*_{A1} & \text{if } p^*_{A1} > p_W > \tilde{p}_{A1} \text{ and } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} \geq p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \\ p_W - \eta & \text{if } p^*_{A1} > p_W > \tilde{p}_{A1} \text{ and } p^*_{A1} \cdot \tilde{\theta}_{Ak}|_{p^*_{A1}} < p_W \cdot \tilde{\theta}_{Wk}|_{p_W} \end{cases}$$

**W's Strategy**

Using analogous arguments to those developed for A's strategy, I derive W's best response to $p_A$. The profit W receives from each set of consumers depends on $p_A$ relative to $p_W$.[17]

$$\Pi_W = \begin{cases} p_W \left( \underbrace{(\theta_k^* - \tilde{\theta}_{Ak})}_{\text{AW}} + \underbrace{(\tilde{\theta}_{Wk} - \theta_k^*)}_{\text{WA}} + \underbrace{(1 - \tilde{\theta}_{Wk})}_{\text{WO}} \right) & \text{if } p_W < p_A \\ p_W \left( \underbrace{(\theta_k^* - \tilde{\theta}_{Ak})}_{\text{AW}} + \underbrace{(0)}_{\text{WA}} + \underbrace{(1 - \tilde{\theta}_{Wk})}_{\text{WO}} \right) & \text{if } p_W = p_A \\ p_W \left( \underbrace{(0)}_{\text{AW}} + \underbrace{(0)}_{\text{WA}} + \underbrace{(1 - \tilde{\theta}_{Wk})}_{\text{WO}} \right) & \text{if } p_W > p_A \end{cases}$$

Altogether

$$BR_W(p_A) = \begin{cases} p_{W1}^* & \text{if } p_A = p_{W1}^* = \tilde{p}_{W1} \\ p_{W1}^* - \eta & \text{if } p_A = p_{W1}^* > \tilde{p}_{W1} \\ p_W - \eta & \text{if } p_A > p_{W1}^* \geq \tilde{p}_{W1} \\ p_{W1}^* & \text{if } p_A < \tilde{p}_{W1} \leq p_{W1}^* \text{ and } p_{W1}^* \cdot (1 - \tilde{\theta}_{Ak}|_{p_{A1}^*}) \geq p_W \cdot (1 - \tilde{\theta}_{Wk}|_{p_W}) \\ p_A & \text{if } p_A < \tilde{p}_{W1} \leq p_{W1}^* \text{ and } p_{W1}^* \cdot (1 - \tilde{\theta}_{Ak}|_{p_{A1}^*}) < p_A \cdot (1 - \tilde{\theta}_{Wk}|_{p_W}) \\ p_{W1}^* & \text{if } p_{W1}^* > p_A > \tilde{p}_{W1} \text{ and } p_{W1}^* \cdot (1 - \tilde{\theta}_{Ak}|_{p_{A1}^*}) \geq p_A \cdot (1 - \tilde{\theta}_{Wk}|_{p_W}) \\ p_A - \eta & \text{if } p_{W1}^* > p_A > \tilde{p}_{W1} \text{ and } p_{W1}^* \cdot (1 - \tilde{\theta}_{Ak}|_{p_{A1}^*}) < p_A \cdot (1 - \tilde{\theta}_{Wk}|_{p_W}) \end{cases}$$

17. Note that $\tilde{\theta}_{Wk}$ differs across the three scenarios as the underlying $p_A$ differs.

# 5 Equilibrium Calibrations

Having derived best responses for A and W given all four scenarios, I analyze Perfect Bayesian Nash Equilibrium (PBNE) results for the pricing subgame given the algorithm investments. All pricing strategies are formulated in terms of $p_A^*, \tilde{p}_A, p_W^*, \tilde{p}_W$, which are functions of $v, \gamma_A, \gamma_W, \sigma_A, \sigma_W$. Note that $\sigma_A$ and $\sigma_W$ varies with algorithm investment decisions, so $\sigma_i$ for firm $i$ depends on whether $i$ invests in the algorithm. For reference, I summarize the best-response prices for A and W in each of the four scenarios.

