

Putting Children on Fixed Points



long, long time ago, a group of children were standing on top of an unstable fixed point.

The point was very small, and the space they stood on was very limited. Squeezed in tightly, their elbows squished to their sides, they climbed one on top of the other and had to stand very still to prevent from falling off the edges of the fixed point. It was a very bizarre situation, but sometimes the universe gives us bizarre situations, and the children dealt with theirs as best they could.

But they couldn't stay that way for very long. Children tend to quickly get bored. Many of them felt like they'd been standing like that forever. One of them, a tall, lanky boy whose pants were two inches too short, lifted his arms to stretch and accidentally pushed a girl standing next to him.

It had only been a little push, but even the slightest shove in that crowded space was enough to throw someone off balance. The girl gave a small gasp, taking a step back, but her foot slipped. The girl had been very strong too; she'd taken it upon herself to carry several of the smaller children on her shoulders, and she'd been carrying quite a few, sitting one atop the other, a tower of youth piled on high. The push caused them all to tip over, the highest ones falling right off the sides of the fixed point,

accelerating away from their known stability. Those who had been lower on the tower fell right atop their fellows, and those fellows, in their instinct to avoid being hit, stepped aside, only to lose their balance as well. A chain reaction started, and panic caused most of them to slip. To catch themselves, some of them would reach out to hold on to others, but only succeeded in dragging them off the edge of the fixed point as well. Even the boy who had originally stretched his arms out didn't get off easy: he ended up stepping on his untied shoe laces, tripping, and falling backwards off the point too. His feet left the ground of the fixed point, and everything around him went dark, and as he fell he thought, "Really now: who's idea was it to put a bunch of restless kids on an unstable point?"

Like marbles falling off a round surface, they all slid away, some bewildered, some resigned, and some arguing over who pushed who. As the commotion ended, and all grew quiet, there was only a single little girl crouched atop the motionless fixed point.

The little girl's name was Ruby, and she slowly lifted her head up and looked around in surprise at the emptiness all around, her mouth shaped in a little 'o'. She had been standing right in the middle of the fixed point, and she had ducked and tucked her feet under her skirt to make herself an even smaller little girl and had avoided being moved from her place during the commotion. She felt quite clever about it too, because she had not panicked like everyone else had.

"They let their fear get the best of them," she said out loud, to no-one in particular. "If they'd all stood still, most of them would have been fine. Now, I have the whole point to myself."

But even after saying that, Ruby stood right where she was, hesitant to move even a teensy inch away from her spot. The

fixed point was dangerously slippery, she knew, and one wrongly placed step could make her slip, just like the rest of the children.

“I’d better stay right here,” little Ruby said, again, to no-one in particular. In fact, her voice sort of echoed back to her, frightening her a bit. *Here, here, here, here, here, here, he...*

Ruby blinked at the noise. “That’s interesting. My name is Ruby. Hello?”

*Hello? Hello? Hello? Hello? Hello?...*

“Can anyone hear me?”

*Me? Me? Me? Me...*

“Where did everyone go?”

*Go? Go? Go? Go...*

“Will I be all alone here forever?”

*Forever, forever, forever...*

*Oh my.* Ruby closed her mouth, not wanting to hear her own voice anymore. *I don’t think I’m going to like it here very much.*

Now that she wasn’t surrounded by a crowd of children, Ruby had a better look of what was around her. The unstable fixed point was a very unusual place. It was dark and foggy, like she were inside a cave, and beyond where she stood, above her, in front of her, all over, all there as to see were gray shadows. She could see herself, her hands, her feet, her little blue skirt, but she couldn’t see anything past herself or the space she stood upon, like a person who could only see what the light of their little candle showed. Little Ruby wasn’t scared in that darkness; after all, there’s no reason to be scared of the dark. However, after

hearing her own voice speaking to her in that darkness slightly disturbed. She couldn't help but picture that someone was out there, and then she thought about the other children.

When her companions had fallen, she hadn't seen where they'd fallen to. She hadn't heard them hit anything as they fell either. At least, she didn't think so.

Curious, she took off one of her shoes—the left one, because it was scuffed and the heel was wearing down—and threw it up into the air away from the fixed point. The shoe arched high above her, going up and up, and right as it started to come back down, it was swallowed by the dark gray fog, disappearing like an actor stepping behind a curtain. Ruby listened very hard, holding out her ears for anything her shoe could hit, but she heard nothing. No thud. No splash. Not even a soft squeak or cry of someone saying, “Ouch! Who threw that shoe?”

