

Not Purely Wasteful:
Exploring a Potential Benefit to Weak Patents

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May 1, 2014

Submitted to the
Department of Economics of Amherst College
in partial fulfillment of the
requirements for the degree of
Bachelor of Arts with honors

Acknowledgments

I would like to thank Professors Jun Ishii and Stanislav Rabinovich for their guidance and advice throughout the process, and Prof. Brian Baisa for his guidance in the Fall. Your comments were much appreciated, and this entire project would not have been possible without your help.

I would also like to thank Professor Jessica Reyes for her guidance to me and all of the other thesis students through the initial steps of this process.

I would also like to thank the entire Economics Department at Amherst College for supporting the thesis writing experience, and for teaching the necessary skills and thought processes to undertake the thesis process.

I would like to thank my friends and classmates for an enjoyable four years at Amherst.

Lastly, I would like to thank my entire family. I would have never made it to this point without their help.

Abstract

The rise of weak patents in recent years has given large firms incentives to try to obtain a large patent portfolio to increase their chances of success in intellectual property disputes. Weak patents provide an incentive for firms to acquire many related patents, because many of the patents provide overlapping property rights. My thesis focuses on the incentives for expenditure on patenting efforts in order to amass such a portfolio. There is an informational asymmetry problem between those applying for patents and the government, so the government runs the risk of either inappropriately granting patent protection or failing to do so when it was warranted. Allowing the level of joint expenditure on patenting efforts to affect the probability of approval for patent protection, the government can use the joint expenditure as a signal of the quality of the underlying innovation. I show that while this leads to a higher level of equilibrium expenditure on patenting efforts, a system where joint patent expenditure acts as a signal and affects approval probability may dominate both a system of guaranteed approval and a system of arbitrary approval.

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1 Introduction

Intellectual property (IP) scholars have noted a rise in what have been termed *weak patents* over the past few decades. Weak patents are patents that have uncertain scope or validity, and therefore do not specify well-defined property rights. Underlying this notion of weak patents is the observation that patents do not provide a guaranteed property right, but rather the right to attempt to enforce such a right, so the validity of a patent is unknown until it is challenged in court (Lemley and Shapiro, 2005). One of the responses to these weak patents is the creation of the patent thicket, “a dense web of overlapping intellectual property rights”, that requires firms to obtain licenses for multiple pieces of technology from multiple sources in order to bring a product to market (Shapiro, 2001). One of the ways in which firms react to this uncertainty is through attempting to acquire a large patent portfolio in order to increase their chances of being awarded patent protection.

There is considerable media coverage of the large expenditure on patent acquisition¹ and patent lawsuits between large firms, especially in the technological sector.² There is a public perception that too much effort and too many resources are expended on trying to obtain monopoly rights through the legal system, and that sentiment is often keyed into by officials at large technology companies.³ This public perception motivates a focus on the allocation of resources away from productive efforts

¹A Stanford study estimated that \$20 Billion was spent on patent litigation and acquisition in 2009-2011. Charles Duhigg and Steve Lohr. 2012. “The Patent Used as a Sword.” *The New York Times*, October 7. <http://www.nytimes.com/2012/10/08/technology/patent-wars-among-tech-giants-can-stifle-competition.html>

²For example, an article that ran on the front page of the August 24, 2012 edition of *The New York Times* carried the headline “Jury Awards \$1 Billion to Apple in Samsung Patent Case”. Nick Wingfield. 2012. “Jury Awards \$1 Billion to Apple in Samsung Patent Case.” *The New York Times*, August 24. <http://www.nytimes.com/2012/08/25/technology/jury-reaches-decision-in-apple-samsung-patent-trial.html>

³Google’s deputy counsel for patents mentioned that one potential benefit of a new licensing agreement with Samsung is that “companies can reduce the potential for litigation and focus on innovation”. Min-Jeong Lee and Jonathan Cheng. 2014. “Samsung, Google Sign Patent-License Deal”. *The Wall Street Journal*, January 27. <http://online.wsj.com/news/articles/SB10001424052702303277704579345791388237228>

and toward patenting efforts as a source of deadweight loss. The allocation away from production toward the contest to acquire a larger patent portfolio is a form of rent-seeking behavior which Krueger (1974) showed leads to deadweight loss.

My thesis focuses on the incentives for expenditure on patenting efforts in order to amass such a portfolio. Weak patents provide an incentive to acquire a large number of related patents, because many of the patents overlap one another. This results in the distribution of the property rights to an underlying innovation distributed among multiple parties. The conflicting property rights make it difficult to determine which firm, if any, is entitled to patent protection for a particular innovation, so this decision is left to a court to decide. It is unknown prior to litigation which individual patent will be key in determining which firm is assigned the rights to the innovation. As a result, firms have an incentive to acquire as many patents as possible in order to improve their probability of being granted the property right.

There is not only uncertainty about the validity of individual patents, but also about whether the innovation for which firms are seeking patent protection should even be granted protection. It is possible for firms to expend resources on patent efforts surrounding a particular innovation to create a patent thicket, only to have the entire thicket found invalid in a patent dispute. Currently, the approval process does little to screen for the quality of an underlying innovation, and therefore is unable to distinguish between innovations that should and should not be granted patent protection. The government currently employs a system in which all decisions of validity are left to the courts, so it grants patents to all underlying innovations.

It is costly for the government to try to ascertain the true quality of an innovation, because of an informational asymmetry problem. Those applying for patents have better information about whether the innovation for which they seek patent protection is deserving of such protection. However it may be possible for the government to use the joint level of rent-seeking activity as a signal of the underlying innovation

quality. Allowing the probability of approval for the underlying innovation to vary with the joint expenditure on patent efforts gives firms an incentive to reveal their private information about patent quality. However such a system could also provide incentives for higher levels of rent-seeking activity.

The familiar story of how patents create deadweight loss centers on the channel of monopoly power rather than the channel of rent-seeking described above. A patent holder is granted a monopoly in order to provide an incentive for undertaking innovative research, and the society is to bear the cost of the monopoly for the life of the patent. There is a considerable amount of research about the tradeoff between incentives for innovative behavior and the social cost of monopoly studied under the existence of strong patents.⁴ In my thesis I consider this tradeoff as well, but with patent protection offered by weak rather than strong patents.

