Wavelet Analysis of Inflation Hedging Abilities of Real Estate Investment Trusts (REITs)

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Abstract

Inflation has been experiencing upward pressure in the recent period and investors have been looking for strategies to protect their portfolios against the risks of increased inflation. Real estate investments have traditionally been regarded as good inflation hedges. Real estate investment trusts (REITs) are a more liquid alternative to investing in the real estate market. This thesis analyzes the short-term inflation hedging abilities of REITs in the US, focusing on short-term lease funds such as self-storage, apartments, lodging, and entertainment between 1973-2022 (depending on the inception time of each REIT).

Continuous wavelet transform and wavelet coherence methods are used on REITs and inflation to examine the dynamic correlation between them at different calendar times and data frequencies. Results can be visualized through heat maps depicting the dynamic correlation and line charts showing changes in lead-lag relationships. The inflation hedging abilities are then compared to those of two long-term lease REITs, WTI crude oil, gold, and the S&P 500 stock index to determine the usefulness of including REITs in a portfolio.

Results for certain REITs indicate comparable hedging abilities with those of gold in the short-to-medium (64–256-day scales) and medium term (256–1024-day scales), representing an advantage over long-term REITs and over a stock-only portfolio. However, they do not necessarily offer protection against changes in inflation during deflationary periods or during the longer term, and they also do not match up to the hedging capabilities of crude oil at any scale. It is also found that REITs within the same sector (i.e. storage facilities, residential, lodging) perform similarly. Finally, benefits are found in the use of a diversified portfolio consisting of REITs, oil, and equity stocks versus just equity stocks.

Keywords: REITs, Inflation, Hedge, Wavelet Analysis

JEL Classification: C58, C60, E31, E44, G11, R30

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1. Introduction

The Covid-19 pandemic introduced a lot of volatility in financial markets and pushed inflation at the end of 2021 to the highest level since 1982, as the consumer price index (CPI) rose by 7% compared to the previous year due to supply chain issues, high energy and shelter costs, labor shortages, and other reasons that may persist in the longer term. CPI continued to maintain an upwards trend through the beginning of 2022, when the Russia-Ukraine tensions pushed the March CPI to 8.5% higher than the year prior. Inflation hedging is an important tool used by investors to protect their portfolios against the risk of diminished purchasing power. In simple terms, it entails adopting a portfolio that includes assets that perform well when inflation increases.

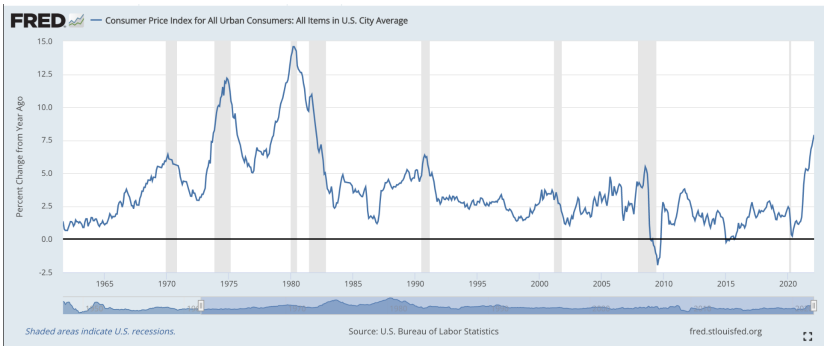

Figure 1. **CPI, Percent Change from Year Ago**

Real estate is typically considered one of the best inflation hedges (Simpson et al., 2007). Real estate investment trusts (REITs) are trusts that own and operate income producing real estate, including many types of residential and commercial real estate. They are more liquid, require less capital upfront, and have lower transaction costs, so they are more frequently traded than physical property. Since real estate tends to be considered a good inflation hedge and REITs own real estate whose value appreciates as inflation rises, REITs could also have the potential to be good inflation hedges.

Inflation is correlated with real estate prices in different ways. In the US many commercial leases are linked to CPI by a periodic "step up" mechanism. When inflation rises, rents and values increase, which supports REIT dividend growth and provides a reliable stream of income for investors. High inflation can also lead to higher construction costs, slowing new development/supply, and driving up rent prices for existing property. Finally, housing prices can influence inflation due to their large weight as part of the CPI.[1] To hedge inflation risk, one must hold an asset that appreciates in value when faced with inflation, to offset the losses that other assets might incur at the time. Real estate can provide this hedge because of price appreciation, rent income growth with CPI, and depreciating debt. There are nuanced distinctions between real estate segments. Most relevant to this study are general lease term timelines. Commercial and residential areas (e.g. hotels, homes, self-storage) have shorter-term leases, allowing for quicker price adjustments to inflationary pressures, which supports their effectiveness as inflation hedges. The industrial sector (e.g. warehouses, shopping malls) adjusts more slowly since leases are typically long-term.

Diversification and hedging are both practices used by all investors to protect their portfolios against risks and improve returns over time. There is a distinction between the two, namely that hedging aims to mitigate one specific risk, while diversification allows investors to smooth out risk across the entire portfolio by combining several uncorrelated assets. This distinction has been clearly approached in existing literature on real estate (Seiler et al., 1999; Amonhaemanon et al.,2013), the properties of gold (Wang et al. 2011), Bitcoin (Bouri et al., 2020), and other alternative investments generally (Hung et al., 2008). Among notable results, real estate investment trusts (REITs) were found to have good diversifying capabilities especially in bull

[1] (32.6% of CPI by weight as of March 2022)

markets, while direct real estate investments can enhance portfolio performance regardless of the economic environment (Hung et al., 2008). Wang et al. (2010) conclude that, while gold does have diversifying properties, it can only serve as a good inflation hedge in periods of high momentum.

It has been documented that during periods of high inflation tangible assets such as commodities or real estate may serve as good hedges against inflation (Bond and Webb, 1995). Fama and Schwert (1977) came up with one of the earliest known empirical tests for the relationship between inflation and multiple asset classes, including Treasury bonds, residential real estate, common stock, and labor income. Using an OLS model, they found that private residential real estate was the only asset on their list that could provide a complete hedge against unexpected and expected inflation, while common stock was a particularly poor performer when inflation was higher than expected. Newer studies such as Bouri et al. (2020) compare the diversification and inflation hedging properties of Bitcoin, gold, and other commodities using a wavelet analysis approach. They find that effects vary in time, although Bitcoin generally exhibits superior properties than the other assets. Therefore, it becomes apparent that timeline is very important when investigating inflation, and differences occur between short-run and long-run effects. For example, Wang et al. (2010) divide the time interval 1970-2010 into short-run and long-run horizons, and only find significant results for the short-run hedging abilities of gold. Financial theory also indicates that investors have a tendency to underreact to new market shocks in the short term but overreact in the longer term (Reboredo and Rivera-Castro, 2012), which would contribute to the varying effects in prices across time.

Real estate research can be split into unsecuritized and securitized real estate. Unsecuritized or direct real estate investments show relative illiquidity compared to other types of assets and

have an extended adjustment period to market shocks. Previous research shows that the relationship between inflation and such real estate investments holds, to a certain extent, for both the short-term and long-term, resulting usually in good hedging properties. Most literature confirms the short-term relationships, while more recent studies focus on their existence in the longer term as well (Stevenson, 2000; Webb and Simpson, 2007; Hong and Lee, 2011). On the other hand, to benefit from higher liquidity, lower capital requirements, and lower costs, investors can opt for securitized real estate such as REITs or exchange traded funds (ETFs). Since the underlying assets of REITs are real estate, it would make intuitive sense that REITs are an equally successful inflation hedge. However, existing research does not offer a straightforward answer. A common finding is that REITs perform like common stock and adjust faster to market shifts (Glascock et al., 2002). Seiler et al. (1999) discuss a few exceptions to this rule and more recent research such as Simpson, Ramchader, and Webb (2007) documents strongly asymmetric responses of REITs to inflation. In other words, they find that REIT returns can rise because of both increases and decreases in expected and unexpected inflation. Market conditions are another differentiator in the strength of real estate alternative investments for hedging purposes. Fluctuating economic conditions were found to affect the inflation-hedging properties of such investments (Amonhaemanon et al., 2013). Both Li (2001) and Le Moigne and Viveiros (2008) indicate that the correlation between inflation and direct real estate investment returns is only strong in high-inflation periods and tends to weaken significantly in low-inflation environments. The Covid-19 pandemic represented a new type of shock to the markets, although it was similar in some ways to past financial crises. De Toro, Nocca, and Buglione (2021) investigate the real estate market in Italy following the pandemic and discover positive structural changes in the demand for housing, as well as increased interest in renovations. This would contribute to higher

shelter prices and potentially higher revenues. Therefore, further analysis of the matter using both older and newer data is justified.

Existing literature also presents conflicting evidence regarding the causality between inflation and real estate returns. There have been Granger causality tests performed to determine whether inflation Granger-causes real estate returns or vice versa, albeit most of them use international data. Stevenson and Murray (1999) find that real estate returns cause inflation in Ireland. Le Moigne and Viveiros (2008) find the same results in Canada, while Chu and Sing (2004) conclude a reversed causality in China. Therefore, there seems to be an intertwined relationship between the two variables that needs to be accounted for in any future modeling. A way to address this is by using wavelet analysis to look at the dependence between time series, which would provide insight into the lead-lag effects between the two factors.

Wavelet analysis is a mathematical procedure that decomposes an original signal for easier processing. Asset return movements are traditionally challenging to analyze because of their non-linear or non-stationary characteristics, so wavelet transforms are used for this type of data. Classical regression models fail or require complex mathematical procedures to recover frequency information from non-stationary trends. Fourier analysis presents the same limitation of accounting for just the time dimension (Reboredo and Rivera-Castro, 2012). Wavelet analysis is a very useful framework in this context, allowing for an efficient detection of dynamic correlations between time series by preserving information from both the time and frequency horizons. Particularly, this method would be able to show whether the dependence between real estate returns and inflation changes over time under market conditions like financial crises or global shocks such as the Covid-19 pandemic. It would also be advantageous in addressing both the short and long-term performance of REITs. Real estate theory indicates there is an adjustment period

between hikes in inflation and rent increases. Therefore, based on the lease duration, there will be a time lag between inflation trends and REIT performance, where REITs might suffer quick losses in the short term but then gain in the longer term. Historically, wavelet methods have been used in statistical analysis in a wide range of areas — one of the most notable advancements being brought by Lau and Weng (1995) in a study about climate. Wavelet analysis was only applied to the specific field of finance and economics more recently. Gallegati (2005) used a discrete wavelet transform to show that the DJIA stock index leads the level of economic activity for periods of 16 months or longer based on data from 1961-2005. In the context of this paper, similar approaches can be used to test whether REITs returns can lead the level of inflation at any time/scale combination. More recent applications of the method were expanded to include two or more time series, as is the case of Grinsted et al. in 2004. This is directly relevant to this thesis, as it permits using wavelet coherence and cross-wavelet transforms to look at co-movement between inflation and REITs returns, to test the confidence level of the causal relationship, and finally to compare the results to a new time series pertaining to a traditional asset. Coherence is a measure of correlation between time series, specifically measuring recurring trends (waves). Coherence results can be visualized through contour heat maps. The coherence strength is shown with different color pallets. Directional arrows then show lead-lag relationships and dependence (Yilmaz et al., 2016; Bouri et al., 2020). When it comes to discussing diversification capabilities, it is helpful to construct portfolios with different allocations of varied assets and see if returns are improved, losses are reduced, or variance/risk are minimized. The reader can encounter this approach in the latter part of the results section. It is possible to expand on this and further quantify results by using a wavelet value-at-risk (VaR) measure which captures portfolio risk across time and frequencies, as seen in Bouri et al. (2020).

2. Data Description

The paper analyzes the historical performance of 13 short-term-lease REITs in contrast with inflation. The diversification and inflation hedging properties of these REITs are then analyzed in comparison with the S&P 500 index, WTI crude oil, gold, and two long-term-lease REITs. REIT data were collected from MarketWatch, while gold, oil, and S&P 500 data were collected from macrotrends.net. To be precise, the SPY ETF is used to track the performance of the S&P 500 index from its inception in 1993 until now.

For all analysis purposes, daily data are used for the stock price for each REIT variable. Because of the way that REITs compensate their shareholders – as a percentage of their Net Operating Income (NOI)[2] – using NOI to measure performance would be the ideal approach. However, NOI is only reported on an annual basis, which does not offer enough flexibility in analyzing changes with inflation for different investment horizons. Therefore, we choose to use the stock price as a measure of performance, which is a reliable metric because investor sentiment does adjust based on the REITs' income statement reports when they happen and will be reflected in the stock price. For convenience, we choose to use CPI data as a percentage change from one year ago, so we use the same metric for the REIT, oil, gold, and stock prices. The time series for daily values of Year-over-Year changes in REITs and assets prices are presented in the appendix. Furthermore, we distinguish between two types of inflation. As is customary in most economics research papers, we first use the Year-over-Year growth in CPI index as a measure for realized inflation. Because CPI is only reported on a monthly basis, we rely on interpolation to create a daily time step similar to the other variables.[3] Then, the daily-reported 13-week (3-month)

[2] NOI = Company income – operating expenses

[3] As further detailed in the conclusion section of this paper, this method of interpolation is useful but perhaps not completely accurate on the very short time scales (less than monthly).

Treasury Bill rate is used next to account for expected inflation. Using short-term T-bill rates to proxy for expected inflation is also in line with previous literature (Fama and Schwert, 1977; Hartzell, Heckman, and Miles, 1987). However, we must be careful when interpreting results from highly volatile periods, as Ratner (1989) and Jones et al. (1995) show that the Treasury bill may only be an appropriate proxy for this purpose over periods of low and stable inflation.

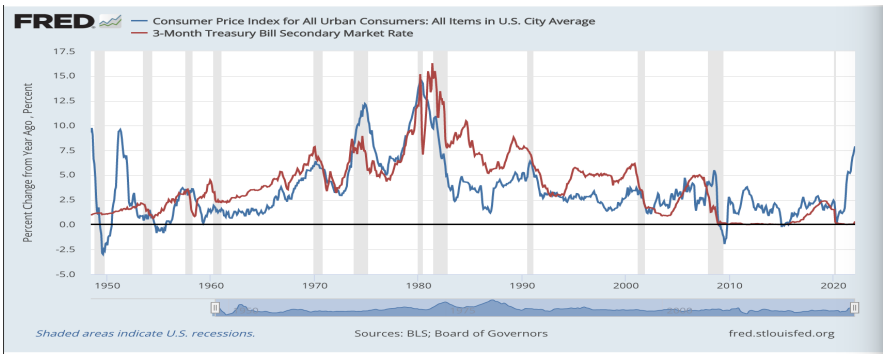

Figure 2. **CPI vs 13-week Treasury Bill rate**

CPI is measured in percentage change from the previous year and is depicted in blue; the 13-week T-Bill rate is measured in percentage and is shown in red.