“It must be a very deep and far fall,” Ruby mumbled, standing even more squarely in the middle of the fixed point in her blue skirt and single shoe. She wondered at what might be beyond the unstable fixed point. Rocks? Hungry wild hyenas? Or was it just a vast chasm waiting to be filled with children?

“It's very good that I stayed here,” Ruby shuddered. “Very, very good.”

However, we all know that it's no fun standing in one place all by yourself with nothing but an echo to talk to. The little girl named Ruby was getting lonely. At first, she tried to distract herself by playing games. She'd yell “Marco!” but her echo never learned to say “Polo!” She tried to play hopscotch, but she had no

chalk and was still scared of moving around too much, so she just ended up hopping up and down in one place until she grew tired. She tried starting a game of I-Spy, but there wasn't much to spy. "I spy with my little eyes," she would say, looking around at the darkness, "something dark and with shadows."

*Shadows*, her echo replied.

Ruby huffed. "You guessed that last time."

She was starting to wonder whether being the only child to not fall was a good thing after all. Where had everyone gone? Should she go after them? The unstable fixed point may be boring, but once she left it, she knew she wouldn't be able to return.

*If I stay here, I'll be all by myself*, she thought. *But if I leave, who knows where I'll end up? What if it's ugly? What if it's too cold? I don't have a jacket to keep me warm, and it's nice and comfortable right here.* Besides, she figured, wherever she'd end up would probably be the same place where everyone else had gone off to, and last time she'd been with them all it had been so crowded. There had hardly been any breathing room, and there had always been someone complaining about an elbow in their eye or about someone having bad breath. Really, staying on the unstable fixed point wasn't so bad.

What Ruby started to think about, though, was that staying there meant staying there *forever*. And not just any kind of forever. This type of forever was long. Longer than "Man, mom is taking forever to finish saying good bye to Aunt so-and-so", and much, *much* longer than "It took me forever to finish my homework." No, this type of forever meant FOR-EVER. Days and weeks and months and years and decades and centuries forever. That's what staying on a fixed point means. Our little girl Ruby could grow into a big girl Ruby and her dress would

become too small and her hair would turn gray and her bones would brittle to ash, but those Ruby ashes would remain in that spot, right there, for all time.

Ruby shuddered, rubbing at her arms to make sure her bones were still hard and strong.

Suddenly feeling wary, she made a slow, careful circle around her spot. She saw only the same gray fog staring back at her. This was where she'd have to stay. Forever.

"But I don't want to die here," she said, refusing. "This won't do, this won't do one bit!" She stomped her shod foot into the ground. "I don't want to stay here, but I also don't want to go to a place I know nothing about. Oh, what should I do?"

She stood for a short time, thinking, wondering. She wracked and scratched and prodded her brain for an answer. She stared into the darkness. She asked her echo three times and her echo asked her three times, too. She made a list of why's and why not's.

Why should I stay here?

1) *It's safe, I know nothing bad will happen here.*

2) *Who else will keep my echo company?*

Why should I leave here?

1) *Nothing will EVER happen here.*

2) *I'm bored.*

3) *I'm scared of becoming an old lady here.*

4) *I quite miss my other shoe.*

5) *And my companions too, of course.*

"That's it!" she declared out loud. "I'm leaving. You can come with me too. Are you in?"

*In, in, in, in...*

"Excellent. Now then."

Getting serious, Ruby steeled herself. She bent over and re-buckled her one shoe and smoothed out her skirt.

Feeling ready, the little girl picked a point in the distance and faced it. She let out a breath and took one small, measly step away from the center of the fixed point.

“That’s one small step for, well, me,” she said—a famous person once said something similar, although who it was, she couldn’t recall. “And one giant leap for—” but before her echo could learn who the leap was for, Ruby, jumping into the air, landed on a patch of the point that became slippery beneath her, as though she’d stepped onto a wet slide at a playground after a rainfall. She lost her footing and fell down, down, down. The fixed point rose and disappeared above her head. The darkness whirled around her. Her hair flew up behind her, and she screamed, first in surprise, but then with laughter.

She had done it! She smiled, feeling proud of herself. There were tears in her eyes, not from emotion, but because wind was blowing into her face, making her eyes water. She’d thought the fall would be scary, but it wasn’t so bad, really. The air around her was warm, and though she still couldn’t see much around her, she secure and snug, like she were falling through a safe tunnel.

However, after a minute or two of falling, she began to worry whether she’d ever stop. Was this her new forever, or would she eventually land somewhere, and if so, where? Hopefully on some nice, comfy pillows, and not a bed of rocks or on top of Laplace’s demon—who, according to one of the children she used to stand with, was all knowing and very smart and quite frankly not someone she’d want to run into.