This tradeoff is an important consideration when the government weighs the costs of either failing to grant approval to deserving innovations or granting approval to undeserving innovations. If the government fails to approve an innovation that is worthy of patent protection it commits a type I error, and if it grants approval to an innovation unnecessarily it commits a type II error. Type II errors create a welfare loss through the familiar channel of monopoly power, and type I errors create a welfare loss due to potential distortions to innovative behavior.

In order to study potential deadweight losses due to the expenditures on patenting efforts, I create a model to explain the tradeoff firms make between productive allocation toward commercializing an innovation and appropriative allocation to acquire patents for that innovation. The property rights afforded by these weak patents are poorly defined, so the traditional treatment of patents providing certain and near costlessly enforced property rights to the holder is not applicable. However, the re-

⁴Examples of topics covered in the literature include, but are by no means limited to, both the timing of innovation (Reinganum, 1989) and cumulative innovation (Scotchmer, 1991) and, more recently, (Bessen and Maskin, 2009) all of which study innovation under a system of strong intellectual property rights.

relationship between poorly defined property rights and expenditure on appropriative technology has been studied extensively in the economics literature about conflict. I am borrowing two key pieces from the literature on conflict; the contest success function (CSF) and the general framework of guns and butter.

The CSF is a function that maps the levels of allocative goods each side holds to a probability of winning a particular contest; the typical example relates the number of weapons each side hold to the probability they would win an open war. I use a contest success function that relates the weapons of IP the patent portfolios of the rival firms, to the probability they would win a particular conflict over IP, an IP suit. The guns and butter model is a basic framework of the tradeoff a decision maker, typically a nation, makes between expenditures on production and appropriation. I am adapting this guns and butter framework to explain the tradeoff firms make between allocations toward patenting and productive efforts.

My thesis uses this model to focus on how expenditure on patenting activities is affected by the probability that the underlying innovation is granted patent approval. Specifically, my thesis analyzes the question of how the use of a system that incorporates rent-seeking activity into the probability of approval affects equilibrium expenditure relative to both a system which always grants approval and a system that arbitrarily grants approval. I find that equilibrium expenditure on patenting efforts is highest under a system incorporating patent expenditure into the probability of approval. My thesis also analyzes the question over whether the value of this revealed information, realized through a relaxation of the tradeoff between a type I and type II error, exceeds the costs of higher allocation toward patenting efforts. I show that depending on a social planner's objectives, a system that incorporates joint expenditure on patenting efforts may emerge as the optimal system.

2 A Model of Allocation to Patent Efforts

The incentive for expenditure on patenting efforts leads firms to make a tradeoff between expenditure on productive efforts and patenting efforts. This tradeoff between productive and appropriative efforts is similar to that faced by the players in the “guns and butter” models from the literature on conflict, and also of the tradeoff in models of rent seeking. In the context of intellectual property, patent efforts play the role of guns in the technology of conflict. The contest success function, typically used to translate the level of guns held by a particular player into a probability of winning a contest, is used here to relate expenditure on patenting efforts to a probability of winning a dispute over the intellectual property rights to a particular innovation. There are two prevalent functional forms for a CSF in the literature: the ratio form and the logistic form. In the context of patent efforts, the ratio form is more applicable because, assuming that the other firm input a non-zero level of effort, the probability of success is zero for a firm that puts in zero effort, whereas for the logistic form it is a positive, non-zero value (Hirshleifer, 1995). The following model is a slight modification of a ‘guns and butter’ model presented in Garfinkel and Skaperdas (2007), which I am naming the *weak patents game*.

2.1 The Weak Patents Game

Suppose that there are two symmetric, risk-neutral firms endowed with an equal level of resources, R , operating in a market competing over the right to the use of a particular innovation. The firms allocate the entire endowment between expenditure on patenting efforts, x , and productive efforts, $R - x$. The decision to undertake research to develop new innovations does not factor into my model. Rather, I am modeling the decision for a firm to commercialize a particular innovation. The firm could allocate toward productive efforts to bring the innovation to market, or it could

allocate toward appropriative efforts to capture the exclusive rights in a particular innovation. Success in litigation over the intellectual property right is assumed to be stochastically related to the level of patenting efforts of the two firms due to weak patents. The relationship between patenting efforts and success in litigation is given by the following contest success function:

$$p_i(x_1, x_2) = \begin{cases} \frac{x_i}{x_1 + x_2} & \text{if } x_1 + x_2 \neq 0 \\ \frac{1}{2} & \text{otherwise} \end{cases} \quad (1)$$

Suppose that dispute over the right to use a particular innovation is a winner-take-all contest, and that the winner receives the joint production of the two firms as the prize. Therefore the expected profit to firm i is the following:

$$\pi_i(x_1, x_2) = p_i(x_1, x_2)(2R - (x_1 + x_2)) \quad (2)$$

I am excluding the possibility of outside agreement that would result in a settlement outcome that is not self-enforcing. If we assume that patenting activities are purely appropriative, then a peaceful equilibrium in which neither firm allocates toward patenting activities would be optimal because it results in no wasteful expenditure. However, as shown in the following lemma, such an equilibrium is not self-enforcing.

Lemma 2.1. *The strategy $x_1 = x_2 = 0$ is not a Nash-equilibrium of the weak patents game.*

Proof. Note that $\pi_i(0, 0) = \frac{1}{2}(2R - (0 + 0)) = R$

Given that firm 2 plays the strategy $x_2 = 0$, then $\forall \epsilon \in (0, R)$ if $x_1 = \epsilon$

$$\begin{aligned} \pi_1(x_1, 0) &= \frac{\epsilon}{\epsilon + 0}(2R - (\epsilon + 0)) \\ &= 2R - \epsilon > R = \pi(0, 0) \end{aligned}$$

Therefore $x_1 = 0$ is strictly dominated by any choice of $x_1 \in (0, R)$, given that $x_2 = 0$. Given that firm 1 plays the strategy $x_1 = 0$, $\forall \epsilon \in (0, R)$ if $x_2 = \epsilon$

$$\begin{aligned}\pi_1(0, x_2) &= \frac{\epsilon}{0 + \epsilon}(2R - (0 + \epsilon)) \\ &= 2R - \epsilon > R = \pi(0, 0)\end{aligned}$$

Therefore $x_2 = 0$ is strictly dominated by any choice of $x_2 \in (0, R)$. The best response for one firm given that the other firm allocates nothing to patenting activities is to allocate the smallest possible non-zero value toward patenting activities, so $x_1 = x_2 = 0$ is not a Nash-equilibrium. \square

This game represents a battle of the giants scenario, so the firms are assumed to be symmetric, therefore I restrict any further analysis to symmetric equilibria.