Table 1. **Data description**

REIT Name	REIT Ticker	Description	Time Frame
Nareit Self Storage	**FN12**	Self-storage units	2012-2022
PSA Public Storage	**PSA**	Self-storage units	1980-2022
Extra Space Storage Inc.	**EXR**	Self-storage units	2005-2022
Nareit Lodging/Resorts	**FN24**	Hotels/Resorts	2012-2022
Host Hotels & Resorts, Inc.	**HST**	Hotels/Resorts	1980-2022
Equity LifeStyle Properties, Inc.	**ELS**	Hotels/Resorts/Vacation homes	1993-2022
Bluerock Residential Growth REIT	**BRG**	Residential REIT	2014-2022
PREIT	**PEI**	Shopping malls	1979-2022
Essex Property Trust	**ESS**	Residential REIT, California	1994-2022
Avalon Bay Communities	**AVB**	Residential REIT, Virginia	1994-2022
Mid-America Apartment Communities, Inc.	**MAA**	Residential REIT	1995-2022
American Homes 4 Rent Class A	**AMH**	Residential REIT	2014-2022
Equity Residential	**EQR**	Residential REIT, luxury apartments	1994-2022

Americold	**COLD**	Long-term lease REIT, warehouses	2018-2022
Store Capital	**STOR**	Long-term lease REIT	2015-2022
Gold	**Gold**	Gold spot prices	1979-2022
SPDR S&P 500	**SPY**	Tracks the S&P500 market index	1992-2022
West Texas Intermediate (WTI)	**WTI**	Global crude oil benchmark	1987-2022

Table 2. **Summary statistics for the change in prices from previous year**

REIT Ticker	Mean	Median	Standard Deviation	Kurtosis	Skewness	Minimum	Maximum
FN12	14.91%	12.40%	20.66%	0.49	0.73	-23.05%	79.43%
PSA	10.63%	11.01%	21.76%	-0.41	-0.13	-50.42%	62.49%
EXR	22.12%	20.88%	32.00%	0.98	0.13	-69.89%	156.86%
FN24	6.79%	7.55%	29.22%	2.79	0.85	-69.37%	194.97%
HST	11.09%	12.03%	38.86%	3.50	0.77	-79.76%	305.30%
ELS	12.37%	13.19%	17.60%	-0.15	-0.19	-46.58%	99.83%
BRG	6.45%	0.77%	35.31%	2.50	1.27	-65.23%	155.96%
PEI	5.96%	7.41%	36.73%	7.08	1.13	-89.73%	251.08%
ESS	14.34%	14.91%	22.26%	0.25	-0.20	-51.86%	85.56%
AVB	12.68%	10.49%	25.61%	-0.07	0.14	-57.71%	107.17%
MAA	10.00%	5.96%	21.12%	1.64	0.78	-52.58%	130.84%
AMH	13.19%	9.21%	19.06%	-0.36	0.61	-20.18%	80.99%
EQR	9.70%	8.16%	25.56%	2.05	0.77	-59.77%	131.88%
COLD	18.43%	10.38%	26.93%	-1.19	0.29	-28.04%	68.84%
STOR	12.43%	12.25%	27.82%	1.65	0.58	-56.22%	128.31%
Gold	8.26%	3.53%	24.93%	8.99	2.15	-37.72%	179.42%
SPY	9.77%	11.90%	16.69%	0.99	-0.59	-46.87%	70.23%
WTI	10.79%	6.18%	38.27%	9.30	1.62	-79.1%	444.95%

3. Methods Overview

Coherence and transform wavelet analysis are used to look at the performance of REITs and their co-movement with inflation and with the traditional asset classes mentioned. Wavelets are wavelike mathematical functions with mean 0 that can decompose a given function or signal into different scale components, conserving both time and frequency information. Unlike sine and cosine functions, wavelets have a variable amplitude that starts at zero, increases to a maximum, and is then amortized back to zero. A brief theoretical introduction of wavelets is below and more details can be found in Macedo (2013) and Foufoula-Georgiou and Kumar (1995).

Mathematically, wavelets take the form $\frac{1}{\sqrt{s}}\Psi(\frac{t-\tau}{s})$, where Ψ is a generating function called the mother wavelet, τ is the position of the wavelet in time and s is the scale, showing how wide or stretched horizontally the wavelet is.[4] Larger scales imply a lower frequency analysis and are used to observe longer term implications of a signal, while smaller scales are helpful in looking at high-frequency short-term implications. While there are several types of wavelets, financial and econometrics literature often use Morlet wavelets in their analysis. Visually, a wavelet can look like the figures below.

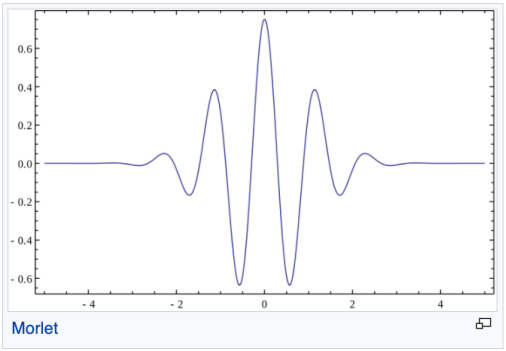

Figure 3. **Morlet wavelet example**

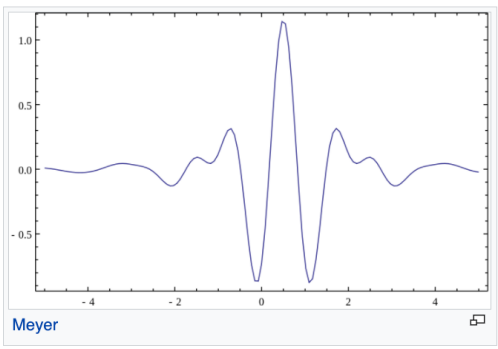

Figure 4. **Meyer wavelet example**

[4] For example, one such generating function can be $\Psi(t) = \frac{sin(2\pi t) - sin(\pi t)}{\pi t}$.

The continuous wavelet transform (CWT) is a way to apply a wavelet function to the initial signal in a continuous manner by varying the scale and translating across the time dimension in order to reduce noise and isolate the important initial components of the signal. In other words, we would start out with an initial generating wavelet function of a given scale and try to see how much of the wavelet is in the signal at any given point. To do this we slide it across the entire signal, then we increase the scale of the wavelet and repeat the process. The CWT takes the mathematical form $W_x(s,\tau) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{s}} \Psi(\frac{t-\tau}{s}) x(t) d(t)$, where x(t) is the initial time series. When the relationship between two different signals is of relevance, such as in this paper, a cross wavelet transform can be used ($W_{x,y}(\tau,s) = W_x(\tau,s) W^*{}_y(\tau,s)$, where * is the complex conjugate — this needs not be considered when using real mother wavelets, but is important if complex wavelets are used, such as the Morlet wavelet). To examine the correlation between two time series (such as movement of two asset classes), we can use the wavelet coherence. In simple terms, wavelet coherence is a dynamic measure of correlation between wave-like time series. Mathematically, coherence is defined as a value between 0 and 1 as follows:

$$R^2(s,\tau) = \frac{|S(s^{-1} W_{xy}(s,\tau))|^2}{S(s^{-1}|W_x(s,\tau)|^2) S(s^{-1}|W_y(s,\tau)|^2)},$$

where W_x, W_y are the wavelet transforms for the two time series, W_{xy} is the cross wavelet transform, and S is just a smoothing operator. Torrence and Compo give a suitable smoothing operator for the Morlet wavelet (1998), which is often used in analysis toolkits, including the packages used for this paper. Wavelet coherence analysis provides a correlation coefficient between the two signals for each scale/frequency throughout the entire time frame. The results can be displayed in a coherence plot, similar to a heat map, that identifies the periods and scales of

high correlation. The same heatmap plot uses arrows to indicate any phase lead/lag relationship between the time series. Let us denote the phase difference between the two time series at a given point by ϕ. If $\phi \in (0, \pi/2)$, the series are in sync or co-move positively with the second time series leading the first one. Between $\phi \in (\pi/2, -\pi)$ they co-move negatively or are out of phase with the first series leading the second. Between $\phi \in (-\pi, -\pi/2)$ they are out of phase with the second one leading, and between $\phi \in (-\pi/2, 0)$ they are in phase with the first one leading. The phase difference is also compatible with a causal relationship at that given point — so if X leads Y then a causal relationship exists from X to Y at that time-frequency. This represents an advantage over the more conventional Granger causality test because the latter assumes a single causal link for the entire sample period and for all frequencies (Grinsted et al., 2004; Tiwari et al ., 2019; Saiti, 2017; Habib et al., 2021). For exemplification purposes, such a coherence plot can be found below.

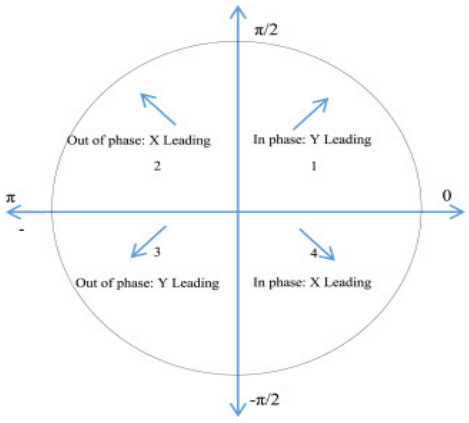

Figure 5. **Phase difference angle and lead-lag relationship meaning**

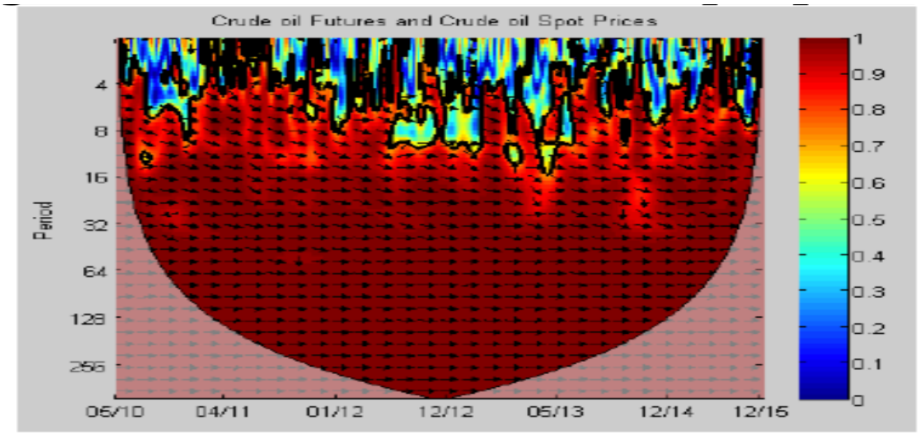

Figure 6. **Crude oil futures and spot prices wavelet coherence/correlation plot (Mauryaa and Thenmozhi, 2021)**

Calendar time is on the x axis and period (the opposite of frequency) in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation (with non-negative values, i.e. between 0 and 1). Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-

Figure 6 looks at the coherence (dynamic correlation) between spot prices and futures prices for crude oil. Daily spot and near-month futures closing prices are considered for a time horizon between 2010 to 2015. The near-month contract is a term used in futures and options trading and refers to the active contract with the shortest time to maturity (usually less than a month). Futures prices and spot prices are related, and futures prices often serve as good forecast measures for spot prices, and they eventually converge to the spot price at maturity. The horizontal axis shows time, in this case ranging from May 2010 (left side of the plot) to December 2015 (right side of the plot). The vertical axis shows the scale in days — lower scales (higher frequency, short-term fluctuations) are at the top of the plot, while larger scales (lower frequency, long-term fluctuations) are at the bottom.

Dark red areas correspond to high correlation (almost perfect), while dark blue would correspond to zero or no correlation. The thick black lines show the 95% confidence level as estimated from Monte Carlo simulations, so everything inside them is significant at a 5% level. It makes sense intuitively that spot prices and near-term futures prices would be significantly correlated. The reason that we see less correlation at scales of 1-16 days has to do with the daily fluctuations in spot price for oil. Oil is typically a volatile asset, and prices change considerably on a daily basis. Because the futures used in this analysis expire later in the month, they are less responsive ahead of time and only become more correlated with the spot price in the last days of the month when they start to converge to a common value. On the other hand, when we consider bigger scales of more than one or two months, the red areas of correlation signify the bigger-picture correlation between the two prices, which is the well-known fact that they generally tend to move together in a similar trend if we make abstraction of any short-term noise.

The grey circular area is the cone of influence (COI). The dataset has a limited length, but the wavelet method assumes data is cyclical, so edge effects will occur at the beginning and end of the time series. Therefore, information contained inside the cone is considered reliable, but any information outside of it may be distorted and should be interpreted cautiously. The black arrows show both correlation and the lead-lag relationship between the two signals. Arrows pointing to the right indicate that the signals are in sync at that position, while arrows pointing to the left indicate anti-phase or negative correlation between the signals at that point. A downward pointing arrow would mean that the first series (here futures) is leading the second one (here spot prices) and vice versa[5]. The angle can give further information about the lag — for example, an arrow pointing north-east at a scale of 32 days would indicate that the second time series leads the first one by pi/4 or an 8th of a cycle, so we would see changes in the first series compared to the second one delayed by 4 days. In this case, we mostly see that the series are in phase or that crude futures lead crude spot prices.

Next, the wavelet power spectrum provides a way to measure the variance of the time series for each moment in time and scale. Red areas correspond to high power (in other words higher variance in the series, so a significant event) and blue areas correspond to lower power. The other plot elements are similar to those of a coherence plot. Figure 7 below represents the WPS for WTI crude oil from 1987 to 2022.

[5] In all coherence plots in this paper involving one asset performance signal and one inflation signal, inflation will be the second time series, so an upward arrow will indicate that inflation is the leading factor.

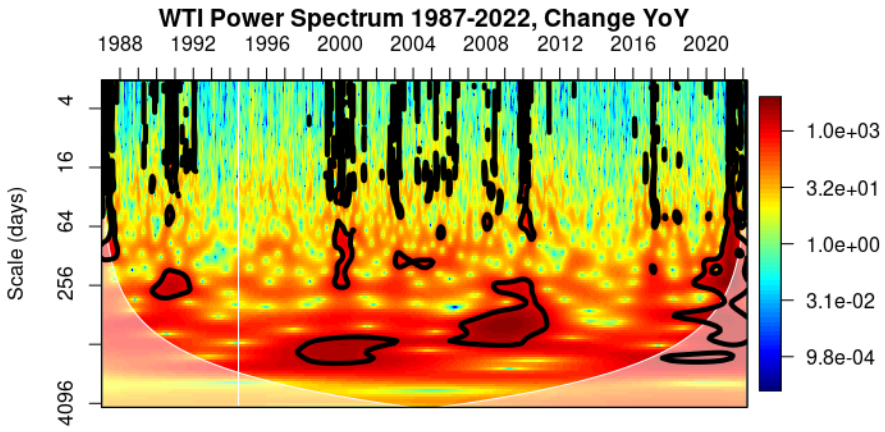

Figure 7. **WTI Crude Oil Wavelet Power Spectrum**

The wavelet power spectrum shows the variance over different frequencies and times for WTI Crude Oil Year-over-Year change in prices. Calendar time is on the x axis, while the scale in days is on the y axis. The power (equivalent to variance) ranges from blue for low values to red for high values. The black contour lines indicate areas of high power significant at a 5% level. The circular grey area is the cone of influence. Each area of statistically significant higher power can be interpreted as a shock or significant event in the time series.