**(Alg, Alg)**:

- A charges $p_{A1}^*$ for first-searchers and conjectured first price $p_W$ for second-searchers.
- W charges $p_{W1}^*$ for first-searchers and conjectured first price $p_A$ for second-searchers.

**(Alg, Non)**:

- If conjectured $p_W > p_{A1}^* \frac{\tilde{\theta}_{Ak}}{\theta_k^*}$, A charges $p_W - \eta$ for first-searchers and $p_W$ for second-searchers. W charges $p_W^*$ for all consumers.
- If conjectured $p_W \leq p_{A1}^* \frac{\tilde{\theta}_{Ak}}{\theta_k^*}$, A charges $p_{A1}^*$ for first-searchers and $p_W$ for second-searchers. W charges $\max\{p_W^*, \frac{p_W^*}{2} + \frac{\sigma_A(1-\tilde{\theta}_{Ak})}{2}\}$ if $p_W^* \leq p_W$ or $p_{A1}^*$ if $p_W^* > p_W$ for all consumers.

**(Non, Alg)**:

- If conjectured $p_A > p_{W1}^*$, A charges $p_A^*$ for all consumers. W charges $p_A - \eta$ for first-searchers and $p_A$ for the second-searchers.
- If conjectured $p_A \leq p_{W1}^*$, A charges $\max\{p_A^*, \frac{p_A^* + \sigma_W \tilde{\theta}_{Wk}}{2}\}$ if $p_A^* \leq p_A$ or $p_{W1}^*$ if $p_A^* > p_A$ for all consumers, W charges $p_{W1}^*$ for first-searchers and $p_A$ for second-searchers.

**(Non, Non)**: Many cases exist.

## 5.1 Market Calibrations

Solving for PBNE results analytically is complicated by many edge cases. To illustrate the main takeaways of my model, in lieu of analytical results, I calibrate PBNE with judiciously chosen values for each variable. I analyze five related markets.

- Market 1 is the symmetric base case. It highlights the difference in $\sigma_i$ with and without an algorithm for firm $i$. The algorithm proportionally lowers $\sigma_i$ for every consumer.
- In Market 2, there exists asymmetry in $\sigma_i$ with algorithm across the two firms. It captures the case that, even when both firms may have algorithms, one's algorithm is more sophisticated and effective than the other.
- In Market 3, there exists a greater difference between algorithmic $\sigma_i$ and non-algorithmic $\sigma_i$ symmetric for each firm. Due to even more reduced search cost, market overlap expands even more, leading to even more potential second-searchers.
- Market 4 considers asymmetry in consumers' belief structure. If consumers expect firm $i$ to have lower prices than its rival, $\gamma_i > \gamma_{-i}$. Different $\gamma$ across A and W introduces vertical differentiation: the firm with higher $\gamma_i$ benefits from a "low price" reputation.
- Market 5 captures how a decrease in consumer value $v$ may narrow the demand overlap, which might result in disjoint markets for some scenarios.

Comparing Markets 2 to 5 with Market 1's results yields insights on how the use of algorithm provides pro-consumer effects, and how much the effects differ in various market conditions.

### Market 1: Base Case

I consider a market where the product is sufficiently valued such that all consumers will search at least once regardless of the algorithms. Moreover, $v$ is large enough such that market overlap exists across all four scenarios. But algorithms can lower search cost significantly such that they can induce a much larger degree of market overlap. To focus on this particular dynamic, I make the two firms, with the possible exception of their algorithm investment, symmetric. I let $v = 5$, $\gamma_A = \gamma_W = 0.4$. For $\sigma_i$,

- With algorithm, $\sigma_A = \sigma_W = 0.8$
- Without algorithm, $\sigma_A = \sigma_W = 2$

Given these parameters, market overlap exists for all four scenarios, as the consumer expects to receive high enough of a surplus from searching and purchasing the good. Table 1 shows PBNE prices for A and W. If firm $i$ is able charge two prices, its pricing profile is shown as an ordered pair ($p_{i1}$, $p_{i2}$).