Lemma 2.2. *In the symmetric equilibrium of the weak patents game, allocation toward patenting efforts is $x_1^* = x_2^* = x_w^* = \frac{R}{2}$.*

Proof. Lemma 2.1 implies that $x_1 \neq 0$ and $x_2 \neq 0$, so this assumption will be maintained for the remainder of the proof. Maximizing π_1 w.r.t. x_1 gives the first order condition (FOC) for firm 1

$$\begin{aligned}\frac{\partial}{\partial x_1} \pi_1 &= \frac{x_2}{(x_1 + x_2)^2}((R - x_1) + (R - x_2)) - \frac{x_1}{x_1 + x_2} \\ &= \frac{x_2(2R - x_1 - x_2) - x_1(x_1 + x_2)}{(x_1 + x_2)^2} \\ &= \frac{2Rx_2}{(x_1 + x_2)^2} - 1 = 0\end{aligned}\tag{3}$$

By symmetry, the FOC for firm 2 is

$$\frac{\partial}{\partial x_1} V_2 = \frac{2Rx_1}{(x_1 + x_2)^2} - 1 = 0\tag{4}$$

Solving for symmetric equilibria, let $x_1 = x_2 = x$. Substituting this in to (3) and (4) gives:

$$\begin{aligned} 0 &= \frac{2Rx}{(2x)^2} - 1 \\ &= \frac{R}{2x} - 1 \\ \Rightarrow x &= \frac{R}{2} \end{aligned}$$

Therefore, we conclude that $x_1^* = x_2^* = \frac{R}{2}$. □

Under the assumptions above, firms allocate half of their resource endowment toward patenting activities, so total allocation is half of the total initial endowment of resources.

2.2 Incorporating Private Information

In the previous model, I assumed that the system of weak patents grants patent protection to all innovations regardless of novelty or obviousness. This leads firms to engage in a race of expenditure on patenting efforts for every innovation. In the specification of p_i in the previous game, I assumed that the underlying innovation had already been approved for patent protection and that expenditure on patent efforts only affected the probability of victory. A reinterpretation of p_i as the probability of winning an intellectual property dispute *conditional* on the underlying innovation being approved for patent protection will allow for an extension to the above model that will capture the effects of a system that attempts to minimize patent protection being granted to innovations that are not novel.

There is an informational asymmetry inherent in the patent application process; those applying for patent protection possess private information about the feasibility of the underlying innovation. The societal costs of patent protection should only

be borne when the incentive to innovate is necessary, so if the innovation does not meet the standards set for patent approval it should not be granted a patent. Due to familiarity with research related to the innovation, applicants have better information of whether an innovation deserves patent protection than the government. One possible solution to the information asymmetry problem is for the government to spend a larger number of resources to more correctly ascertain the quality of the underlying innovation for which firms are seeking patent protection.⁵ Another possibility, explored in this section, is for the government to draw upon the firm's private information about the innovation through a competitive patenting process in order to more accurately ascertain the feasibility of the underlying innovation. It is costly for the government to obtain the private information about the feasibility of the innovation, but it can induce the firms to reveal that information by allowing private expenditure on patenting activities to affect the probability of approval.

In order to study how such a system would affect expenditure on patenting activities, it is first necessary to develop a formal notion of innovation quality. I use the term quality to capture the aspects of novelty and non-obviousness. High quality innovations are more novel and less obvious, and therefore more difficult to commercialize than low quality ones. High quality innovations are the types of innovations the government is trying to encourage by providing incentives in the form of patents. Innovations that are not novel or are obvious will be easier for firms to implement because they are not drastically different than those presently available. However, there is a certain level of novelty or non-obviousness for which the incentives provided by patent protection are necessary to induce firms to undertake that innovation.

Suppose that the quality of an innovation is a random variable uniformly distributed between 0 and 1; $\phi \sim U[0, 1]$. Let $\phi^* \in (0, 1)$ represent the feasibility

⁵Some scholars, most notably Lemley (2000), maintain that it is preferable for the U.S. PTO to remain "rationally ignorant. So few patents end up being of commercial significance, that it is not worth the cost of more intensely reviewing each individual patent.

threshold. If $\phi < \phi^*$, then the innovation is of low quality and would be undertaken and commercialized in the absence of patent protection, so patent protection is not necessary for such an innovation. If $\phi \geq \phi^*$, then the innovation is of high quality and would not be developed without patent protection because the private costs of doing so outweigh the private benefit of commercializing the innovation without monopoly rights.

I assume that government has information about the distribution of ϕ , but does not observe ϕ directly, so it has to make an estimate of the worthiness of patent protection for a particular innovation. Armed only with the knowledge of the distribution of ϕ , a simple estimate would assess $(1 - \phi^*)$ fraction of all innovations worthy and therefore confer patent protection onto that fraction of underlying innovations arbitrarily, regardless of whether or not the innovation is truly valid.⁶ I define a type I error to be the failure to grant patent protection to a worthy innovation, and a type II error to be the granting of unnecessary patent approval to an unworthy innovation.

The system above that grants patent protection arbitrarily would lead to the following probabilities of Type I and Type II errors

$$P(\text{Type I}) = P(\text{no approval} \mid \text{high quality}) = \phi^* \tag{5}$$

$$P(\text{Type II}) = P(\text{approval} \mid \text{low quality}) = 1 - \phi^* \tag{6}$$

It is easy to see from the above probabilities that it is not possible for such a system to reduce the probability of one type of error happening without raising the probability that the other occurs. However, if the system incorporated information about the quality of the underlying innovation, then this tradeoff need not occur. The govern-

⁶Arbitrarily granting patent protection to exactly $1 - \phi^*$ fraction of all innovations might not be optimal from the social planner's perspective. It is possible that the ideal arbitrary rejection level is higher or lower than this depending on the social planner's loss function, but the answer to that question is not the subject of my analysis. Adding this additional piece adds complexity without adding much insight into the question that is at hand.

ment cannot directly incorporate information about the underlying quality because it does not observe ϕ , but it can use the firms' private expenditure on patenting activities as a signal of the underlying quality.