But the writer of this story would never do that to little Ruby. Soon afterwards, she noticed that the rate at which she was falling was starting to slow down. The tunnel around her grew lighter, the

dark grays being chased away by softly lit greens. Some force began to push her sideways too, so that she was falling in a sort of diagonal manner. She moved her feet like she were walking, and it felt like she were travelling on a smooth walkway. As she walked, the force pushing her slowed to a stop. She was walking completely on her own now, no longer falling, and heading towards a green light at the end of the tunnel. In its direction she could hear a myriad of voices that sounded very familiar to her.

Suddenly, in the middle of a step, Ruby emerged out from the tunnel and into a valley of light and color. She had to cover her eyes at first and duck her head, the change from monochromatic darkness to an onslaught of vibrant life too much to process. Squinting between her fingers, she saw that she was standing on green grass, soft beneath her shoeless foot. Slowly, still holding her hands to her face, Ruby looked up, blinking her eyes.

She was at the bottom of a bowled valley, encircled by what looked like mountains, green at their bases but fading into gray as they rose into the gray fog that domed the valley. That fog seemed farther away than usual now, no longer resembling an all-encompassing shadow. And the noises. There were voices, children laughing around her, the sound of running and playing and excitement filling her ears. There was a breeze on her skin and it felt quite inviting, like it was giving her a welcoming hug. Ruby thought that she'd never felt so at peace. It was all so new and fresh and exciting that she didn't know what to do with herself.

So overwhelmed was she by her sensations, she wasn't able to process much after those first few impressions. Thus, when someone shouted, "Watch out!" Ruby didn't register the warning and didn't notice the spinning red frisbee that was flying straight



towards her. It smacked into the center of her forehead, making a very interesting *thwap!*

“Ouch!” Ruby exclaimed, more stunned than hurt. She looked down at the frisbee, blinking the pain out of her eyes. Someone was walking towards her, and she raised her eyes to see a boy around her age standing in front of her. Something about him seemed familiar.

“Are you okay?” he asked. “I tried to warn you.”

Ruby rubbed her throbbing head. “Where did you get that?”

“The frisbee? It was already here when we arrived.”

“We?”

Her eyesight having adjusted to the brightness, Ruby dropped her hands and saw the last feature of the land. In that valley, dozens of children were amok, and she recognized many of them. Children who had stood motionless with her on the unstable fixed point were now either running free, calmly walking, or just rolling around goofily on the grass, tearing it up with their hands and throwing handfuls at one another like confetti. Some of the kids were even laying down on their stomachs, taking a nap, while others played a game of jumping over them. As she watched, one of the napping kids suddenly woke up and attempted to sit up, not seeing a jumping kid who resultingly fell on top of him. The group around them started laughing. Ruby looked from child to child. She didn't recognize some of them, perhaps they had been here before the others had arrived, just like the red frisbee, but the entire group interacted with each other as equally as only children knew how to. A group of them were trying to run up the sides of the valley, climbing the mountains, holding hands to tug each other along. They would get a running start, climbing up as fast as they could, but as soon

as they reached a certain point, they'd slow and gently slide back down, laughing as they tumbled to the bottom.

"You're from the unstable fixed point!" she exclaimed to the boy who'd hit her with a frisbee.

The boy gave her a curious look and nodded. "I'm Niko. Welcome to the *stable* fixed point."

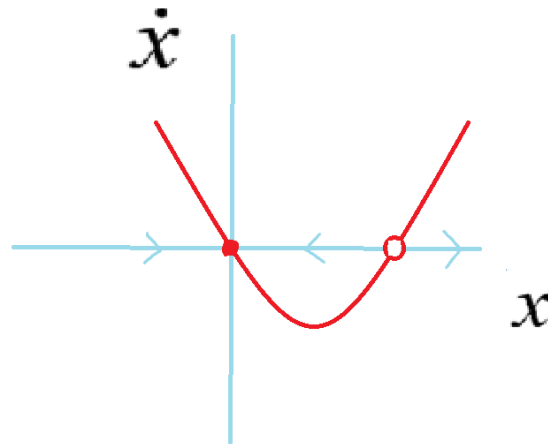
"Stable," she repeated, feeling the word being said. "Is everyone who fell of the unstable one here?"

The boy's face changed and he shook his head. "Not everyone was as lucky as us. Some kids fell off the wrong side of the fixed point."

"The wrong side? There's a wrong side?"

"Well, unless you like flying forever, then yes. We are in the  $r = -5$  stage of the system  $\dot{x} = rx + x^2$ , after all, and that means there's only two ways to fall off the unstable fixed point: either you fall towards the stable fixed point, or you fly off into infinity."