For the purposes of the analysis conducted and explained in future sections of this paper, the time frames will be separated into short term investment horizons (less than 256 days, around 9 months), medium term (between 256-1024 days), and long term (more than 1024 days, around 3 years). As will be noted later as well, for the short-term we will also differentiate between periods lower than 64 days and those between 64-256 days, the latter band showing more promising results and being worth paying more attention to than the former.

We also identify some key periods of significant changes in US inflation or real estate trends that will be used as a reference during the analysis and interpretation steps. More details about these periods are in Hamilton (2011), Gilchrist et al. (2015), Nersisyan and Wray (2022), Labonte and Weinstock (2021), Cheng et al. (2014).

Consistent deflation can be seen:

- 1981-1984 and 1991-1992 (end of oil shocks)
- End of 2014 to mid-2015
- Mid-2008 to 2009

High inflation happens between:

- 1973-1981 (due to oil shocks)
- 1989-1991 (the Gulf war led to oil shocks)
- Beginning to mid-2008 (financial crisis combined with oil shocks)
- 2021-2022 (Covid-19 pandemic)

Some real estate trends happening throughout the years include, but are not limited to:

- 2001-2012 (housing bubble formed in 2001, peaked in 2005, burst in 2008, and a general downward trend continued until 2012)
- 2016-2017 (US housing market boom)

4. Results

4.1 Individual REIT Discussion

The wavelet power spectrum (shown entirely in section A4 of the appendix) depicts localized volatility. In this paper, the side-by-side wavelet power spectrum plots show changes in the trends of inflation and of prices of different types of REITs throughout the given time frames. In all of these, we see common bands of raised power between mid-2007 and 2009, as well as between 2020-2022. These bands are particularly prevalent at longer periods of 3 months or more, but still present in the shorter-term horizon as well. For the time series extending over longer time frames we see common areas of high power around the years 1978-1982, 1989-1991, and the

beginning of the year 2000. Notice that all these time intervals have been previously identified in the methods section as significant macroeconomic moments. One small exceptions from these similarities is in the power spectrum of oil, which also shows higher volatility around the year 2000 and which is not that prominent in the CPI spectrum. The gold plot is similar to both oil and CPI, being representative of gold's properties as both a commodity and an asset that varies with inflation. Therefore, the shared bands of high variation indicate similar shocks in certain REITs, gold, and the S&P500 index at key moments in time. This is a promising step regarding the inflation hedging abilities of these REITs and gives us confidence to proceed with the next steps of the analysis.

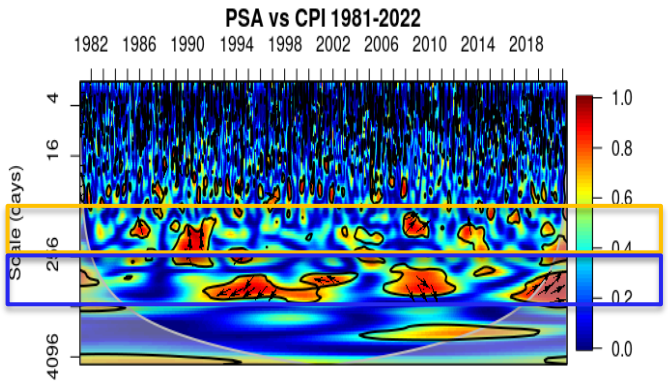

Figure 8. **PSA vs CPI wavelet coherence plot**

Time is on the x axis and period in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation. Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-lag relationship between the PSA prices and CPI (both measured in YoY changes) (rightward pointing arrow = positive correlation, leftward pointing = negative correlation).

The 64-256-day and 256-1024-day period bands are marked in the coherence plot and correspond to the yellow and blue line from Figure 9, respectively.

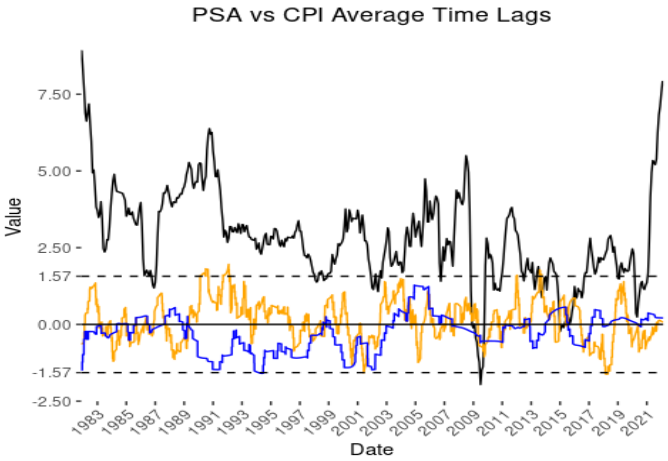

Figure 9. **PSA vs CPI lead-lag phase difference angle**

The black line shows CPI growth YoY. The yellow line shows the phase difference angle between PSA and CPI averaged over the 64-256-day scale band (short term). The blue line shows the same but averaged over the 256-1024-day scale band instead (medium term).

A positive phase difference angle means the two time series are in sync/positively correlated at that point; a negative angle means they are negatively correlated. Values between (0,1.57), (-3.14, -1.57) mean that inflation leads changes in the REIT; values between (1.57, 3.14),
(-1.57, 0) mean that the REIT is leading changes in inflation.

The next step is analyzing localized potential for REITs to serve as hedges for real and expected inflation using wavelet coherence analysis. Because instead of raw prices we are using percentage change from previous year for each asset, the time horizon actually included in the analysis starts one year later than each REIT inception and ends in March 2022. A detailed discussion of the public storage PSA REIT is found below, along with the corresponding coherence heatmap in Figure 8. The complete set of coherence plots can be found in section A5 of the appendix. As a reminder, the horizontal axis shows the time horizon, while the vertical axis shows the scale in days.

In addition to the heatmaps, it is helpful to consider the average phase difference between CPI and PSA trends over the two scale bands mentioned above to determine the direction of movements and any causality effects. This can be seen above in Figure 9. The YoY change in CPI is pictured in black, while the phase difference between the time series is shown in orange for the shorter-term band and in blue for the medium-term band. For reference only, there is a continuous baseline at 0 and two dashed lines at $\pm\pi/2$. The same charts are available for the rest of the dataset in section A6 of the appendix.

Looking at the coherence plot between PSA and CPI in Figure 8, we can identify consistent areas of high correlation in the short-to-medium (64–256-day scales) and medium term (256–1024-day scales). In the short-to-medium term we see a pronounced significant correlation patch between 1989-1992. This corresponds to a period of high inflation peaking in 1991 after the gulf war oil shocks, followed by a stabilizing period. Over the shorter term, the two time series are in sync with an average phase between 0 and $\pi/2$, which indicates that higher inflation is leading to higher PSA returns. Over the same shorter term, we see another significant patch around the 2008-2009 financial crisis, another one between 2014-2016, and a final one starting in late 2019- early

2020 around the Covid-19 pandemic. During the high-inflation part of 2008, the two series are in phase with high CPI leading high REIT returns. As inflation starts decreasing towards 2009, the phase slowly becomes negative, indicating that the REITs start adapting to the inflationary changes and mitigating losses. Between 2014-2016 we experienced both lower inflation and lower REIT returns with a positive phase angle, and from 2020 onwards the higher CPI levels encourage higher correlation levels and increasingly positive lead angles, meaning that PSA returns are again adapting to the new environment and minimizing losses. Then, in the medium term, we have significant areas of high correlation between 1991-2002, 2007-2009, and 2017 onwards. During 1991-2002, CPI was mostly stable or decreasing, during which the two series are in anti-phase, meaning that the REIT is performing well. PSA slightly underperformed during the slight inflation peak in 2000 when the two series became almost in sync, but not quite. Between 2007-2011 and 2017-2022, the two trends were moving in opposite directions when inflation dropped (in 2008) and mostly positively when inflation increased (2010-2011, 2017-2022). This analysis suggests that there are slight inflation hedging benefits for REITs around the 2–9-month investment horizons and even more promising/stable ones in the medium term (9 months- 3 years).

After analyzing the entire dataset individually and as a whole, a few observations become apparent. Overall, results do not show any hedging capabilities for the short-term REITs in the very short- or long-term investment horizons. First, all short-term REITs present higher and more correlation with CPI than with expected inflation as measured by the 3-month treasury bill rate. The opposite is true for the long-term lease REITs (STOR and COLD). Given this, we will focus mostly on the relationship between REITs and CPI in the rest of the results section of this paper. We look at the three divisions of the investment horizon mentioned in previous sections: short (less than 256 days, approximately 9 months), medium (256 - 1024 days), and long term (more

than 1024 days, approximately 3 years). We also compare overall correlation patterns with those observed in the long-term REITs, gold, the S&P 500, and WTI crude oil.

In the very short term, for scales up to 2-3 weeks (16-21 days), there are very small periodic areas of significant correlation between the short-term REITs and CPI, however they fluctuate between positive and negative signs in what seems like an aleatory manner. There is a distinguishable band of increased significant correlation in several of the REIT heatmaps between the 64–256-day periods (PSA, EXR, ELS, PEI, ESS, AVB, MAA, EQR). In the interest of distinguishing more general trends among all REITs, the simplified plot below can be considered[6]. The plot was created by averaging out the coherence between each REIT and CPI at the same time over the entire 64–256-day scale band and over half-year calendar time frames. The cells highlighted in yellow represent the half year during which the average coherence exceeded 0.5. Correlations are consistent for most REITs at this scale between 1997-1998, 2008-2009, 2014, and 2020-2022.

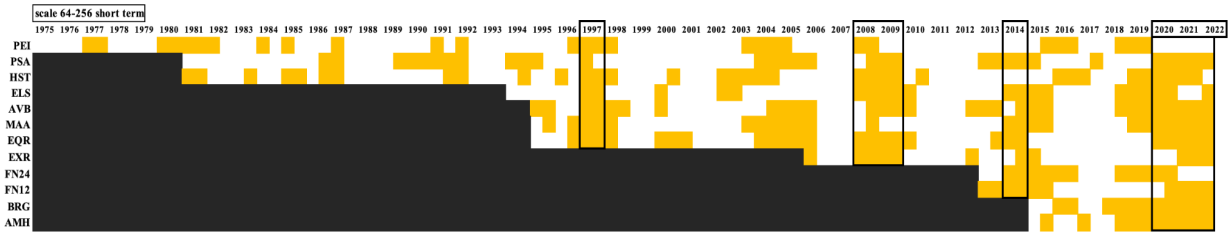

Figure 10. **Average correlation between REITs and CPI higher than 0.5 for 64–256-day scales (short-term investment horizons)**

Correlation was averaged across the first and the second half of each calendar year, respectively, and simultaneously across the 64-256-day scales. Correlations higher than 0.5 are considered noteworthy and are highlighted in yellow in this figure. The cells highlighted in black indicate that no data is available for those REITs at those times. The rectangles contoured in black indicate areas where several of the REITs exhibit high correlation with CPI.

[6] The black area means no data exists for a certain asset during that period. The color yellow is just meant to represent the short-term horizon and has no connection with correlation levels or other analysis aspects.

The medium term seems to be slightly more promising for several of the REITs, with larger patches of high correlation corresponding to important changes in CPI trends. Among the most remarkable is a band of frequent increased periodic correlation centered around the 256-day mark but extending at times downward towards the 1024-day period (seen for PSA, EXR, FN24, HST, ELS, PEI, AVB, MAA, AMH, EQR). Consistent correlations at this scale are common for most of the REITs between 1982-1983, 1990-1992, 2007-2012, and 2018-2022, as seen in the figure below.[7]

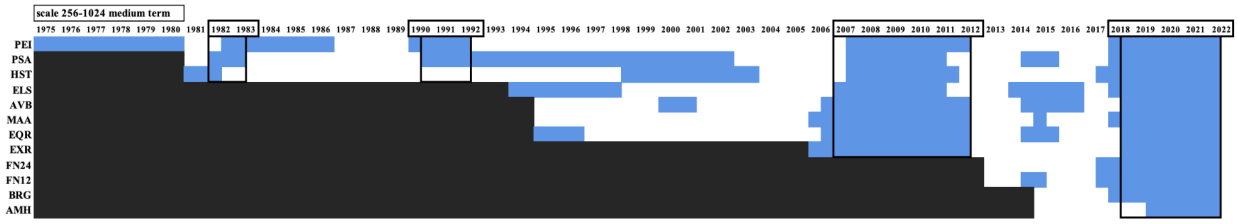

Figure 11. **Average correlation between REITs and CPI higher than 0.5 for 256-1024-day scales (medium-term investment horizons)**

Correlation was averaged across the first and the second half of each calendar year, respectively, and simultaneously across the 256-1024-day scales. Correlations higher than 0.5 are considered noteworthy and are highlighted in blue in this figure. The cells highlighted in black indicate that no data is available for those REITs at those times. The rectangles contoured in black indicate areas where several of the REITs exhibit high correlation with CPI.

Given the inconsistent (and relatively short) time frames available for each REIT, fluctuations at bigger scales (more than 1024 days) are difficult to interpret reliably in the context of this paper. However, we can briefly observe the big picture for the 8 REITs that date back to before the 2000s. As a reminder, these are PSA, HST, ELS, PEI, ESS, AVB, MAA, EQR. For most of these, all areas of high correlation end right above the 1024-day mark during all years except the patch corresponding to the time around the Covid-19 pandemic. This means that the analysis predicts longer lasting effects (or more spaced-out delays between them) due to the recent

[7] The black area means no data exists for a certain asset during that period. The color blue is just meant to represent the medium-term horizon and has no connection with correlation levels or other analysis aspects.

events. Some correlations show outside of the cone of influence or as a continuous band at the very bottom of some plots which we avoid interpreting due to data distortions at the ends of the dataset. Beyond that, there are only very few areas of importance at scales larger than 1024 days: PSA (between 2005-2018), HST (2005-2016), PEI (2006-2015), ESS (2003-2021), AVB (2000-2019), MAA (2005-2022), EQR (2004-2021), ELS (2010-2021). For all these areas, the arrows are pointing north-east for the most part, indicating that changes in CPI are leading changes in REITs by $\pi/4$ or around 4 months.

We can also look at the REITs group by group — leaving aside PEI which deals with shopping malls, we are left with self-storage facilities, hotels/resorts, and residential. It is interesting to note that the REITs within each group tend to behave similarly, especially during the periods of relevant importance we have previously identified. Figure 12 shows correlations higher than 0.5 averaged over each of the two period bands, while figure 13 shows the phase differences between REITs in each group and CPI averaged over the short term.