Under this informational asymmetry, there is no incentive for firms to reveal their private information unless the government can make a credible commitment toward awarding patents under a competitive patent scheme that incorporated private expenditure. This commitment becomes credible if the quality of the patent factors into the firms' expected profits regardless of whether the government were awarding patent protection arbitrarily or through a scheme incorporating private information. In the weak patent game analyzed above, the firms were exogenously endowed with a level of resources that was independent of any other exogenous factors. If the endowment of resources is allowed to vary with the quality of the underlying innovation, then the expected profits to the firms depend directly on the quality of the innovation.

It is not necessary to specify an exact relationship between R and ϕ for the purposes of my analysis, only that $R(\phi) > 0$, and $R(\phi)$ is increasing in ϕ for all values of ϕ . The first property is necessary in order for there even to be a tradeoff for firms to make. If the resources available to a firm are zero, then there is no decision to make between the two activities, because allocation to both is zero. The second property, that $\frac{\partial R}{\partial \phi} > 0$, captures the effect of an increase in quality on the private value of the innovation. High values of ϕ represent radical innovations and low values represent incremental innovations.⁷ I am assuming that when firms are trying to commercialize a more novel innovation, they have a larger pool of resources available to them.⁸ One way in which this could occur is increased access to credit due to a higher perceived commercial value of a radical innovation relative to an incremental innovation. Note that $R(\phi)$ does not vary with respect to the decision the firms make between

⁷For a survey of the many terms used to describe the novelty of an innovation, including radical and incremental, see Garcia and Calantone (2002).

⁸The exact mechanism through which that happens is does not factor into the main question I am considering

allocation toward patenting, because it is only a function of ϕ which is exogenously determined.

2.3 Arbitrary Weak Patents Game

The government could choose not to incorporate expenditure into the probability of approval, and just award patent protection arbitrarily as discussed above. This implies that the probability of approval is $f = 1 - \phi^*$, so the weak patents game is modified by this probability of approval to give the *arbitrary weak patents game* with the following expected profits:

$$\begin{aligned}\pi_i &= p_i(x_1, x_2)f(2R - (x_1 + x_2)) \\ &= (1 - \phi^*)p_i(x_1, x_2)(2R - (x_1 + x_2))\end{aligned}\tag{7}$$

Lemma 2.3. *The strategy $x_1 = x_2 = 0$ is not a Nash-equilibrium of the arbitrary weak patents game.*

Proof. Note that $\pi_i(0, 0) = \frac{1}{2}(1 - \phi^*)(2R - (0 + 0)) = (1 - \phi^*)R$

Given that firm 2 plays the strategy $x_2 = 0$, then $\forall \epsilon \in (0, R)$ if $x_1 = \epsilon$

$$\begin{aligned}\pi_1(x_1, 0) &= \frac{\epsilon}{\epsilon + 0}(1 - \phi^*)(2R - (\epsilon + 0)) \\ &= (1 - \phi^*)(2R - \epsilon) > (1 - \phi^*)R = \pi(0, 0)\end{aligned}$$

therefore $x_1 = 0$ is strictly dominated by any choice of $x_1 \in (0, R)$.

Given that firm 1 plays the strategy $x_1 = 0, \forall \epsilon \in (0, R)$ if $x_2 = \epsilon$

$$\begin{aligned}\pi_1(x_1, 0) &= \frac{\epsilon}{0 + \epsilon}(1 - \phi^*)(2R - (0 + \epsilon)) \\ &= (1 - \phi^*)(2R - \epsilon) > (1 - \phi^*)R = \pi(0, 0)\end{aligned}$$

therefore $x_2 = 0$ is strictly dominated by any choice of $x_2 \in (0, R)$. The best response for one firm given that the other firm allocates nothing to patenting activities is to allocate the smallest possible non-zero value toward patenting activities, so $x_1 = x_2 = 0$ is not a Nash-equilibrium. \square

Once again, a peaceful outcome is not self-enforcing, so I solve for non-zero self-enforcing equilibria to this game.

Lemma 2.4. *In the symmetric equilibrium of the arbitrary weak patents game, allocation toward patenting efforts is $x_1^* = x_2^* = x_a^* = \frac{(1 - \phi^*)R}{2}$.*

Proof. The payoffs to the two firms are:

$$\pi_1(x_1, x_2) = (1 - \phi^*) \frac{x_1}{x_1 + x_2} (2R - (x_1 + x_2)) \quad (8)$$

and

$$\pi_2(x_1, x_2) = (1 - \phi^*) \frac{x_2}{x_1 + x_2} (2R - (x_1 + x_2)) \quad (9)$$

Maximizing π_1 w.r.t x_1 given x_2 gives the first order condition

$$\begin{aligned} \frac{\partial}{\partial x_1} \Pi &= (1 - \phi^*) \frac{x_2}{(x_1 + x_2)^2} (2R - (x_1 + x_2)) - \frac{x_1}{x_1 + x_2} \\ &= (1 - \phi^*) \frac{x_2(2R - (x_1 + x_2)) - x_1(x_1 + x_2)}{(x_1 + x_2)^2} \\ &= (1 - \phi^*) \frac{2Rx_2}{(x_1 + x_2)^2} - 1 = 0 \end{aligned} \quad (10)$$

and maximizing π_2 w.r.t x_2 given x_1 gives the first order condition

$$\begin{aligned}
\frac{\partial V}{\partial x_1} &= (1 - \phi^*) \frac{x_1}{(x_1 + x_2)^2} ((R - x_1) + (R - x_2)) - \frac{x_2}{x_1 + x_2} \\
&= (1 - \phi^*) \frac{x_1(2R - (x_1 + x_2)) - x_2(x_1 + x_2)}{(x_1 + x_2)^2} \\
&= (1 - \phi^*) \frac{2Rx_1}{(x_1 + x_2)^2} - 1 = 0
\end{aligned} \tag{11}$$

Solving for a symmetric equilibrium, substitute $x_1 = x_2 = x$ to obtain the following:

$$\begin{aligned}
0 &= (1 - \phi^*) \frac{2Rx}{(2x)^2} - 1 \\
&= (1 - \phi^*) \frac{R}{2x} - 1
\end{aligned}$$

This implies $x_1^* = x_2^* = \frac{(1 - \phi^*)R}{2}$. □

Awarding patent protection arbitrarily allows the government to lower equilibrium allocation toward patenting activities. This control over the equilibrium comes with the risk of a type I error that arises due to patents only being awarded to a fraction of innovations at random.