Table 1: PBNE Prices: Base Case

| Scenario | A | W |
|---|---|---|
| (Alg,Alg) | (3.4, 3.4) | (3.4, 3.4) |
| (Alg,Non) | $(3.57, 3.57 - \epsilon)$ | 3.57 |
| (Non,Alg) | 3.57 | $(3.57, 3.57 - \epsilon)$ |
| (Non,Non) | 4 | 4 |

The prices in Market 1 depict pro-consumer effects of the adoption of algorithms. In the (Non, Non) case, each firm charges $p_i = p_{-i} = 4$ for all consumers. Even without algorithms, the price is still below monopolistic pricing due to the market overlap. If only one firm has an algorithm, the price drops to 3.57 or $3.57 - \epsilon$ for all consumers regardless of their search sequence or number of searches. The drop in the price for a firm with an algorithm is a result of all three effects of algorithm. Recommendation effect reduces the search cost and expands the overlapped market, which the second-searched firm with algorithm can capture by matching its rival's first price offering. The sequence tracking and rival price monitoring effects drive the price down in the overlapped markets due a variation of Bertrand's competition. What appears surprising is that the price offering by the firm without an algorithm also drops. This can be explained by the theory of credible threat: any firm with an algorithm can credibly threaten its rival that it matches the first price for any second-searchers. The firm without an algorithm is incentivized to deter its first-searchers from second search via a discount. Prices drop more when both retailers use an algorithm, as the market overlap is the most extensive and both firms face credible threat of price matches from their rival.

**Market 2: Greater difference in Alg $\sigma_i$ and Non $\sigma_i$ for A**

Similar to Market 1 except that I let:

- With algorithm, $\sigma_A = 0.7 < \sigma_W = 0.8$
- Without algorithm, $\sigma_A = \sigma_W = 2$

Intuitively, these parameters reflect scenario where some firms can adopt more sophisticated algorithms than others. The more sophisticated the algorithm, the greater reduction in search cost, and the greater expansion of market overlap.

Table 2: PBNE Prices with Asymmetric Algorithmic $\sigma_i$

| Scenario | A | W |
|---|---|---|
| (Alg,Alg) | (3.34, 3.34) | (3.34, 3.34) |
| (Alg,Non) | (3.46, $3.46 - \epsilon$) | 3.46 |
| (Non,Alg) | 3.57 | (3.57, $3.57 - \epsilon$) |
| (Non,Non) | 4 | 4 |

Asymmetry in algorithmic $\sigma_i$ creates different sizes of the market overlap when only one firm has the algorithm. When A has an algorithm and W does not, the greater expansion of overlap as a result of lower $\sigma_A$ leads to more second-searchers, which requires A to lower the price for first-searchers to discourage them from searching twice. In the (Non, Alg) case, W's algorithm reduces search cost less and causes less market overlap. The equilibrium price in (Alg, Non) is lower than in (Non, Alg). Ceteris peribus, consumers benefit more from adoption of more sophisticated algorithms which lower the search cost.

**Market 3: Symmetric $\sigma_i$ with Smaller Difference**

Similar to Market 1 except that I let:

- With algorithm, $\sigma_A = \sigma_W = 1$
- Without algorithm, $\sigma_A = \sigma_W = 2$

In this market, there exists a smaller difference in $\sigma_i$ for with and without algorithm.