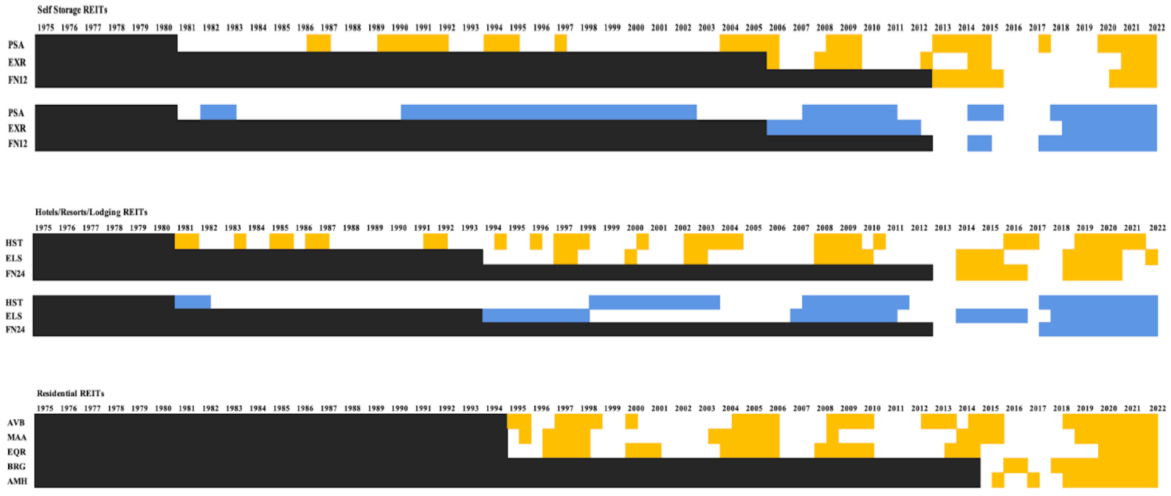

Figure 12. **Average correlation between REITs and CPI higher than 0.5 over short- and medium-term horizons, divided by type of REIT (self-storage, lodging, residential)**

Correlation was averaged across the first and the second half of each calendar year, respectively, and simultaneously across the 64-256-day or 256-1024-day scales. Correlations higher than 0.5 are considered noteworthy and are highlighted in yellow (short term) or blue (medium term) in this figure. The cells highlighted in black indicate that no data is available for those REITs at those times.

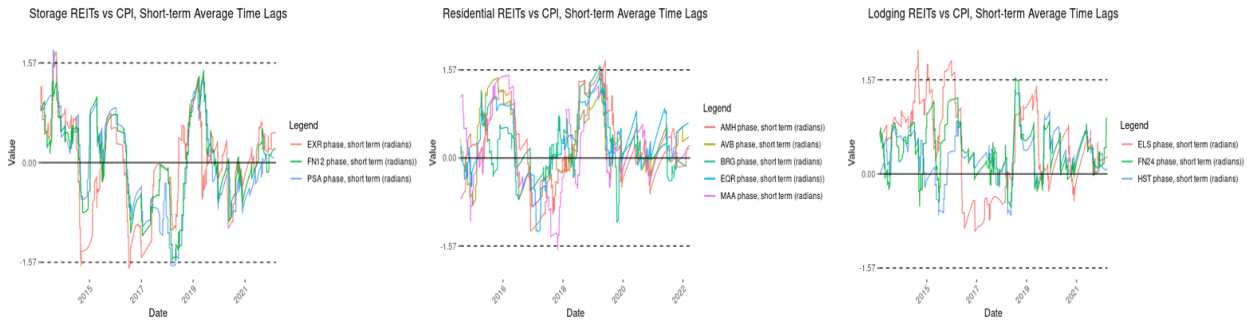

Figure 13. **Average phase difference angle averaged over the 64-256-day scales, presented by REIT group**

The three charts above show the phase difference angles between each REIT and CPI, averaged over the 64-256-day scales (short-term investment horizon) for every calendar day, and grouped by type of REIT (storage, residential, lodging). The charts only include the common time period were data exists for all REITs in each group, respectively. Notice that overall, REITs in the same group tend to perform similarly in relationship to CPI over the given time frames.

A positive phase difference angle means the two time series are in sync/positively correlated at that point; a negative angle means they are negatively correlated. Values between (0,1.57), (-3.14, -1.57) mean that inflation leads changes in the REIT; values between (1.57, 3.14), (-1.57, 0) mean that the REIT is leading changes in inflation.

As a last step, we compare the co-movement between short term REITs and inflation with the longer-term REITs (see Figure 14 below), as well as the behavior of gold, the S&P500 index, and WTI crude oil. Both long term lease REITs, COLD and STOR, display significantly fewer correlation areas in the very short term as well as between the 64–256-day bands compared to the other 13 REITs that were analyzed. Moreover, STOR and COLD seem to be more dependent on changes in the treasury bill rate than CPI, which is the opposite of what we see for the other REITs.

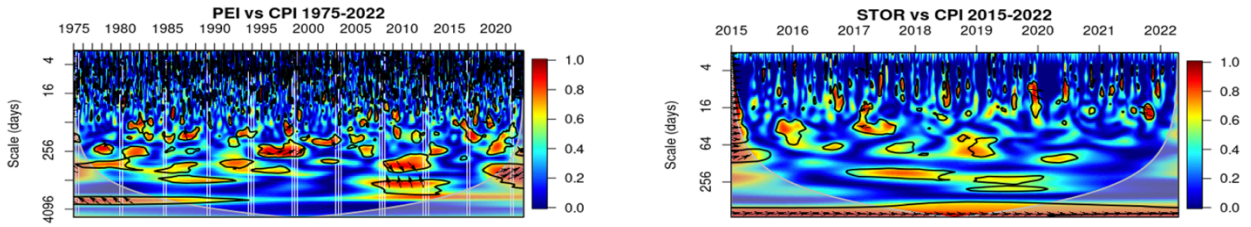

Figure 14. **Side-by-side wavelet coherence plots for PEI and STOR (versus CPI)**

Time is on the x axis and period in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation. Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-lag relationship between the REIT prices and CPI (both measured in YoY changes) (rightward pointing arrow = positive correlation, leftward pointing = negative correlation).
Notice more frequent patches of high correlation for PEI than for STOR across most scales.

The coherence heat maps for the 13 REITs generally look very similar to the coherence plot for gold (see Figure 15 below), especially because we can identify the same distinctive band of correlations around the 256-day scale. This is an indication that towards the medium-term investment horizon, (some) REITs have the potential to react the same way that gold does. If we are to trust literature and accept that gold is a good inflation hedge, that would imply that these select REITs share the same hedging properties for that scale and during those periods of time.

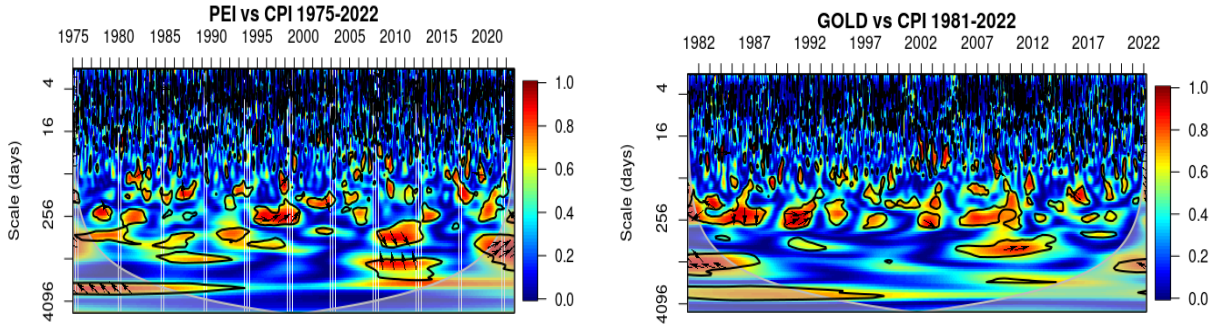

Figure 15. **Side-by-side wavelet coherence plots for PEI and gold (versus CPI)**

Time is on the x axis and period in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation. Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-lag relationship between the REIT or gold prices and CPI (both measured in YoY changes) (rightward pointing arrow = positive correlation, leftward pointing = negative correlation).

Notice similar patterns for the presence of areas of high correlation across the 64-256-day and 256-1024-day scales.

The plot for SPY (Figure 16), tracking the S&P 500 index, appears at first to include more correlation areas than the REITs and gold do. Upon inspection, it becomes apparent that inflation and SPY move out of phase often (for example between 1994-2004 and 2007-2010) , meaning that increased inflation can cause stock prices to destabilize or fall, which is undesirable. We do not see the same pronounced effect with REITs. However, it is to be expected (and in line with literature) that the two share some common characteristics due to being traded in similar ways on public exchanges.

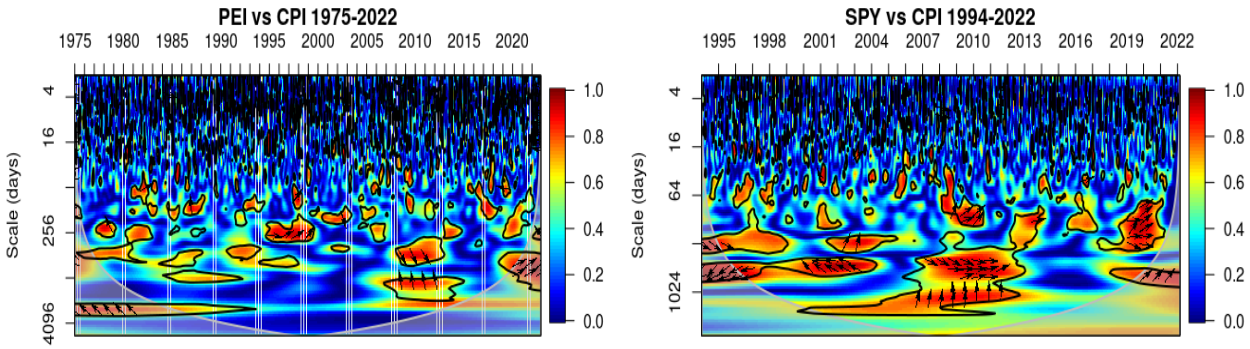

Figure 16. **Side-by-side wavelet coherence plots for PEI and SPY (versus CPI)**

Time is on the x axis and period in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation. Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-lag relationship between the REIT or SPY prices and CPI (both measured in YoY changes) (rightward pointing arrow = positive correlation, leftward pointing = negative correlation).

Notice more frequent patches of high correlation for SPY than for the REIT, especially across the 256-1024-day scales. However, SPY is often out of phase with CPI when CPI is rising, which means the more often correlation is in fact not desirable in this case.

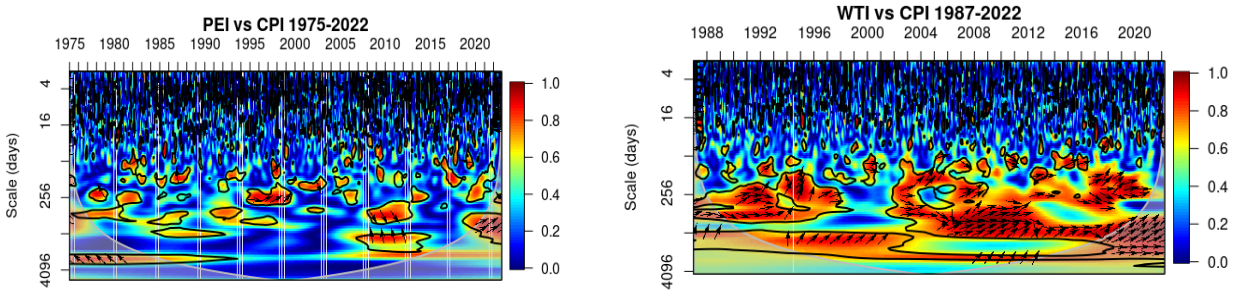

Figure 17. **Side-by-side wavelet coherence plots for PEI and WTI Crude Oil (versus CPI)**

Time is on the x axis and period in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation. Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-lag relationship between the REIT or oil prices and CPI (both measured in YoY changes) (rightward pointing arrow = positive correlation, leftward pointing = negative correlation).

Notice more frequent patches of high correlation for oil than for the REIT across most scales. In a nutshell, this is due to oil being one of the commodities most influenced by and influential to inflation.

Finally, in Figure 17, WTI crude oil shows the highest and most frequent correlations with changes in CPI all throughout the 64–2048-day intervals. This is because oil and inflation are highly dependent on one another, and factors that drive inflation also encourage demand for oil.[8] Typically, rising inflation can happen because of either increased money supply or supply chain disruptions and shortages. Both of these help push demand for oil up. Oil is priced in dollars, so in the first scenario an increased dollar supply for the same global oil supply will lead to higher prices. In the second case, oil production is complex. Any disruptions in transportation, workforce, political relations, or natural resources can put pressure on the oil supply available. If supply stops being able to meet demand, prices will rise. While this is true in the long run for many goods, it can more easily become an issue for commodities since natural resources are very limited, and oil is one of the ones most prone to this. We have seen both of the prior scenarios take place throughout the Covid-19 pandemic and Russia-Ukraine war. Oil is also less sensitive to changes in interest rates which usually happen when there are changes in inflation. For these reasons, investors and researchers tend to consider oil a better hedge against inflation than gold. REITs on the other hand do not share the same strong relationship with inflation, so while they behave similarly to gold and better than traditional equity stocks, they do not match the hedging capabilities of oil.

This is an interesting and important fact to note, which we will circle back to in the conclusion section. Housing is a large part of CPI, direct property returns is positively correlated with inflation, and REIT performance is positively correlated with direct property returns. However, the correlation between REITs and CPI is not consistent across all calendar time periods because of the equity component of REITs which makes them fluctuate more when the stock

[8] (TradeSmith Investment Services, 2022)

market is impacted. On a qualitative scale, this places REITs somewhere in between equity stocks (like the S&P 500) and direct property investments or direct commodity investments.

4.2 Diversified Portfolio Construction

After analyzing all coherence heatmaps and correlation phase plots, we were able to construct portfolios to better visualize the benefits of including REITs in an investment strategy. Based on the individual analysis, 5 REITs were identified that seemed to best match our hedging interest: PSA, HST, ELS, EQR, and FN24. In the interest of a larger time frame and because FN24 only dates back to 2013, we are using PSA, HST, ELS, and EQR for the portfolios. Four such portfolios were created, as follows:

1. Benchmark Portfolio 1: SPY-only (100% of the portfolio is invested in SPY stock)
2. Benchmark Portfolio 2: REITs-only (100% of the portfolio is invested into an equally weighted mix of PSA, HST, ELS, and EQR, so each REIT makes up 25% of the portfolio)
3. Diversified Portfolio 1: 80/20 REITs/SPY (80% of the portfolio is invested in the REITs mix, 20% in SPY stock)
4. Diversified Portfolio 2: 70/15/15 REITs/SPY/Oil (70% of the portfolio is invested in the REITs mix, 15% in SPY stock, 15% in WTI crude oil)

The weights were chosen through trial and error for exploration purposes rather than mathematically chosen to best optimize portfolio returns. However, the components of each portfolio are indeed informed by the results of the wavelet analysis and are the ones that showed most promise as a potential inflation hedge.