2.4 The Revealed Information Patents Game

Using the level of expenditure on patenting efforts as a signal of the feasibility of the underlying innovation, the government can allow the probability of approval to vary with the level of expenditure. In order to provide the correct incentives the probability of approval should be increasing in the value of the innovation, and therefore should be increasing in x_i which the government is using as its measure of ϕ . The probability of approval should be larger than that of the arbitrary level for any positive level of expenditure in order to provide an incentive for firms to reveal their private information. If that was not the case, then the firms would have no incentive

to reveal their private information because they would do strictly better if they left it up to chance. The government does not observe $R(\phi)$, for if they did they could deduce the quality of innovation, but suppose they know that $R(1) = M$. Therefore, they can conclude that $x_1 + x_2 \leq 2M \forall \phi$ because R is increasing in ϕ . Suppose that the government grants approval of patent protection according to the following form⁹:

$$f(x_1, x_2) = (1 - \phi^*) + \phi^* \left(\frac{x_1 + x_2}{2M} \right) \quad (12)$$

Lemma 2.5. $f(x_1, x_2)$ satisfies the following properties:

1. $0 \leq f(x_1, x_2) \leq 1 \forall x_1, x_2 \in [0, M]$
2. $\frac{\partial}{\partial x_i} f > 0$
3. $f(x_1, x_2) \geq 1 - \phi^* \forall x_1, x_2 \in [0, M]$

Proof. 1. If $x_1, x_2 \in [0, R]$, then $0 \leq x_1 + x_2 \leq 2M$. So

$$0 \leq f(x_1, x_2) = (1 - \phi^*) + \phi^* \left(\frac{x_1 + x_2}{2M} \right) \leq 1 - \phi^* + \phi^*(1) = 1$$

$$2. \frac{\partial}{\partial x_i} f = \frac{\phi^*}{2M} > 0$$

3. If $x_1, x_2 \in [0, M]$, then $0 \leq x_1 + x_2 \leq 2M$. So

$$f(x_1, x_2) = (1 - \phi^*) + \phi^* \left(\frac{x_1 + x_2}{2M} \right) \geq 1 - \phi^* + \phi^*(0) = 1 - \phi^*$$

□

Incorporating expenditure on patenting activities as a signal of patent quality allows the government to reduce the probability of a type I error because by lemma

⁹This function represents a “reduced form” solution to the signaling game. An explicit solution to the signaling problem is left for future research

2.5 the probability of type I error is now

$$P(\text{Type I}) = 1 - f < 1 - (1 - \phi^*) = \phi^*$$

Firms must now take account of how their expenditure on patenting activities affects not only their probability of winning litigation, but also the probability that the underlying innovation is approved for patent protection. Therefore the expected profits in this game, which I am calling the *revealed information patents game*, are:

$$\begin{aligned} \pi_i &= p_i(x_1, x_2)f(x_1, x_2)(2R - (x_1 + x_2)) \\ &= \left((1 - \phi^*) + \phi^* \left(\frac{x_1 + x_2}{2M} \right) \right) p_i(x_1, x_2)(2R - (x_1 + x_2)) \end{aligned} \quad (13)$$

Lemma 2.6. *The strategy $x_1 = x_2 = 0$ is not a Nash-equilibrium of the revealed information patents game.*

Proof. Note that $\pi_i(0, 0) = \frac{1}{2}(1 - \phi^*)(2R - (0 + 0)) = (1 - \phi^*)R$

Given that firm 2 plays the strategy $x_2 = 0$, then $\forall \epsilon \in (0, R)$ if $x_1 = \epsilon$

$$\begin{aligned} \pi_1(x_1, 0) &= \frac{\epsilon}{\epsilon + 0} \left(1 - \phi^* + \frac{\epsilon\phi^*}{2M} \right) (2R - (\epsilon + 0)) \\ &= (1 - \phi^*)(2R - \epsilon) + \frac{\epsilon\phi^*}{2M}(2R - \epsilon) > (1 - \phi^*)R = \pi(0, 0) \end{aligned}$$

therefore $x_1 = 0$ is strictly dominated by any choice of $x_1 \in (0, R)$.

Given that firm 1 plays the strategy $x_1 = 0, \forall \epsilon \in (0, R)$ if $x_2 = \epsilon$

$$\begin{aligned} \pi_1(x_1, 0) &= \frac{\epsilon}{0 + \epsilon} \left(1 - \phi^* + \frac{\epsilon\phi^*}{2M} \right) (2R - (0 + \epsilon)) \\ &= (1 - \phi^*)(2R - \epsilon) + \frac{\epsilon\phi^*}{2M}(2R - \epsilon) > (1 - \phi^*)R = \pi(0, 0) \end{aligned}$$

therefore $x_2 = 0$ is strictly dominated by any choice of $x_2 \in (0, R)$. The best response for one firm given that the other firm allocates nothing to patenting activities is to

allocate the smallest possible non-zero value toward patenting activities, so $x_1 = x_2 = 0$ is not a Nash-equilibrium. \square

Like in the two games analyzed previously, the peaceful equilibrium is not self-enforcing. I therefore solve for a non-zero symmetric equilibrium like before.