Table 3: PBNE Prices: Symmetric $\sigma_i$ with Smaller Difference

| Scenario | A | W |
|---|---|---|
| (Alg,Alg) | (3.5, 3.5) | (3.5, 3.5) |
| (Alg,Non) | $(3.67, 3.67 - \epsilon)$ | 3.67 |
| (Non,Alg) | 3.67 | $(3.67, 3.67 - \epsilon)$ |
| (Non,Non) | 4 | 4 |

Smaller algorithmic effect to reduce search cost leads to a smaller overlap in the market, hence less threat of second search from consumers. A smaller recommendation effect results in higher prices than the base case.

**Market 4: Asymmetry in $\gamma_i$**

Similar to Market 1 except that I let:

- $\gamma_A = 0.5 > \gamma_W = 0.4$.

Asymmetry in consumers' belief structure introduces difference in expected price, $\bar{p}_A$ and $\bar{p}_W$. This asymmetry implies that, ceteris peribus, all consumers expect A's price to be lower than W's: $\bar{p}_A = (1 - \gamma_A)v < \bar{p}_W = (1 - \gamma_W)v$. Ceteris peribus, this leads consumers to favor A over W, a vertical differentiation. But the difference in consumer's search cost with respect to their location on the Hotelling line incentivizes some consumers to search at W. The indifferent consumer of first search should be located to the right of the indifferent consumer in Market 1 across all scenarios.

Table 4: PBNE for Prices with Asymmetry in $\gamma_i$

| Scenario | A | W |
|---|---|---|
| (Alg,Alg) | (3.15, 3.15) | (3.15, 3.15) |
| (Alg,Non) | (3.21, 3.21 $-\ \epsilon$) | 3.21 |
| (Non,Alg) | 3.42 | (3.42, 3.42 $-\ \epsilon$) |
| (Non,Non) | 3.75 | 3.75 |

When consumers expect A's price to be cheaper than W's before initiating the search, all prices– including the equilibrium price where neither firm has algorithm– drop relative to the prices in Market 1. The increase in the expected surplus a consumer gains from purchasing the good induces more consumers to search at A than in Market 1. With large enough of an AO demand, A can still gain significant sales even if W gains all second-searchers. When A has an algorithm, the marginal benefit of first-searching and second-searching at A increases, and the marginal cost of search decreases. Ceteris peribus, it costs W more to deter W's first-searchers to second-search A. Therefore, W offers a lower price than in Market 1 for each scenario, regardless of whether A has an algorithm or not.

**Market 5: Decrease in Value, Disjoint Markets**

The market is similar to Market 1 except for a smaller $v$. Algorithmic effect in reducing search cost and tracking search sequence is less pronounced when consumers do not necessarily commit to searching. That is, if the value they could gain from the good is small, search cost appears relatively high. Even if some consumers would search twice and incur search costs for both firms, the additional surplus they obtain from consuming the good at a lower price is small.

To demonstrate how a smaller $v$ may lead to disjoint markets for different scenarios, I calculate the equilibrium prices for two $v$s: $v = 2$ and $v = 1.3$. When $v = 2$, (Non, Non) scenario forms a disjoint market, and all other scenarios form overlapped markets. In contrast, when $v = 1.3$, only (Alg, Alg) scenario has an overlapped market.

Table 5: PBNE for Prices with $v = 2$ and $v = 1.3$

| $v$ | $v = 2$ | | $v = 1.3$ | |
|---|---|---|---|---|
| Scenario | A | W | A | W |
| (Alg,Alg) | (1.6, 1.6) | (1.6, 1.6) | (1.18, 1.18) | (1.18, 1.18) |
| (Alg,Non) | $(1.77, 1.77 - \epsilon)$ | 1.77 | $(1.3 - \epsilon, 1.3 - \epsilon)$ | $1.3 - \epsilon$ |
| (Non,Alg) | 1.77 | $(1.77, 1.77 - \epsilon)$ | $1.3 - \epsilon$ | $(1.3 - \epsilon, 1.3 - \epsilon)$ |
| (Non,Non) | $2 - \epsilon$ | $2 - \epsilon$ | $1.3 - \epsilon$ | $1.3 - \epsilon$ |