While the SPY-only and REIT-only returns are mostly similar, there are a few periods of wide gaps between the two, where one exhibits negative returns and the other positive (in Figure 18, see 1999-2001, 2001-2003, and 2016-2019). We aim for a more stable return, either positive across the entire time frame, or at least less negative during those periods. Therefore, we resort to mixed/diversified portfolios.

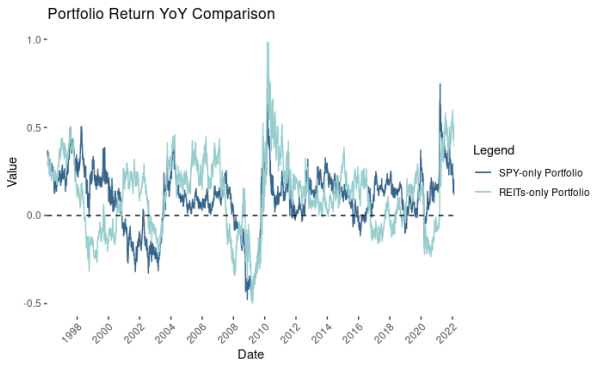

Figure 18. **Year-over-Year Return Comparison Between SPY-only and REITs-only Portfolios**

The figure depicts the YoY change in prices for the two benchmark portfolios – SPY-only (which is the same as the individual SPY stock chart found in appendix A2) and REITs-only, consisting of equal parts PSA, HST, ELS, and EQR (which is the average of their individual YoY changes in stock prices calculated for every calendar day).

The REITs/SPY and REITs/SPY/Oil combinations also appear similar, with the main exception of the 2008-2009 period, where the oil portfolio outperforms the former, and the 2014-2016 period where the opposite happens. Because in the 2008-2009 case only one portfolio shows positive returns, while in the 2014-2016 one both are somewhat positive, we prefer to use the oil portfolio for the rest of the analysis.

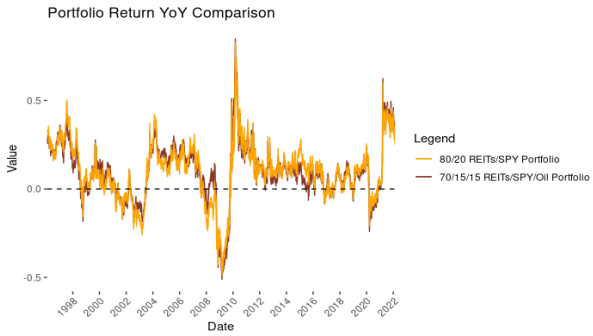

Figure 19. **Year-over-Year Return Comparison Between Diversified Portfolios**

The figure depicts the YoY change in prices for the two suggested diversified portfolios – one consisting of 80% the REITs mix and 20% SPY stock, and the other consisting of 70% the REITs mix, 15% SPY stock, and 15% oil.

Notice that the newly constructed portfolio consisting of REITs, SPY, and oil is able to considerably reduce losses versus the single-asset portfolios. As with all hedging strategies, reduced losses come at the price of reduced gains, but even then, the overall performance of the mixed portfolio appears better than that of a stock-only (or REITs-only) investment.

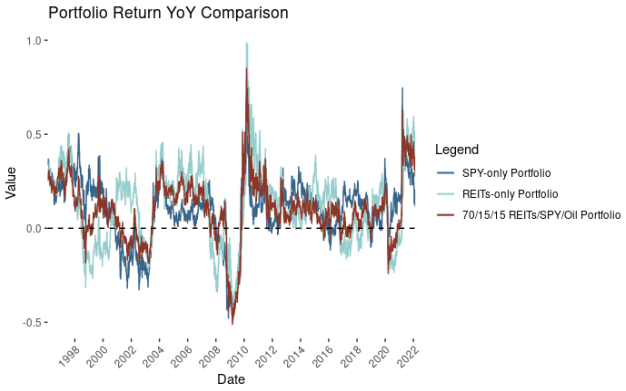

Figure 20. **Year-over-Year Return Comparison Between SPY-only, REITs-only, and 70/15/15 Mixed Portfolio**

The figure depicts the YoY change in prices for the second diversified portfolio in comparison with the benchmark SPY-only and REITs-only portfolios.

From a wavelet standpoint, we can again break down the analysis into short-to-medium term (64-256-day period bands) and medium term (256-1024-day period bands). Figure 21 is helpful here. In the short term, while the changes of the mixed portfolio with inflation are slightly more precise than SPY, we do not necessarily see much consistent improvement over a stock-only portfolio. However, we can confirm the benefits of adding REITs and oil to the portfolio for medium-term investment horizons. On the heatmap, more correlation spots can be identified below the 256-day mark in the 70/15/15 portfolio than the SPY one. Moreover, in Figure 22, if we carefully look at the high correlation areas (shaded below), the direction of inflation, and the phase between each portfolio returns and CPI for the medium term (blue line), we notice that SPY is always either in phase with CPI, meaning that it drops with low inflation and it rises with high inflation, or in anti-phase with rising CPI, meaning that it drops when

inflation rises between 1996-1997, 2014-2015, and 2015-2017. Meanwhile, the 70/15/15 mix does a much better job adapting to changing inflationary environments.

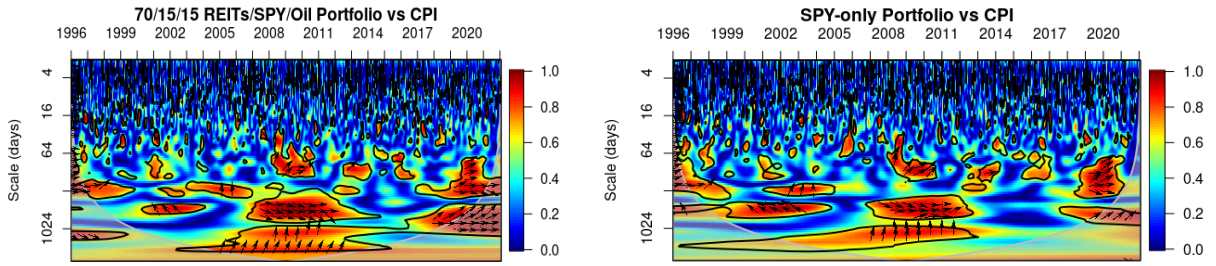

Figure 21. **Side-by-side wavelet coherence plots for the 70/15/15 mixed portfolio and the SPY-only portfolio (versus CPI)**

Time is on the x axis and period in days is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation. Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence. Arrows show the lead-lag relationship between the portfolio or SPY prices and CPI (both measured in YoY changes) (rightward pointing arrow = positive correlation, leftward pointing = negative correlation).

Notice similar patterns of correlation between the diversified portfolio and CPI and the benchmark SPY-only portfolio and CPI in the short term (up to 256 days). However, we can see benefits of including REITs and oil in the portfolio for the medium-term, shown through increased areas of correlation as well as more favorable lead-lag relationships (the latter effect can be more easily seen in Figure 21).

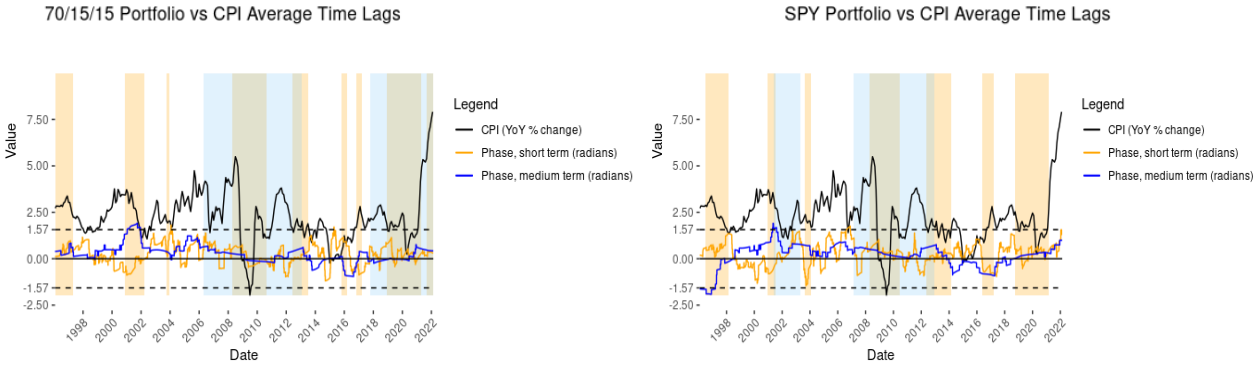

Figure 22. **Side-by-side phase difference angle comparison for the 70/15/15 mixed portfolio and the SPY-only portfolio.**

The black line shows the YoY change in CPI from 1996 to 2022. The yellow line shows the lead-lag phase difference angle for each portfolio, averaged at any given time over the entire 64-256-day scale band, corresponding to part of the short-term investment horizon. The blue line shows the lead-lag phase difference angle for each portfolio, averaged at any given time over the 256–1024-day scale band, corresponding to the entire middle-term investment horizon. The vertically shaded areas represent periods of time where the correlation between the portfolio in question and changes in inflation is above 0.5 (same significance for colors: yellow = short term, blue = medium term). Lower correlations reduce the meaningfulness of the phase difference angle since effects of one time series on to the other are minimal. Higher correlations mean the effects would be significant, so those are periods of particular interest in our case.

5. Conclusion

Real estate is frequently proposed as a way to hedge against rising inflation. Equity REITs are a more liquid and accessible way of investing in real estate, however empirical evidence about the effectiveness of using REITs as an inflation hedge in the US is limited. This paper analyzes the dynamic inflation hedging properties of US REITs with short term leases that specialize in either storage facilities, hotels/resorts, and residential properties across the US. The motivation behind analyzing REITs instead of direct property investments is that if REITs are indeed good inflation hedges, then investors from a wider range of geographic locations and net-worth brackets would have access to this portfolio protection strategy. It is hypothesized that short-term lease REITs would better suit this goal than long-term ones because of their ability to adjust rents more quickly and minimize losses during market shocks. By using a Morlet wavelet continuous transform and the wavelet coherence method, we are able to localize co-movement between inflation and REIT performance and describe hedging abilities for the short, medium, and long term at any moment in time.

Findings show that several of the REITs considered act as a short-to-medium- and medium-term inflation (CPI) hedge at several points in calendar time. This does not hold for expected inflation proxied by short term treasury bill rates. Results also show that REITs within the same group (storage, hotels/resorts, residential) behave similarly to each other despite small differences from group to group. When compared to other assets, it is found that REITs are similar to gold in the short and medium term, both assets reacting favorably when CPI goes up at a higher pace, but none of them protecting against drops in inflation or during the very short or very long term. REITs also react differently (but better) than equity stock and long-term REITs. However, they do not

match up to the inflation hedging potential of crude oil as measured by the WTI. In terms of responding to inflation shocks, REITs inherit some of the variability of equity stocks, as well as some of the benefits of direct real estate properties. We assert that diversified portfolios consisting of equity stocks, short-term lease REITs, and potentially oil, deliver more consistent returns, reducing losses and adapting better to inflationary changes than a stock-only portfolio. In the context of this paper, two examples are suggested: an 80%-20% mix of REITs (PSA, HST, ELS, EQR), and the SPY ETF and a 75%-15%-15% mix of the same REITs, oil, and the SPY ETF. While the weights are not strategically chosen to optimize portfolio returns, the components of each portfolio are informed by the results of the wavelet analysis. The suggested portfolios show improved correlation and lead-lag relationships with CPI during key moments, as well as reduced losses in critical moments with only small reductions in potential gains at the peaks.

The results of the current study do not contradict existing literature, in that mixed results are found and the answer to the question "Are REITs a good inflation hedge?" depends on the specific REIT and investment horizon. This paper adds on to existing literature by applying a dynamic wavelet analysis method to the study of specifically short-term lease REIT investments specifically in the US and pertaining to four categories of the real estate industry (residential, storage, retail/shopping malls, and hotel and lodging). There are advantages and drawbacks to using wavelets to treat this problem. On the one hand, it has the benefit of accounting for both time and frequency domains, pointing out fluctuations in causality relationships as opposed to other causality tests, and not having to pre-define a causality structure between the two variables which is unavoidable with typical regression/OLS models and that changes so often in real world. The ability to distinguish between different frequencies is especially relevant when looking at financial data or investment decisions. Time horizons are very important to investor when devising a

strategy and using wavelets is one method of providing information in this direction. In our particular case, for example, we notice no clear benefits for the very short or very long term, which we might not be able to deduce from a regular regression application. On the other hand, this is a relatively new method, so it is perhaps not yet perfected, not as much methodology is available, and it does not currently allow for the inclusion of any categorical variables into the nonparametric tests. Another drawback or uncertainty of the current study is due to the monthly reporting of the CPI index. Linear interpolation is used to bring CPI data to a daily frequency to match up with the changes in REIT prices, which may not be completely accurate in the very short term (but should not change anything for the bigger scales). Lastly, similar to other advanced asymmetric time-series analysis methods, this method is also constrained by having to select only a limited number of variables at most to include in the model (Habib et al., 2021).

Several areas of future research can be identified at the end of this study. For example, partial wavelet coherence and partial phase difference extensions could be used in conjunction with any third variables that could be impacting both CPI and the real estate or REIT market at the same time. This would eliminate the effects of the third confounding variable and confirm the real relationship between the two main variables. Also, the concept of wavelet value at risk could be adapted to investigate if more volatility is added to a given portfolio (at any time or frequency) through the addition of REITs based on changes in inflation. In terms of data, a possible improvement would involve developing a method of analyzing the relationship between CPI and the Net Operating Income of each REIT instead of stock price. Deeper qualitative research into the macroeconomic factors affecting each individual time interval would also be valuable.

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Appendix[9]

A 1. Individual REIT description

This section consists of brief descriptions for each REIT and their background, along with the initial reasoning for considering them potential inflation hedges and choosing to include them in the analysis. Information summarized below is mostly collected from the individual REIT websites or from the The National Association of Real Estate Investment Trusts (Nareit) website.

PREIT (PEI)

PREIT (PEI) is a REIT long established as a developer and operator of retail and entertainment destinations, specializing among others in differentiated shopping malls. This includes several short-term lease opportunities such as temporary inline space and pop-ups, as well as carts and kiosks, all of which were reported to be important revenue generators.

Nareit Self Storage (FN12)

The National Association of Real Estate Investment Trusts (Nareit) represents most real estate investment trusts (REITs) in the US. The FTSE Nareit US Real Estate Index Series is a comprehensive family of REIT-focused indexes that span the commercial real estate industry and can be used as a benchmark or to gain exposure to different real estate sectors in the US. FN12 is part of this index series and focuses solely on self-storage REITs. Self-storage units offer short-term leases which allows for the increase of rent prices in order to adjust for inflationary pressures.