Lemma 2.7. *In the symmetric equilibrium of the revealed information patents game, allocation toward patenting efforts is*

$$x_1^* = x_2^* = x_r^* = \frac{\phi^*R - (1 - \phi^*)M + \sqrt{(\phi^*R + (1 - \phi^*)M)^2 - RM\phi^*(1 - \phi^*)}}{3\phi^*}$$

Proof. For notational simplicity let $\alpha = 1 - \phi^*$, so the expected profits to firm i are:

$$\begin{aligned} \pi_i &= \left(\frac{\alpha x_i}{x_1 + x_2} + \frac{\phi^* x_i}{2M} \right) (2R - (x_1 + x_2)) \\ &= \frac{2R\alpha x_i}{x_1 + x_2} + \left(\frac{\phi^* R}{M} - \alpha \right) x_i - \frac{\phi^* x_i (x_1 + x_2)}{2M} \end{aligned}$$

Maximizing π_1 w.r.t x_1 given x_2 gives the first order condition:

$$\frac{\partial}{\partial x_1} \pi_1 = \frac{2R\alpha x_2}{(x_1 + x_2)^2} + \frac{\phi^* R}{M} - \alpha - \frac{\phi^*}{2M} (2x_1 + x_2) = 0$$

By symmetry:

$$\frac{\partial}{\partial x_2} \pi_2 = \frac{2R\alpha x_1}{(x_1 + x_2)^2} + \frac{\phi^* R}{M} - \alpha - \frac{\phi^*}{2M} (2x_2 + x_1) = 0$$

Solving for a symmetric equilibrium, I impose the constraint that $x_1 = x_2 = x$, so the first order condition is now

$$\frac{R\alpha}{2x} + \frac{\phi^* R}{M} - \alpha - \frac{\phi^*}{2M} (3x) = \frac{RM\alpha + (2R\phi^* - 2\alpha M)x - 3\phi^* x^2}{2Mx} = 0$$

I assume $x \neq 0$ by lemma 2.6, which implies the following:

$$3\phi^*x^2 - (2\phi^*R - 2\alpha M)x - RM\alpha = 0$$

Solving this quadratic for x gives:

$$\begin{aligned} x_1^* = x_2^* = x_r^* &= \frac{2R\phi^* - 2\alpha M \pm \sqrt{(2R\phi^* - 2\alpha M)^2 + 12RM\alpha\phi^*}}{6\phi^*} \\ &= \frac{(2R\phi^* - 2\alpha M) \pm \sqrt{4(R\phi^*)^2 + 4RM\alpha\phi^* + 4(\alpha M)^2}}{6\phi^*} \\ &= \frac{2(R\phi^* - \alpha M) \pm 2\sqrt{(R\phi^* + \alpha M)^2 - R\phi^*\alpha M}}{6\phi^*} \end{aligned}$$

substituting for α

$$= \frac{2(R\phi^* - (1 - \phi^*)M) \pm 2\sqrt{((R\phi^* + (1 - \phi^*)M)^2 - R\phi^*(1 - \phi^*)M)}}{6\phi^*}$$

taking the positive root

$$= \frac{\phi^*R - (1 - \phi^*)M + \sqrt{(\phi^*R + (1 - \phi^*)M)^2 - RM\phi^*(1 - \phi^*)}}{3\phi^*}$$

□

3 A Welfare Analysis

The allocation toward patenting activities differs under the three schemes discussed above, with equilibrium allocation toward patenting efforts independent of ϕ^* in the weak patents game, and dependent on ϕ^* in the other two games. This is not surprising because under weak patents there is no threshold value for patent quality, so allocation should not be dependent on a threshold value. However, the equilibrium allocation under all three schemes is affected by a change in the the quality of the underlying innovation.

Lemma 3.1. *The equilibrium allocation toward patenting activities in all three of the*

above games is increasing in the underlying quality, ϕ , of the innovation.

Proof. Recall that I have assumed $\frac{\partial R}{\partial \phi} > 0$. Therefore the partial derivatives with respect to ϕ of the various equilibrium expenditures are as follows:

$$\begin{aligned}\frac{\partial}{\partial \phi} x_w^* &= \frac{1}{2} \frac{\partial R}{\partial \phi} > 0 \\ \frac{\partial}{\partial \phi} x_a^* &= \frac{1 - \phi^*}{2} \frac{\partial R}{\partial \phi} > 0 \\ \frac{\partial}{\partial \phi} x_r^* &= \frac{1}{3} \frac{\partial R}{\partial \phi} + \frac{1}{6\phi^*} \left(\frac{2\phi^* \frac{\partial R}{\partial \phi} + M\phi^*(1 - \phi^*) \frac{\partial R}{\partial \phi}}{\sqrt{(\phi^* R + (1 - \phi^*)M)^2 - RM\phi^*(1 - \phi^*)}} \right) \\ &= \frac{1}{3} \frac{\partial R}{\partial \phi} \left(1 + \frac{2 + M(1 - \phi^*)}{2\sqrt{(\phi^* R + (1 - \phi^*)M)^2 - RM\phi^*(1 - \phi^*)}} \right)\end{aligned}$$

□

The most interesting of the three partial derivatives above to consider is $\frac{\partial}{\partial \phi} x_r^*$. If $x_1 + x_2$ is, as I am arguing, a credible signal for the underlying quality of the innovation, then it should be increasing in the quality of the innovation. In equilibrium, this is true, so this result supports my claim of the government using joint expeditor as a signal of the quality of the underlying innovation.

A comparison of the equilibrium allocation across the three games is necessary to conduct a welfare analysis. For this I need to look not at the rates of change of the equilibrium expenditure, but at the levels of expenditure. The relationship between equilibrium allocations is as follows:

Lemma 3.2. *Allocation toward patenting activities is highest in the revealed information patents game, followed by the weak patents game, and lowest in the arbitrary weak patents game.*