In scenarios without market overlap, the indifferent consumer does not obtain a positive payoff for searching at either A or W. There exist consumers who do not search at all. A and W each act as a monopoly among their searchers without the need to worry about poaching, leading them to a price right below $v$. The only "escape" from monopolistic pricing is for algorithms to reduce the search costs for some consumers enough to induce searching twice. Given a low value of consuming the good, the effect of the algorithm is small due to the small marginal benefit of second search for most consumers. Ceteris peribus, the lower $v$, the less likely overlap, and the less algorithmic effect on improving consumer welfare to escape monopolistic pricing.

## 5.2 Algorithmic Investment

PBNE prices I just calibrated are good indicators for consumer welfare, as consumers are generally better off with lower prices.[18] It remains to show, however, that algorithms confer any competitive advantage to the firm. If the adoption of algorithm improved consumer surplus but decreased producer surplus, no firm would be incentivized to offer the service in the first place (Stage 0). To demonstrate the margin of algorithmic advantage to the firms, I calculate firm's expected payoff given the PBNE pricing strategy in Market 1. Let $K$ be the upfront fixed cost to investment to have an algorithm for both firms.

18. In overlapped markets, all consumers end up buying the good in PBNE.

Table 6: Payoffs for Algorithmic Investment Game in Market 1

| | | $W$ | |
|---|---|---|---|
| | | $Non$ | $Alg$ |
| $A$ | $Non$ | $(2, 2)$ | $(1.02, 2.55 - K)$ |
| | $Alg$ | $(2.55 - K, 1.02)$ | $(1.7 - K, 1.7 - K)$ |

The algorithm investment game has a payoff structure similar to Prisoner's Dilemma with investment in algorithm a dominant strategy as long as $K$ is sufficiently small ($K < 0.55$). The firm gains the most profit when it invests in an algorithm but its rival does not ($2.55-K$). The firm gains the least profit when its rival adopts an algorithm but it does not (1.02). When both firms invest in an algorithm, they may get lower profits ($1.7-K$) than in the case where neither firm invests in an algorithm (2.2). Therefore, consumers are the main beneficiary of mutual adoption of algorithms. However, if either firm is willing to invest more to create asymmetry in $\sigma_i$ values between the two firms, their algorithmic advantage expands, leading to an increase in the (Alg, Alg) profit that asymptotically approaches (Non, Non) profit. As in Prisoner's Dilemma, what drives investment is less the advantage of having an algorithm but more the *disadvantage* of *not* having one when its rival does.

# 6 Discussion and Conclusion

My sequential search duopoly model demonstrates the under-discussed pro-competitive potential of algorithmic pricing from the joint effect of the three features: search cost reduction through recommendation, search sequence tracking, and rival price monitoring. When either or both firms develop pricing strategy with algorithms, the resulting competitive equilibrium may increase consumer welfare for all consumers. That second-searchers can enjoy the benefit of algorithmic pricing due to search-sequence-based price discrimination is intuitive. Perhaps surprising is that the first-searchers may obtain a similar magnitude of benefit merely from the possibility that some of them may be poached by a rival.

The size of the pro-consumer effect largely depends on the extent of market overlap. By stating that firms charge a price approximate to consumer's value $v$ in the disjoint market, we assume that firms already have some technology that enables them to do near first degree price discrimination in a monopoly. Empirical evidence supports this assumption. Calvano et al. (2018) showed the ability of algorithms to identify each consumer's reservation price, enabling them to sell at each consumer's willingness to pay. But in overlapped market, my model shows how strategic price discrimination can lead to price discounts for all consumers, depending on the extent of overlap. The greater the overlap, the more consumers considering searching twice, and the greater the price discount has to be offered to deter second searches. As my calibrations have shown, a greater difference in algorithmic and non-algorithmic $\sigma_i$ may lead to a greater reduction of search cost and greater improvement of consumer welfare. A higher $\gamma_i$ and a lower expected price may lead to greater marginal benefit of second search, which also improves consumer welfare. A lower consumer value of consuming the good, however, may cause narrower market overlap or disjoint market conditions, hence smaller algorithmic effect on reducing equilibrium prices.