Public Storage (PSA)

PSA REIT is the world's largest owner and developer of self-storage facilities, with more than 2500 facilities in the US, and they are a member of the S&P 500 and FT Global 500. Similar to FN12, they are able to adjust rent quickly.

Extra-Space Storage (EXR)

EXR is the largest self-storage management company in the US and the second largest owner/operator of these units in the country. They are also a member of the S&P 500. Similar to FN12 and PSA.

Nareit Lodging/Resorts (FN24)

Similarly, FN24 is part of the same FTSE Nareit US Real Estate Index Series with a focus on lodging/hotel/resort REITs. The nature of the lodging industry allows for quick adjustment of leases/prices, which is desirable in inflation hedging strategies.

[9] For clarity, it is recommended to the reader to view the charts in the appendix in the pdf version on a desktop.

Host Hotels and Resorts (HST)

HST is an S&P 500 company and is the largest lodging real estate investment trust (REIT) and one of the largest owners of luxury and upper-upscale hotels. Their properties are mostly situated in the heart of business/tourist districts. Like FN24, they allow for quick adjustments to inflation.

Equity Lifestyle Properties (ELS)

ELS holdings are concentrated in Manufactured Home Communities with many leisure amenities (gold courts, tennis, etc.), Resorts, and Campgrounds in North America. Reasoning for including it in analysis is similar to FN24 and HST.

Avalon Bay Communities (AVB)

AVB owns and operates apartments in key metropolitan areas such as New England, New York City, Washington, D.C., Seattle, and California. They benefit from flexibility in term of leases, making it a good contender for inflation hedging purposes. It is also more geographically diverse than other residential REITs included in this thesis. Around 60% of their holdings are in suburban areas and 30% in urban areas.

Mid-America Apartment Communities (MAA)

As the name suggests, MAA is focused on acquiring and redeveloping multifamily residential units in mid-America.

Equity Residential (EQR)

EQR typically acquires residential communities of at least 100 apartments or develops new units. It is an S&P 500 company and it is known for targeting urban and high-density suburba coastal gateway markets inhabited by mostly affluent renters.

Bluerock Residential Growth (BRG)

BRG acquires and operates institutional-quality apartment buildings in demographically attractive growth markets across all of the US.

American Homes 4 Rent (AMH)

AMH is known for acquiring and developing high-quality single-family homes across the US.

Americold Realty Trust (COLD)

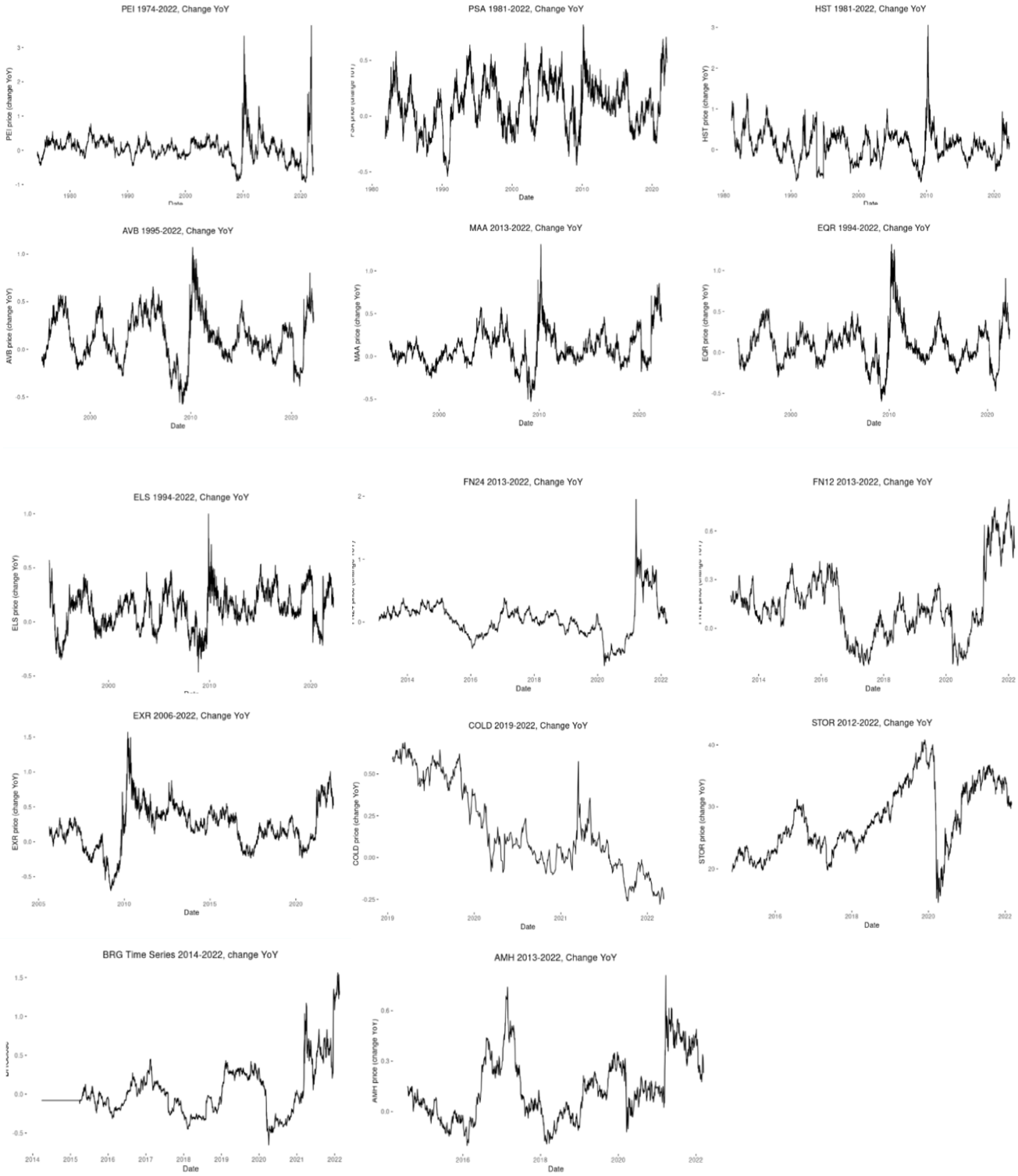
COLD manages and operates long-term-lease temperature-controlled warehouses. Because the contracts for this type of storage facilities are usually signed for longer periods of time, we use COLD as one of the two REITs to which to compare the short-lease ones. COLD's units play a big role in the supply chain for food producers, distributors, and consumers.

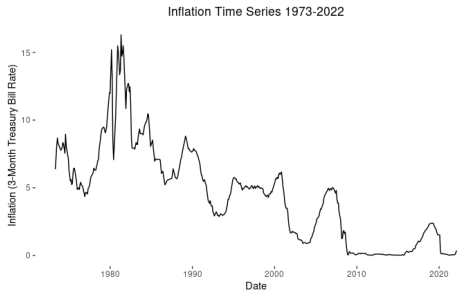
Store Capital (STOR)

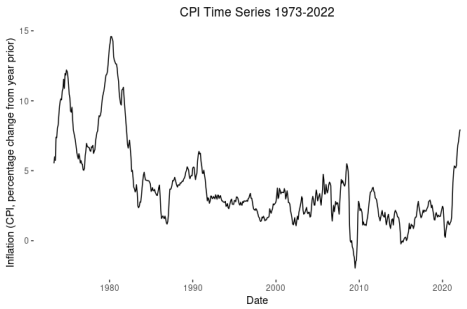
STOR is a REIT focusing on commercial real estate, specifically longer-term net-leases (meaning the consumer is responsible for paying most of the taxes and maintenance fees). Due to its longer-term objectives, we use it alongside COLD as an object of comparison.

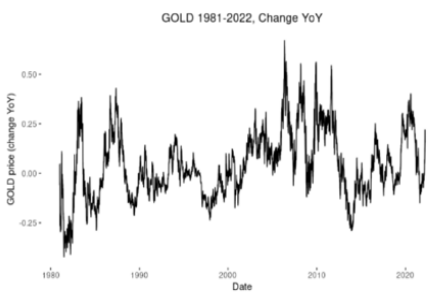
A 2. Year-over-Year Changes in Inflation and Assets

This section includes the daily year-over-year changes in prices for each asset, as well as the daily year-over-year changes in CPI and the daily 13-week T-bill rate. The x axis for each chart represents calendar time and the y axis represents the year-over-year change in prices (i.e. the percentage change in prices from one year prior).




Inflation Time Series 1973-2022
Inflation (3-Month Treasury Bill Rate)
15
10
5
0
1980
1990
2000
2010
2020
Date


CPI Time Series 1973-2022
Inflation (CPI, percentage change from year prior)
15
10
5
0
1980
1990
2000
2010
2020
Date

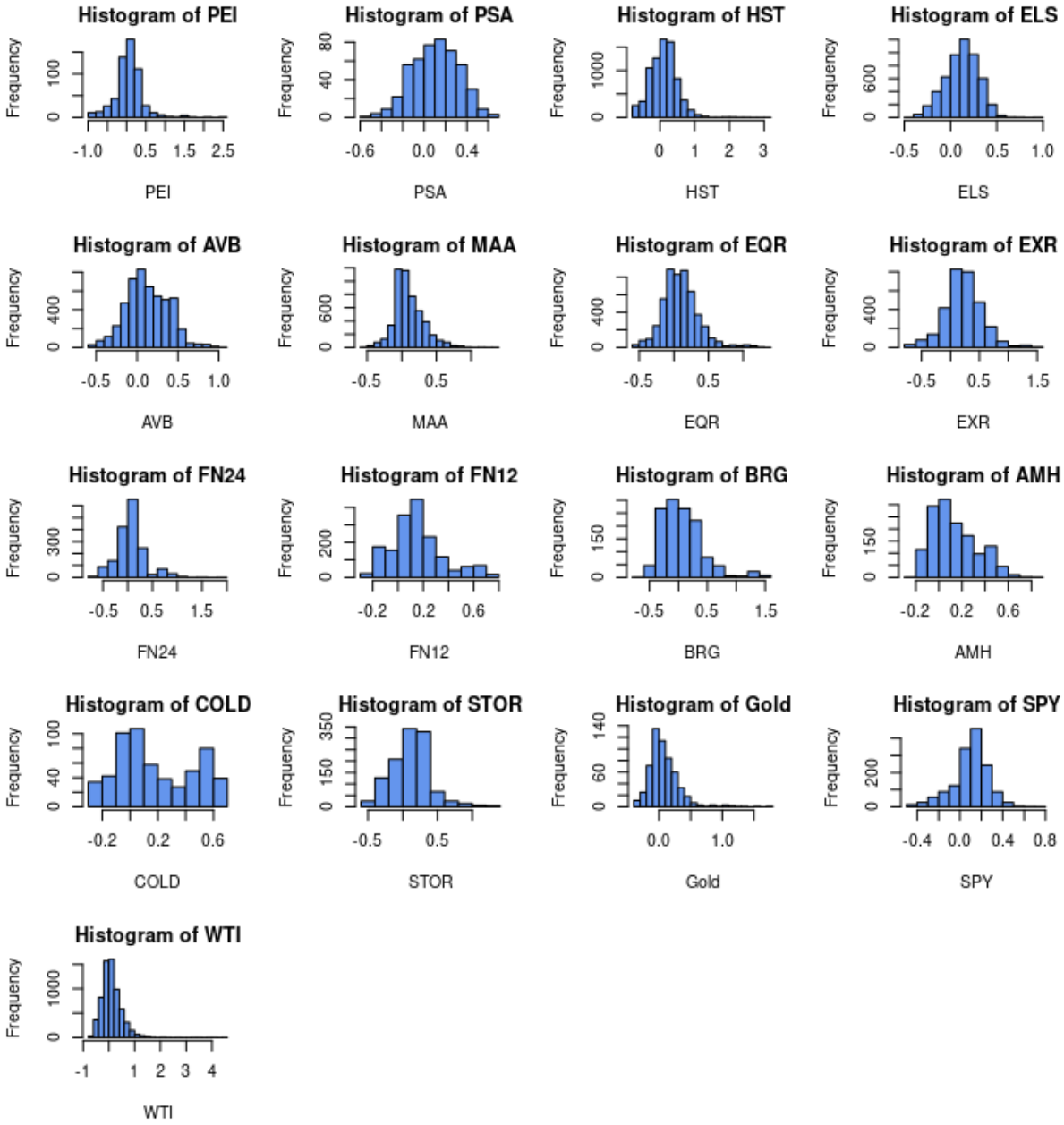

GOLD 1981-2022, Change YoY
GOLD price (change YoY)
0.50
0.25
0.00
-0.25
1980
1990
2000
2010
2020
Date


SPY 1994-2022, Change YoY
spy price (change YoY)
0.75
0.50
0.25
0.00
-0.25
-0.50
2000
2010
2020
Date


WTI 1987-2022, Change YoY
WTI price (change YoY)
4
3
2
1
0
-1
1990
2000
2010
2020
Date

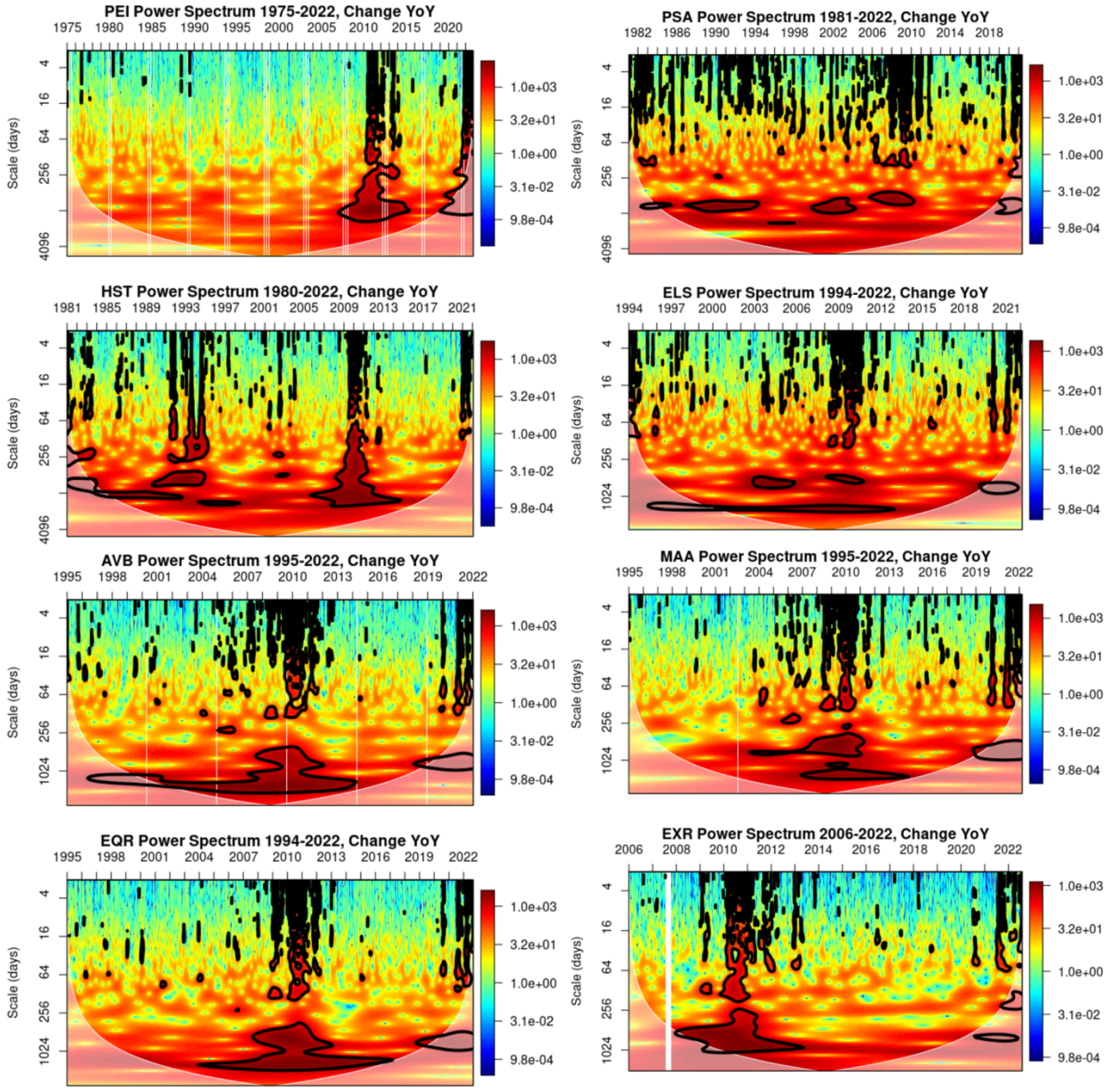
A 3. Year-over-Year Asset Returns Histograms

This section includes the histograms of the YoY initial returns for each asset (i.e. for the percentage change in prices from one year prior). Except for the COLD REIT, all distributions are unimodal. Notice, however, that they are all centered around different return percentages, and several of them include extreme values well above 100% yearly changes in price.