Proof. In order to show the above statement true it is equivalent to show $x_r^* > x_w^* >$

x_a^* . Recall that $\phi^* < 1$, so

$$\begin{aligned}
\frac{R\phi^*}{2} < R\phi^* &\Rightarrow \frac{(R\phi^*)^2}{4} < (R\phi^*)^2 \\
&\Rightarrow \frac{(R\phi^*)^2}{4} + R\phi(1 - \phi^*)M + ((1 - \phi^*)M)^2 < (R\phi^*)^2 + R\phi(1 - \phi^*)M + ((1 - \phi^*)M)^2 \\
&\Rightarrow \sqrt{\left(\frac{R\phi^*}{2} + (1 - \phi^*)M\right)^2} < \sqrt{(R\phi^*)^2 + R\phi(1 - \phi^*)M + ((1 - \phi^*)M)^2} \\
&\Rightarrow 0 < -\frac{R\phi^*}{2} - (1\phi^*)M + \sqrt{(R\phi^*)^2 + R\phi(1 - \phi^*)M + ((1 - \phi^*)M)^2} \\
&\Rightarrow \frac{3R\phi^*}{2} < R\phi^* - (1\phi^*)M + \sqrt{(R\phi^*)^2 + R\phi(1 - \phi^*)M + ((1 - \phi^*)M)^2} \\
&\Rightarrow \frac{R}{2} < \frac{R\phi^* - (1\phi^*)M + \sqrt{(R\phi^*)^2 + R\phi(1 - \phi^*)M + ((1 - \phi^*)M)^2}}{3\phi^*} = x_r^*
\end{aligned}$$

Therefore $x_r^* > x_w^*$, and $\frac{1}{2} > \frac{1 - \phi^*}{2}$, so $x_w^* > x_a^*$. Therefore by transitivity, $x_r^* > x_w^* > x_a^*$. \square

A simple measure of welfare reduction through expenditure on patenting activities does not account for potential welfare gains or losses due to changes in the probabilities of type I and type II errors. A lower probability of a type I error could lead to a welfare gain by increasing incentives to undertake innovative activity. The incentives to innovate increase when the probability of a type I error decreases because firms know that they are less likely to sink resources into activities for which they will be unable to recover the private costs of bringing an innovation to market. The main mechanism through which patents provide incentives to innovate, the promise of a monopoly in that innovation, is diluted if firms face uncertainty over whether or not they will receive that benefit. Society benefits from the increase in innovative activity, because innovations that formerly would not have been commercialized are now more likely to be so.

The cost incurred by the distortion of incentives to innovate is a dynamic cost of

a particular patent scheme. The dynamic costs are lowest in a scheme that awards patents to every innovation, because firms know they will always be granted patent protection. The probability of a type I error is zero under a scheme that grants patents to every innovation, so such a scheme incurs the lowest dynamic cost. This is a beneficial feature of the system of approval in the weak patents game. However, the higher dynamic benefit incurred under such a system comes at the expense of static costs incurred through both the probability of a type II error and through allocation away from productive activities.

A type II error, in which an innovation that would have been realized in the absence of patent protection is granted protection, imposes a welfare loss due to the granting of an unnecessary monopoly. The government would ideally reserve the incentive of monopoly protection for innovations that would not have been realized without it. Society bears unnecessary deadweight loss due to monopoly in the case of a type II error, because the incentive that monopoly protection provides was unnecessary.¹⁰ The system of approval in the weak patents game will always incur the costs of approving an undeserving innovation when the innovation is of low quality, because everything is approved. This is a large detriment of a system that approves patents for every innovation; the dynamic costs incurred are low, but the costs of unduly granted monopoly power are incurred with a higher frequency.

Not only are the equilibrium levels of expenditure a function of ϕ , but also the probabilities of both a type I and type II error. The probability of those two types of errors are direct functions of the probability of approval, which is a function of ϕ . In the case of the system of approval in both the weak patents and arbitrary weak patents game, the probability of approval is a constant that does not vary with ϕ . The system of approval in the private information patents game is a function of total expenditure, $x_1 + x_2$, which is a function of ϕ . For notational simplicity, and also to

¹⁰I am assuming that the monopolist is unable to perfectly price discriminate, so that there is a deadweight loss due to the firm being awarded monopoly protection.

reinforce the idea that the probabilities of both type I and type II errors are functions of ϕ , allow $\lambda(\phi)$ to represent the probability of a type I error, and $\delta(\phi)$ to represent the probability of a type II error. While both λ and δ are constant with respect to ϕ in both the weak patents and arbitrary weak patents game, the relationship between λ , δ , and ϕ in the revealed information patent game is given in the following lemma.

Lemma 3.3. *1. The probability of a type I error is decreasing in the quality of the underlying innovation and the probability of a type II error is increasing in the quality of the underlying innovation under the system that incorporates joint expenditure into the probability of approval.*

2. The probability of a type I error in the system under which the revealed information patents game is played is bounded below by the probability of a type I error in the system under which the weak patents game is played and bounded above by the probability of a type I error in the system under which the arbitrary weak patents game is played.

3. The probability of a type II error in the system under which the revealed information patents game is played is bounded below by the probability of a type II error in the system under which the arbitrary weak patents game is played and bounded above by the probability of a type II error in the system under which the weak patents game is played.

Proof. 1. Recall that $f = 1 - \phi^* + \phi^* \left(\frac{x_1 + x_2}{2M} \right)$. Allow $\bar{x} = x_1 + x_2$, so $\frac{\partial}{\partial \phi} \bar{x} = \frac{\partial x_1}{\partial \phi} + \frac{\partial x_2}{\partial \phi} > 0$ by lemma 3.1. $\lambda(\phi) = 1 - f$, so $\frac{\partial}{\partial \phi} \lambda = -\frac{\partial f}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \phi} = -\frac{\phi^*}{2M} \frac{\partial \bar{x}}{\partial \phi} < 0$, and $\delta(\phi) = f$, so $\frac{\partial}{\partial \phi} \delta = \frac{\partial f}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \phi} = \frac{\phi^*}{2M} \frac{\partial \bar{x}}{\partial \phi} > 0$

2. Given that the innovation is worthy of patent protection, the probability of a type I error in any of the three systems is $\lambda = 1 - f$. Recall that in the weak patents game $f = 1$, so $\lambda_{weak} = 0$. In the arbitrary weak patents game,

$f = 1 - \phi^*$, so $\lambda_{arbitrary} = \phi^*$. In the revealed information patent game, $1 - \phi^* \leq f \leq 1$ by lemma 2.5. Therefore $1 - (1 - \phi^*) \geq 1 - f \geq 1 - 1$, so $\lambda_{weak} \leq \lambda_{revealed} \leq \lambda_{arbitrary}$.

3. Given that the innovation is not worthy of patent protection, the probability of a type II error is $\delta = f$. Because $1 - \phi^* \leq f \leq 1$, $\delta_{arbitrary} \leq \delta_{revealed} \leq \delta_{weak}$.