The model does not predict that algorithms are universally pro-competitive, nor will all firms adopt them solely for their competitive advantage. The cost of search versus the value of purchasing the good determines the extent to which consumers can benefit from strategic algorithmic competition. The model is most applicable to consumers who highly value the good but lack a concrete idea of its prices. Algorithms encourage firms to offer lower prices to consumers inclined to purchase the good from that firm. Even with the recommendation effect, consumers who derive limited value from the good receive a relatively small algorithmic benefit. When consumers expect high prices and face high search costs, they may choose not to search twice and receive quasi-monopolistic pricing. The algorithmic effect would be limited for consumers who already have sufficient knowledge of the good's price before conducting a search, as they would not expect significant welfare gain from second search.

## Regulatory Insights

Combined with existing research on algorithmic facilitation of tacit collusion, my thesis may inform regulatory authorities to adopt a more nuanced approach to assessing the competitive implications of algorithmic pricing. Most economic analyses on the anti-competitiveness of algorithms implicitly assumes market overlap. They conclude that algorithms' tracking technologies provide greater convenience for the firms to monitor each other's pricing behaviors and punish deviations from price coordination. However, monitoring rival prices only makes sense if the firms face contested demand.

With market overlap, there is a tension between the pro-competitive benefit and tacit collusion concern. The U.S. antitrust law's "consumer welfare standard" doctrine dictates that antitrust law is only violated if the conduct's anti-consumer effect outweighs its pro-consumer effect.[19] Although overt collusion is illegal, tacit collusion is evaluated based on its impact on consumer welfare. When market characteristics make collusion difficult to sustain, such as low discount factor, high product differentiation, or high cross-group externalities, regulators should avoid blanket outlawing of algorithms as the net effect may be pro-competitive.

The legal implications of strategic algorithmic price discrimination depend on legal insti-

19. "Pricing Algorithms: The Antitrust Implications — Advisories," Arnold & Porter, https://www.arnoldporter.com/en/perspectives/advisories/2018/04/pricing-algorithms-the-antitrust-implications.

tutions and different doctrinal focuses. The EU antitrust regulatory framework, in particular, does not view unilateral improvements in consumer welfare as a compelling justification for collusion.[20] While recommendation and tracking technologies improve consumer welfare, they also raise privacy concerns that regulatory authorities are aware of. Some consumers who are skeptical of tracking may turn off Cookies, clear browsing history, or prefer shopping in person, which renders the algorithmic pro-consumer effect almost negligible in the model. In such cases, the authorities may have stronger grounds for corporate scrutiny as the reduction in search cost comes at the expense of personal privacy.

## Possible Extensions

My model builds upon three key sets of assumptions. The first set of assumptions concerns my particular characterization of pricing algorithms, focusing on recommendation, search sequence tracking, and rival price monitoring. The second set of assumptions involves consumers behaving as expected price-takers, which is supported by some behavioral studies. The third set of assumptions involve abstracting away the prospect of collusion and other coordination among firms, as the model is one-shot. I consider possible extensions and relaxations of each assumption.

### Additional Algorithmic Characterizations

Apart from the investment decision, algorithms are static in my model. The characterization is innocuous in a one-shot game. But in a repeated game framework, it may be desirable to model algorithms as evolving, motivated by the observation that algorithms can get more sophisticated with continual use. This suggests an even stronger incentive for firms to adopt algorithms due to the cumulative advantage they may enjoy. Firms that invest in algorithms earlier than their rivals may gain a "first-mover advantage," as their algorithms may improve inference precision by collecting more data compared to algorithms created later. As a result,

20. "Sector inquiry into e-commerce."

the algorithmic overlap expansion may become more pronounced over time.