A 4. Full Set of Wavelet Power Spectrum Plots

Each chart shows the wavelet power spectrum of all the REITs analyzed in this paper, as well as the two measures of inflation, gold, oil, and the S&P 500 index. Time is displayed on the x axis, while the scale (the opposite of frequency) is on the y axis. Red areas indicate higher power (i.e. higher variance, which can be interpreted as a signal for a significant event/shock affecting the asset). Blue areas indicate lower power. The black contour line surrounds areas significant at a 5% level. The grey circular area is the cone of influence — due to dataset limitations, everything outside of the cone should be interpreted with caution.



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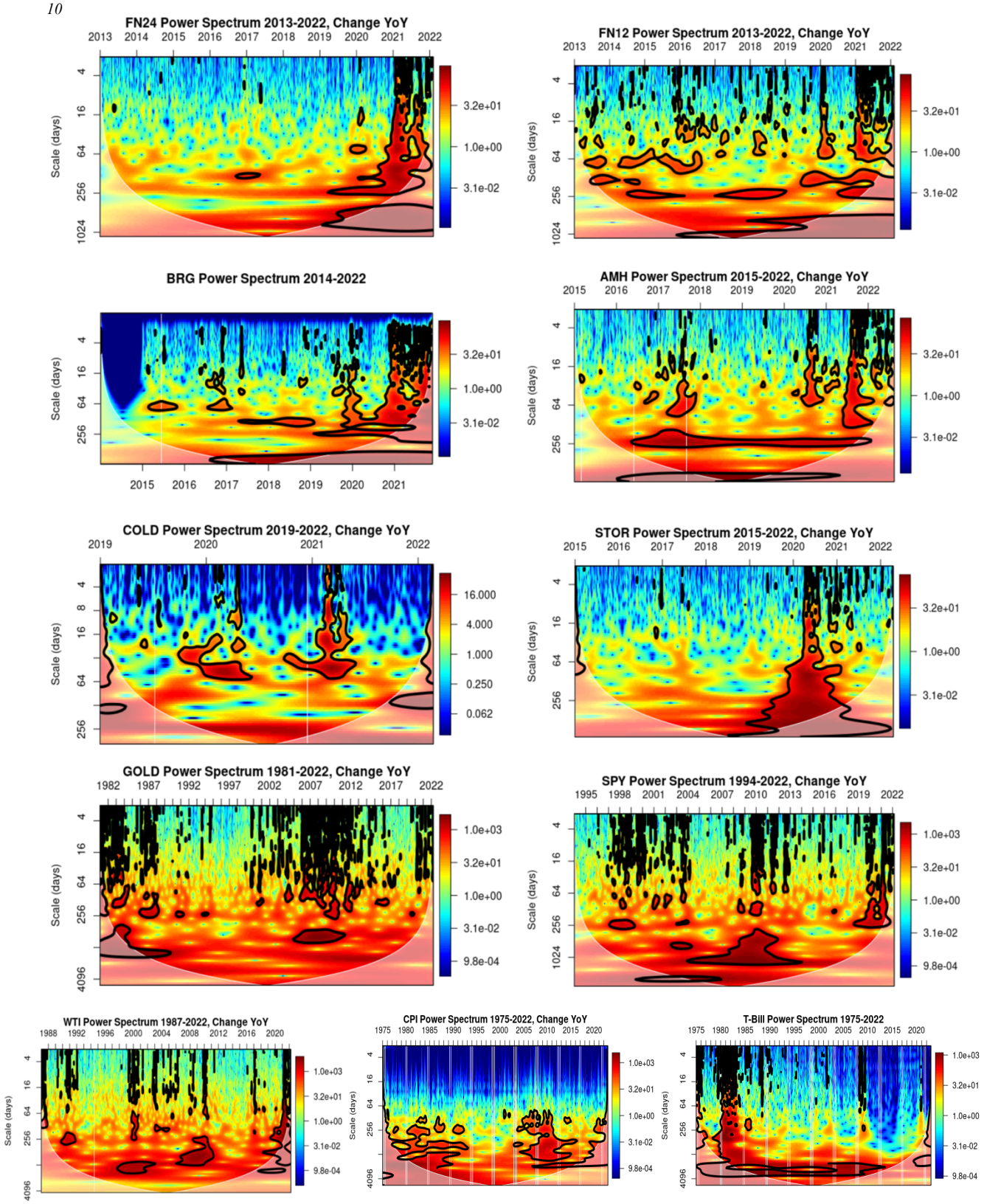
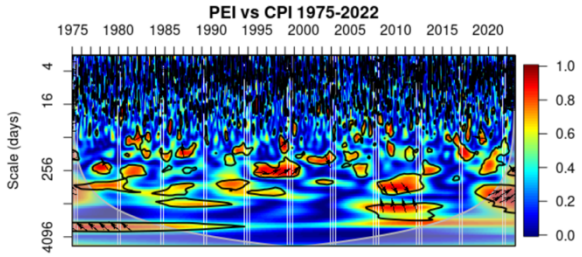
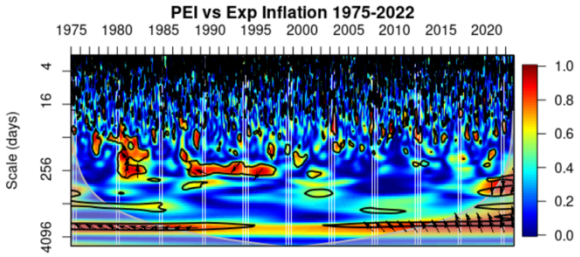


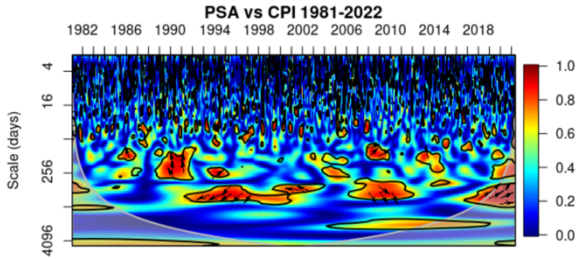
[10] **For the CPI power spectrum**, we attribute the blue band of very low power at the top of the plot (scales of 30 days and less) to the interpolation method used to go from monthly sampling frequency to daily. Because linear interpolation is used, no "significant events" can happen throughout those scales, so no higher power will show on the chart.

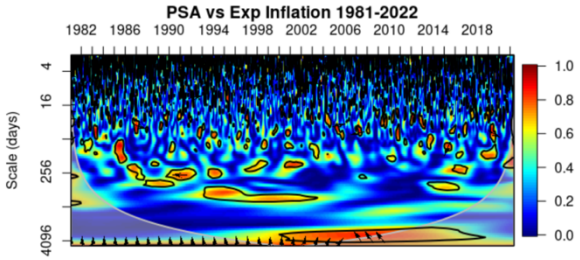
A 5. Full Set of Wavelet Coherence Plots

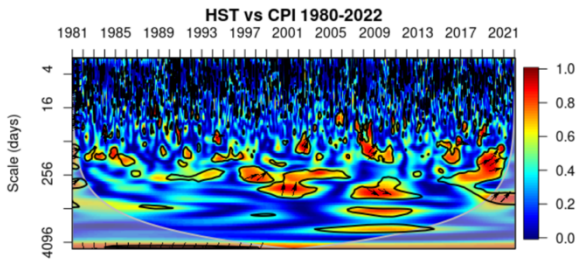
Each chart shows the coherence plot of all the REITs analyzed in this paper, as well as gold, oil, and the S&P 500 index. The left column shows coherence with the CPI index, while the right column shows coherence with expected inflation proxied by the 13-week T-bill rate. Time is displayed on the x axis, while the scale (the opposite of frequency) is on the y axis. Red areas indicate higher coherence, which can be interpreted as high correlation (with values between 0 and 1). Blue areas indicate lower correlation. The black contour line surrounds areas significant at a 5% level. The gray circular area is the cone of influence — due to dataset limitations, everything outside of the cone should be interpreted with caution. Arrows show the lead-lag relationship between asset and CPI. Arrows pointing to the right indicate positive correlation and those to the left negative correlation. The methods theory section can be consulted for specific interpretation of the angles.

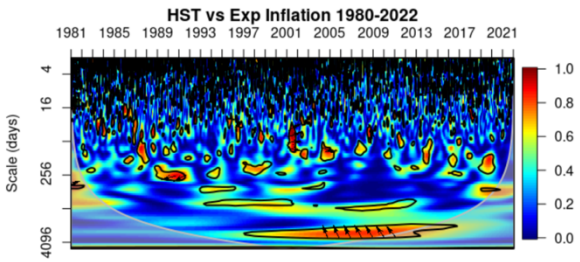


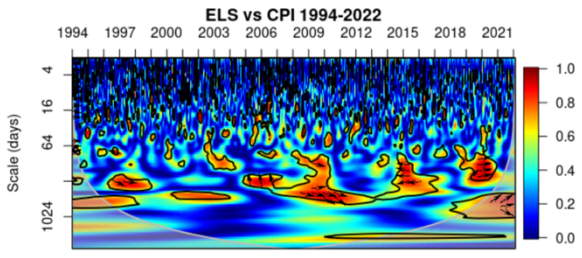


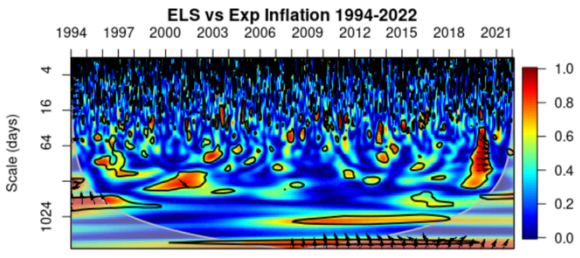


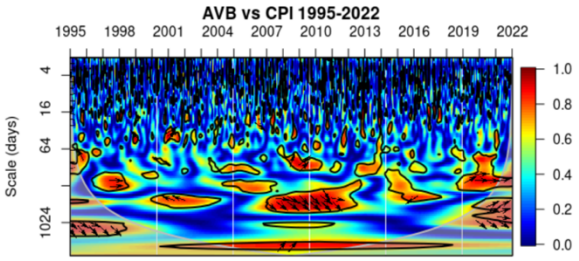


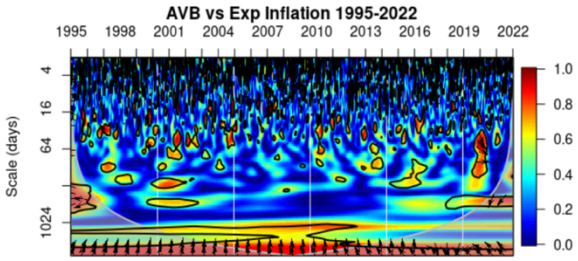









AVB vs CPI 1995-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
4 16 64 1024
1.0 0.8 0.6 0.4 0.2 0.0


AVB vs Exp Inflation 1995-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
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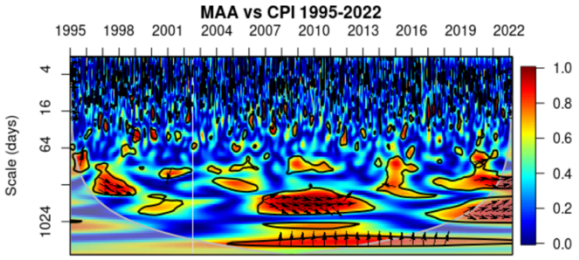
MAA vs CPI 1995-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
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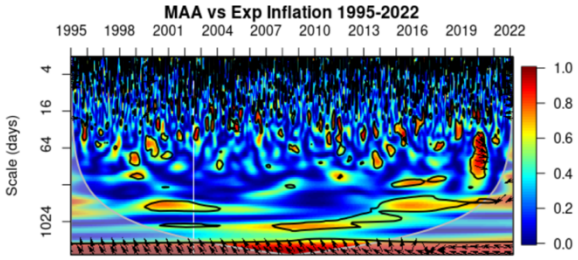
MAA vs Exp Inflation 1995-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
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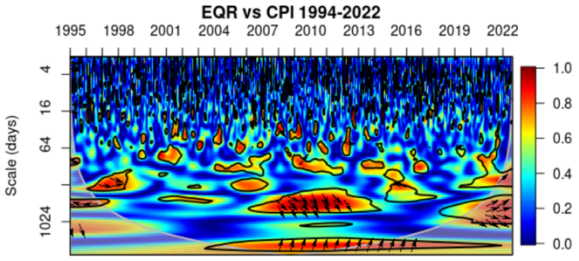
EQR vs CPI 1994-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
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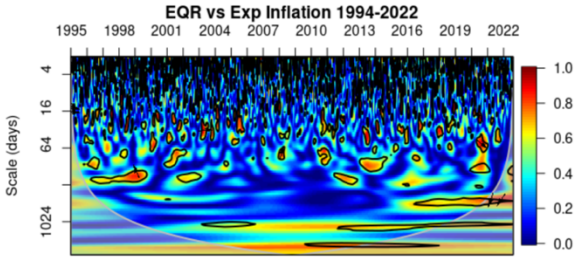
1.0 0.8 0.6 0.4 0.2 0.0