□

Even though the probability of a type II error is increasing in ϕ , there is less of a concern surrounding a type II error at higher levels of ϕ . The probability of a type II error is only increasing for $\phi < \phi^*$. If ϕ is greater than ϕ^* , then there is no concern of a type II error. The difference in the probabilities of the two types of errors across the three systems shows that the system of approval incorporating expenditure offers a middle road between the two extremes offered through a system granting approval to every innovation and one granting approval to an arbitrary proportion. This middle road provides a possible escape from the strict tradeoff between a type I and type II error, but it comes at the cost of higher expenditure on patenting efforts.

There is no simple solution to the problem of choosing an ideal system. A social planner must weigh the relative costs and benefits of the probabilities of both a type I and type II error and the equilibrium expenditure on patenting activities. An ordered ranking for those three criteria across the three systems is summarized in table 3.1.

If the social planner had lexicographic preferences, then a simple solution to the question of the optimal system arises. If the social planner has either a low probability of type II error, or low expenditure on patenting activities as his primary objective, then the system in which patent protection is arbitrarily granted would be the clear choice. If the social planner were only concerned with the probability of a type I error, then the system under which the weak patents game is played would emerge as the best choice. Regardless of which consideration the social planner preferred, a

	P(type I)	P(type II)	\bar{x}
Lowest	Weak	Arbitrary	Arbitrary
Middle	Revealed	Revealed	Weak
Highest	Arbitrary	Weak	Revealed

Table 3.1: A summary across the three games. Weak, Arbitrary, and Revealed refer to the system under which patent approval is granted in the respective games

system of approval incorporating private information would not emerge as the optimal choice. However, if the social planner desired a system that sought to minimize the welfare loss through a reduction in a combination of type I and type II errors, then a system such as that from the private information patent game might be optimal. Even though the static costs of allocation toward patenting activities are highest under a system that incorporates private information about innovation quality, it is entirely possible that they are offset by a reduction in costs due to type I and II errors.

The social planner's decision might be influenced by the location of the threshold value, ϕ^* . If ϕ^* is close to 1, then there is a greater likelihood that an innovation is bad. The social planner would be more likely, on average, to commit a type II than a type I error under a system that granted patents to every innovation. This might lead a social planner to avoid such a system, and therefore choose between the two alternatives. The choice of system would depend on the social planner's valuation of a type I error. If innovative activity is not sensitive to the probability of a type I error, then the gains made by reducing the probability of a type I error might not merit the higher costs incurred to do so, and the social planner would choose to grant patents arbitrarily. If the incentives to innovate are highly sensitive to the probability of a type I error, then it is possible that the benefits to a reduction in a probability of a type I error exceed the higher costs of allocation away from productive activities, and the social planner would choose a system that incorporates joint expenditure into the

probability of approval.

The line of reasoning followed above illustrates just one possible way by which a social planner could arrive at a particular scheme. The ease at which a scenario could be constructed that leads the social planner to choose a particular system demonstrates a larger point about the social planner's decision about the optimal patent system. The social planner could be lead to choose any of the three systems depending on the valuation of the probabilities of type I and type II errors. Choosing one system over another always incurs a tradeoff between the three considerations discussed above, so the valuation of each consideration determines which system is optimal.

4 Conclusion

The growing concern over the issuance of weak patents has some intellectual property scholars calling for a reform of the patent system. While I have demonstrated that the existence of weak patents causes an allocation away from production toward patenting efforts, I have also explored a potential benefit to this patenting activity. When issuing patents, the government could make an error either granting patents to innovations that are not worthy of them, or failing to grant them to innovations that are. Without information about the quality of the innovation under consideration, the government faces a strict tradeoff between the probability of committing these two types of errors. However, if the government takes joint patenting activity around a particular innovation as a signal of the underlying quality, it can escape this strict tradeoff by allowing the approval of patents for a particular innovation to depend on this expenditure. The government could also try and escape the strict tradeoff by conducting a more rigorous review process of each application to try and uncover the private information about the underlying innovation. Monitoring the level of joint

expenditure around a particular innovation would be less costly than expending more resources on review of applications. The incorporation of joint expenditure into the probability of approval provides an incentive for a higher allocation toward patenting activities, but it is possible that those costs are outweighed by the ability to escape the strict tradeoff between a type I and type II error.

Intellectual property scholars have noted that certain industries, especially software and technology, suffer from the use of patent thickets.¹¹ In industries in which the rate of innovation is high, and many innovations build directly off of pre-existing ones, it is impractical for the government to try and keep up with the pace of innovation. This suggests that the informational asymmetry problem is greater amongst patent applications from particular industries. A one size fits all approach to a mechanism of patent approval would not be ideal due to the differing degree of informational asymmetry across industries. A system such as the one I propose, in which the government allows the probability of approval to vary with joint expenditure, might lead to better outcomes if used as a standard for the industries where it is impractical for the government to stay informed of the state of the art.

My model adds to the argument that the government is rationally ignorant in continuing to issue weak patents. This argument against strengthening patents centers around the idea that the costs of a stronger review process outweigh the benefits because many weak patents are never challenged. My model suggests that the P.T.O need not remain entirely ignorant. Using the joint expenditure on patenting efforts as a signal of innovation quality is a low cost means of improving patent strength. Each individual patent is no stronger, but the innovations that maintain patent protection are more likely to be worthy of it. The possibility of using weak patents as a signaling mechanism, suggests that the focus on the allocation of resources toward patenting activities distracts from more harmful aspects of weak patents.

¹¹For a more complete look at patent policy and behavior in particular industries see (Burk and Lemley, 2003).

In my thesis I looked at symmetric self-enforcing equilibria in a one-shot framework. In reality, firms are repeatedly involved in disputes with their rival firms over different technologies. The existence of repetition allows for the possibility of collusive agreements. These collusive agreements are facilitated by weak patents. Firms enter cross-licensing agreements with rival firms as a response to the uncertainty surrounding weak patents. In these licensing agreements firms allow rival firms access to some of the patents in their portfolio. While these cross licensing agreements are viewed positively because firms avoid costly legal battles, they are also seen as a collusive agreement. This form of legal, tacit collusion suggests that weak patents may create more welfare loss through this channel than through a misallocation of resources.

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