Alternatively, regulatory authorities in some economies have restricted some of the features considered in my thesis. For example, regulators in Australia have preemptively limited algorithms from actively collecting certain information about consumers, including demographic characterizations that could result in algorithmic discrimination on race and gender.[21] Similarly, Malaysian regulators are considering outlawing the tracking of consumers' browsing history, which could make it difficult for the second-searched firm to undercut its rival's first price offering precisely.[22] A valuable avenue for future research could be to develop a version of my model with a subset of the considered algorithmic effects and examine how the distinction affects my pro-competitive results.

### Sophisticated Consumers

The model assumes that consumers form their price expectations exogenously and invariant of their location on the Hotelling line. The expected price for a consumer depends solely on their value of consuming the good, and not on their position on the demand line or $\sigma_i$. However, consumers may form more sophisticated price expectations. They may be fully rational economic agents who consider both firms' strategies to develop an optimal strategy for themselves. For instance, a fully rational consumer may intentionally click on multiple websites before making a purchase to take advantage of heavier discounts offered to multiple searchers. Thus, how consumers position themselves in the algorithmic game can affect their consumer welfare. However, empirical surveys reviewed earlier suggest that such full rationality may be unrealistic.

Alternatively, consumer sophistication could be framed as a partition of the consumer group into attentive and inattentive learners. In a one-shot game, consumer attention is

21. "AI, Machine Learning & Big Data Laws and Regulations — Australia — GLI," GLI - Global Legal Insights - International legal business solutions, Text, https://www.globallegalinsights.com/practice-areas/ai-machine-learning-and-big-data-laws-and-regulations/australia.

22. "Antitrust and Competition Laws in Malaysia," Global Compliance News, https://www.globalcomplioncenews.com/antitrust-and-competition-laws-in-malaysia/.

captured in the relative search costs they need to pay to access a retailer's product. In a repeated game, an attentive consumer may update their belief about a firm's price offering by observing their previous interactions with the firm. This memory becomes important when a consumer "guesses" the pattern of algorithmic pricing and becomes increasingly skeptical about the pricing of the recommended products. In this case, price discrimination becomes more severe, as inattentive consumers may turn into more attentive consumers over time. Therefore, the firm needs to devise a strategy that not only pays the second-searchers to switch but also compensates the first-searchers to stay.

### Collusion

An obvious direction for future research is to extend the current model to a repeated game framework that allows for possible collusion. Firms could renew their commitment to collusion and their investments in algorithms in each period, resulting in repeated investment and pricing strategy subgames. To ensure that collusion is incentive-compatible, the firms could re-negotiate the surplus transfer required for both to stay in the collusive scheme.

If both firms have algorithms, monitoring deviation is not costly, which would provide the firms with greater incentive to remain in the collusive game. However, if one firm has an algorithm and the other does not, one-way monitoring can potentially destabilize the firm with the algorithm, as its deviation cannot be easily detected, and thus it would be difficult to enforce punishment for deviation. Moreover, firms' temptation to deviate from the collusive scheme may increase by possible strategic algorithmic price discrimination. This temptation could be especially strong if the gains from such price discrimination by poaching most consumers in the market are greater than the expected punishment from the colluding partner(s).

Future research could delve deeper into how a possible strategic algorithmic price discrimination may affect incentive compatibility of tacit collusion, as well as the ways in which the terms of collusion might, in turn, impact algorithm investments. Following these research topics, it is crucial to show how collusion could potentially undermine the benefits that algorithmic pricing brings to consumers. A more comprehensive understanding of these issues can inform policymakers and firms alike in designing effective strategies to regulate algorithmic pricing and prevent anti-competitive behavior.

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