EQR vs Exp Inflation 1994-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
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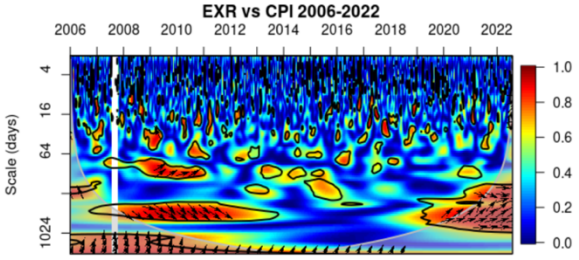
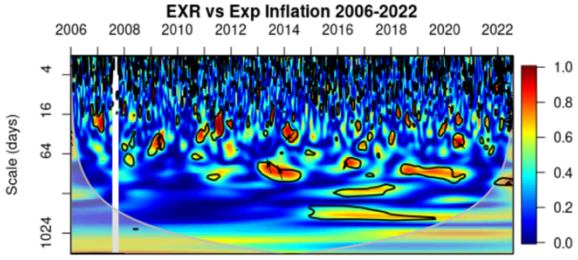

EXR vs CPI 2006-2022
2006 2008 2010 2012 2014 2016 2018 2020 2022
Scale (days)
4 16 64 1024
1.0 0.8 0.6 0.4 0.2 0.0


EXR vs Exp Inflation 2006-2022
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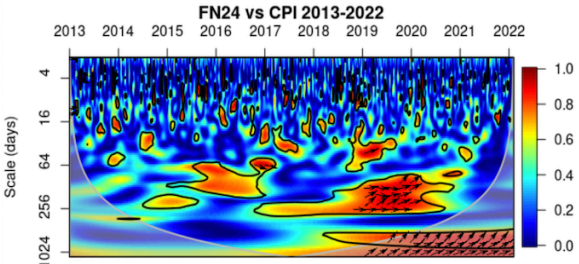
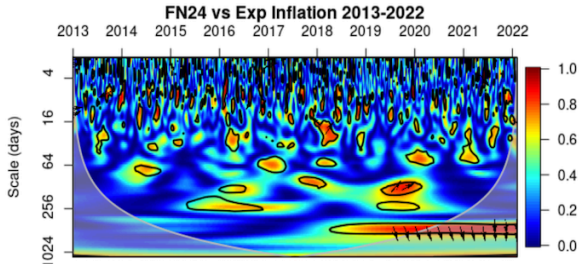
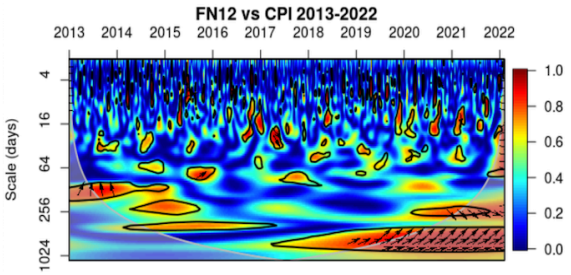

FN24 vs CPI 2013-2022
2013 2014 2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
4 16 64 256 1024
1.0 0.8 0.6 0.4 0.2 0.0


FN24 vs Exp Inflation 2013-2022
2013 2014 2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
4 16 64 256 1024
1.0 0.8 0.6 0.4 0.2 0.0


FN12 vs CPI 2013-2022
2013 2014 2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
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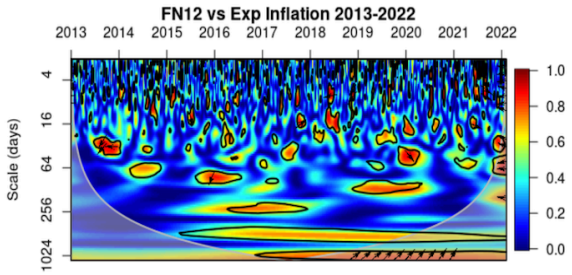
FN12 vs Exp Inflation 2013-2022
2013 2014 2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
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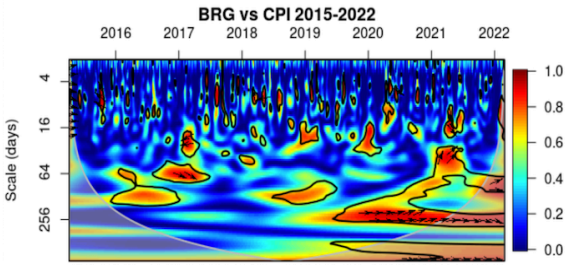
BRG vs CPI 2015-2022
2016 2017 2018 2019 2020 2021 2022
Scale (days)
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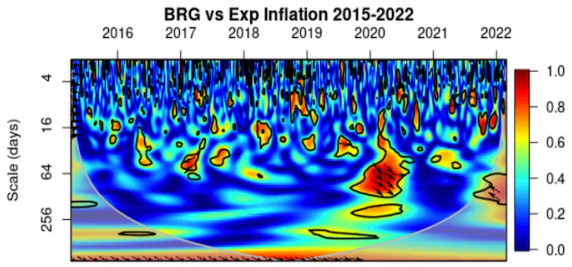
1.0 0.8 0.6 0.4 0.2 0.0


BRG vs Exp Inflation 2015-2022
2016 2017 2018 2019 2020 2021 2022
Scale (days)
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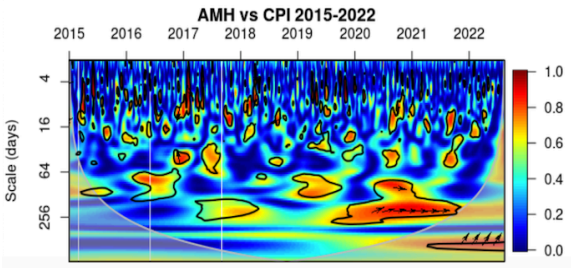
1.0 0.8 0.6 0.4 0.2 0.0


AMH vs CPI 2015-2022
2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
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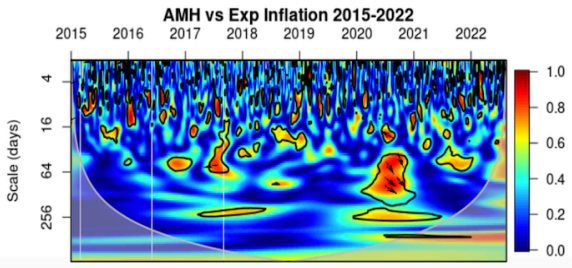
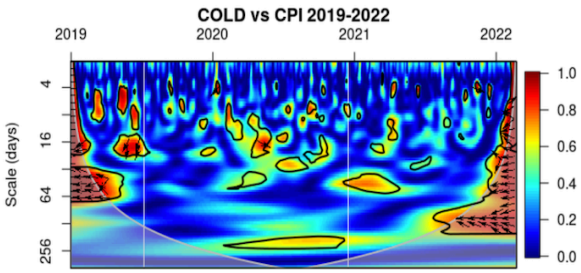
1.0 0.8 0.6 0.4 0.2 0.0


AMH vs Exp Inflation 2015-2022
2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
4 16 64 256
1.0 0.8 0.6 0.4 0.2 0.0


COLD vs CPI 2019-2022
2019 2020 2021 2022
Scale (days)
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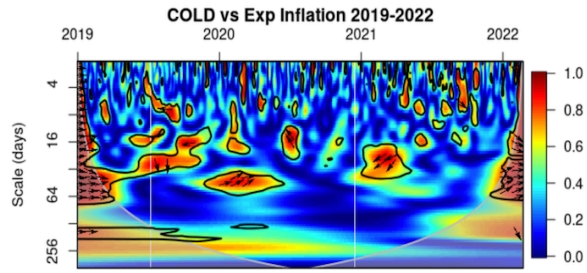
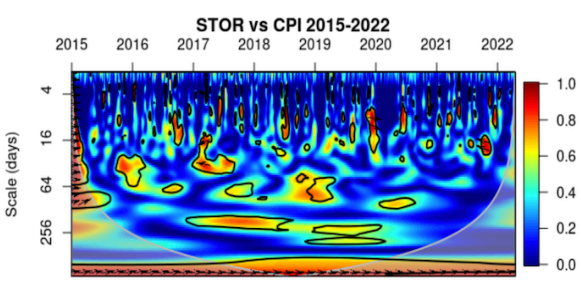
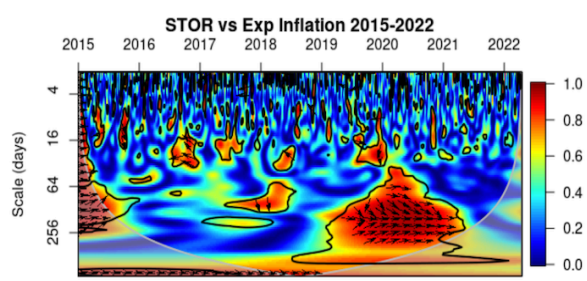
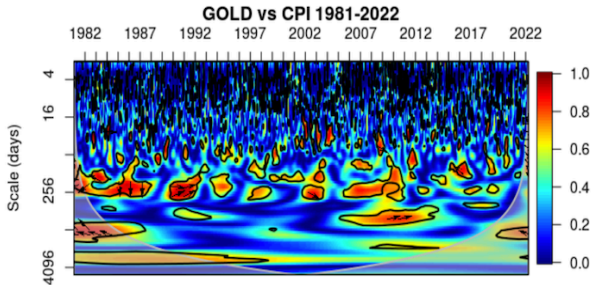
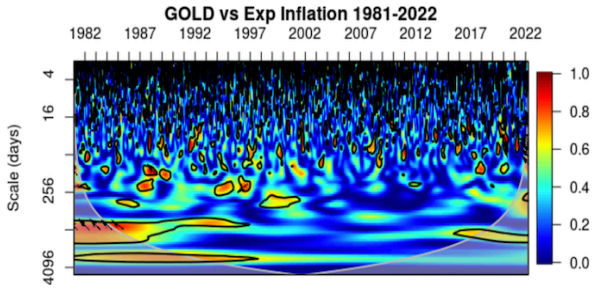
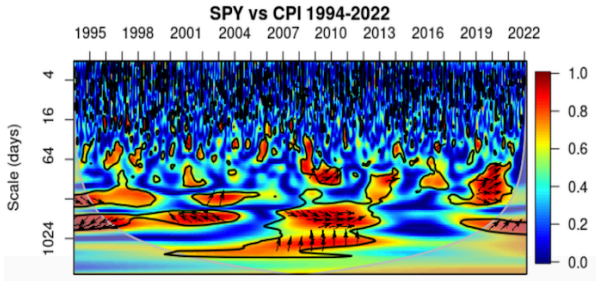

COLD vs Exp Inflation 2019-2022
2019 2020 2021 2022
Scale (days)
4 16 64 256
1.0 0.8 0.6 0.4 0.2 0.0

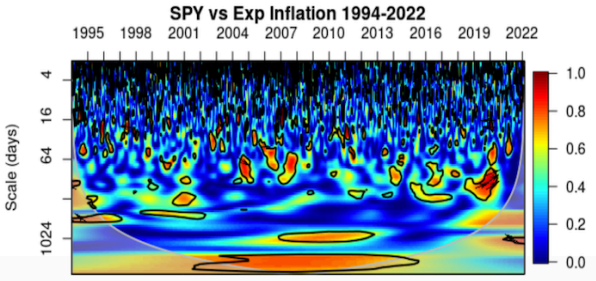

STOR vs CPI 2015-2022
2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
4 16 64 256
1.0 0.8 0.6 0.4 0.2 0.0

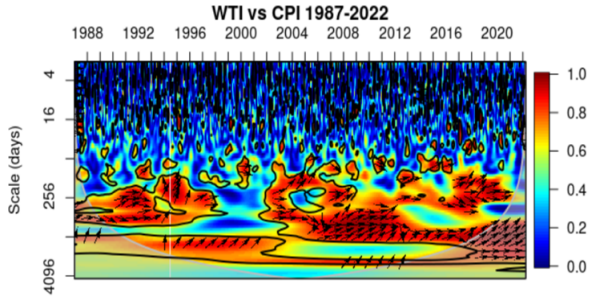

STOR vs Exp Inflation 2015-2022
2015 2016 2017 2018 2019 2020 2021 2022
Scale (days)
4 16 64 256
1.0 0.8 0.6 0.4 0.2 0.0


GOLD vs CPI 1981-2022
1982 1987 1992 1997 2002 2007 2012 2017 2022
Scale (days)
4 16 256 4096
1.0 0.8 0.6 0.4 0.2 0.0


GOLD vs Exp Inflation 1981-2022
1982 1987 1992 1997 2002 2007 2012 2017 2022
Scale (days)
4 16 256 4096
1.0 0.8 0.6 0.4 0.2 0.0


SPY vs CPI 1994-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
4 16 64 1024
1.0 0.8 0.6 0.4 0.2 0.0


SPY vs Exp Inflation 1994-2022
1995 1998 2001 2004 2007 2010 2013 2016 2019 2022
Scale (days)
4 16 64 1024
1.0 0.8 0.6 0.4 0.2 0.0


WTI vs CPI 1987-2022
1988 1992 1996 2000 2004 2008 2012 2016 2020
Scale (days)
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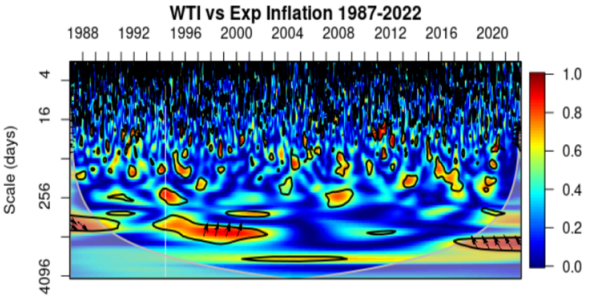
1.0 0.8 0.6 0.4 0.2 0.0


WTI vs Exp Inflation 1987-2022
1988 1992 1996 2000 2004 2008 2012 2016 2020
Scale (days)
4 16 256 4096
1.0 0.8 0.6 0.4 0.2 0.0

A 6. Phase Difference Angles Between REITs and CPI

Each chart shows the YoY change in CPI (in **black**), along with the short-term (in yellow) and medium-term (in blue) phase difference between the REIT and CPI at any given calendar time. The last chart shows the same thing for the final suggested portfolio of 70% REITs/15% SPY/15% Oil. The phase difference in this case is measured in radians and is calculated as the average over the periods of bandwidth 64-256 days and 256-1024 days, respectively. A positive phase angle means that the two time series (REIT/portfolio and CPI) are in phase/positively correlated at that time, and a negative phase means they are negatively correlated. For inflation hedging purposes we would want a positive angle when inflation is rising (and possibly a negative angle when inflation is falling). The phase differences are more relevant during periods of high significant correlation between the two time series, as those are the only moments when effects of one series on another are noticeable, therefore the time intervals between effects become more meaningful.