Syllabus for the Multivariable Calculus Portion of the Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Elementary vector analysis

- Dot or scalar product
- Cross or vector product
- Lines and planes
- Tangent vectors and tangent lines to parametrized curves

Definitions and computations involving functions of several variables

- Partial derivative
- Directional derivative
- Gradient
- Tangent plane to a surface

Maxima and minima of functions of several variables

- Finding critical points
- The second derivative test for local maxima/minima and saddle points
- The method of Lagrange multipliers

Double integrals

- Cartesian and polar coordinates
- Finding area and volume

Triple integrals

- Cartesian, cylindrical and spherical coordinates
- Finding volume

Line integrals of vector fields

- Fundamental Theorem of Calculus for Line Integrals
- Green's Theorem

Syllabus for the Comprehensive Examination in Linear Algebra (MATH 271/2)
Department of Mathematics and Statistics, Amherst College

Basic Definitions

- Vector space
- Subspace
- Span of a finite subset
- Linear independence of a finite set of vectors
- Basis and dimension
- Linear transformation
- Kernel or null space
- Image or range
- Inverse of a matrix or linear transformation
- Determinant
- Characteristic polynomial
- Eigenvalues and eigenvectors of a matrix
- Diagonalizability

Computational Techniques

- Determine when a subset is a subspace
- Basic matrix manipulations
- Row operations on matrices
- Solving systems of linear equations
- Find the inverse of a matrix
- Find a basis of a given subspace
- Find the nullity, rank, and determinant of a matrix
- Find the null space $N(T)$ and range $R(T)$ of a linear transformation T
- Given bases of V and W, find the matrix of a linear transformation $T : V \to W$
- Given a matrix
  - Compute its characteristic polynomial
  - Find its eigenvalues and eigenspaces

Basic Results to Know

- dim $N(T)$ + dim $R(T)$ = dim V
- nullity(A)+rank(A)=number of columns of A
- Criteria for A^{-1} to exist
- Criteria for A to be diagonalizable

Write short proofs for problems involving subspaces, linear maps, linear independence, spanning sets, null spaces and ranges.

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Syllabus for the Algebra Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Sets, Functions, and Integers

- One-to-one, onto, and bijective maps
- Equivalence relations and equivalence classes
- Division algorithm, gcd and lcm, primes and unique factorization

Groups

- Uniqueness of identities and inverses
- The order of an element

Subgroups

- Lagrange's Theorem and its consequences
- Cosets
- Normal subgroups
- Quotient groups

Group Homomorphisms

- Kernels and images
- Isomorphisms
- The Fundamental Theorem of Group Homomorphisms: $G/\mathrm{Ker}(\phi) \simeq \mathrm{Im}(\phi)$

Permutations

- S_n and disjoint cycle decomposition
- Transpositions and A_n

Rings

- Commutative rings; Rings with unity; Fields
- Polynomial rings

Ideals

- Ideals
- Quotient rings

Ring Homomorphisms

- Kernels and images
- Isomorphisms
- The Fundamental Theorem of Ring Homomorphisms: $R/\mathrm{Ker}(\phi) \simeq \mathrm{Im}(\phi)$

Quotient Rings and Fields

- Criteria for R to be a field
- Maximal ideals
- Criteria for R/M to be a field

Polynomial Rings $k[x]$, for a Field k

- The division algorithm
- Every ideal in $k[x]$ is principal
- Irreducible polynomials and maximal ideals in $k[x]$

Syllabus for the Analysis Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Mathematical Induction

The Real Numbers

- Rational and irrational numbers
- Real numbers and the axiom of completeness

Sequences

- Convergence
- Convergence of bounded monotone sequences
- Cauchy sequences
- The Bolzano-Weierstrass Theorem for sequences

Point-Set Theory

- Limit points (also called cluster points or accumulation points)
- The Bolzano-Weierstrass Theorem for sets
- Open and closed sets
- Compact sets and the Heine-Borel Theorem

Infinite Series

- Convergence
- p-series and geometric series
- Absolute and conditional convergence
- Comparison, ratio, and alternating series tests

Limits and Continuity

- The limit of a function
- The definition of continuity and relation to sequences
- Continuity of sums, products, quotients, and compositions
- The Intermediate Value Theorem
- Boundedness
- Attainment of extreme values
- Uniform continuity

Differentiability and Derivatives

- Limit definition of derivative
- Derivatives at local extreme points
- The Mean Value Theorem and Rolle's Theorem

Sequences of Functions

- Pointwise and uniform convergence
- Continuity of the limit function
- Proving uniform convergence

Series of Functions

- Pointwise and uniform convergence
- The Weierstrass M-test
- Power series
- Radius and interval of convergence, behavior at endpoints
- Continuity and differentiation of power series

Integration

- Definition of the Riemann integral
- Properties of the Riemann integral
- Integrability of a continuous function over $[a, b]$
- Integration of sequences and series
- Integration of power series