Ruby and Niko proceeded to have a long conversation concerning the nature of differential equations and the shapes of graphs and the direction and magnitude of an object away from a fixed point. At this segment, you shouldn't feel intimidated if you're not sure about what these two are saying. Remember, they exist in a mathematical universe, so they know math as naturally as you and I know that chairs are for sitting and that we should drink water when we're thirsty. So don't worry if you don't understand their conversation. Instead, picture this:



This is the mathematical universe, or, system that the children currently exist in. See that empty dot on the right? That's where the children were in the beginning of the story, the infamous unstable fixed point. When something moves away from that empty dot, it either goes left, towards the filled red point, which is the stable fixed point (where Ruby currently is), or they could go towards the right. Notice, there isn't *any* fixed point, stable or unstable, to the right of the unstable fixed point. The little blue arrow on the y axis means that once something moves to the right of the unstable fixed point, it will move in that direction forever. Not only that, but see how the red graph seems to go up and up? That represents the velocity of the object that's moving. Overall, falling off the right side (the wrong side, according to Ruby and Niko) means that not only will you be unable to stop, but your speed will always be increasing, moving faster and faster towards far, far infinity.

“What happens when someone reaches infinity?” Ruby asked Niko. In response, he gravely shook his head.

“It's a place far beyond us,” he said. “We don't know what's there, and anyone heading there has never come back to tell us.”

“Oh. Do you think my other shoe is headed there?”

“Only if it’s not here. Let me check.” He took a deep breath, cupped his mouth with his hands, then shouted, “Has anyone seen a shoe?”

“It looks like this one!” Ruby lifted her foot for display.

The entire valley, kids that were spinning, running, walking, talking, paused in what they were doing to look at Ruby’s shoe then at the ground around them. Everyone reported that they had not seen her shoe.

“Thanks anyway,” she told them all with a little frown, and they all resumed their play. She looked back to the Niko. “That’s alright. I can live without a shoe. Maybe it will keep someone company on their way to infinity.”

But the boy could see that Ruby was still a little upset, so he said, trying to cheer her up, “Don’t worry too much about it. We’re lucky. At least our system isn’t one that will undergo subcritical bifurcation, then we really would have been flying off into infinity, or worse—” his voice dropped “—negative infinity.”

Ruby blinked, and she too whispered, “What’s negative infinity?”

Even though she’d whispered it, every child in the valley, sensing something wicked being mentioned, stopped dead in their tracks again, this time their smiles peeling off their faces. They all turned towards her, and in unison said, “You don’t want to know.” Then they went back to playing, as though nothing had happened at all.

Ruby shivered a little, slightly weirded out. “Does everyone always talk in unison like that?”

Niko shrugged. “Only for dramatic purposes.” He stooped to pick up his fallen frisbee, which Ruby had forgotten about. “But remember: You’re lucky. This is the stable fixed point!”

Where we can do whatever we want and never have to worry about falling off—because that’s impossible here.”

“Does that mean we have to stay here forever?” Ruby grimaced. She really hated that word.

“Only until bifurcation day.”

“Until whose birthday?”

A few meters away, a group of kids were calling Niko over to start a game of ultimate frisbee. Niko told them he’d join them in a minute and looked back to Ruby. “No, I mean *bifurcation* day, when our  $r$ -level changes and the stability of the fixed points of our universe change.” He formed a cup with his hands, holding the palms up. “Right now, we’re safe in a little pocket where we can’t move, but on bifurcation day, we’ll undergo transcritical bifurcation and that pocket will be turned inside out—” he mimicked the action with his hand “—making this place an unstable fixed point. But that won’t be for a long time.”

Ruby squinted her eyes skeptically. “How would you know?”

“I did the math. Now come on, wanna play frisbee?”

Ruby still felt uncertain, but she didn’t want to spoil his fun. She joined the group in their game, at one point even kicking off her other shoe and ran barefoot, her toes kicking up soft grass. The air around her felt fresh and the gray fog around them was so light it was almost blue, like a sky. Ruby wondered how she knew what a sky was, but then someone threw the frisbee in her direction and she let go of her question in pursuit of the flying disk. She felt happy and let herself feel happy. She wasn’t alone anymore. Niko was right: she was lucky. Had she leapt in the wrong direction, she wouldn’t have been there. Let bifurcation day come, she thought, she was content with this part of forever.

But, theoretically, what would have happened if Ruby *had* stepped off the other side of the fixed point?

What if instead of landing at the stable fixed point, she had zoomed out forever and forever, without stop, towards infinity?

Unfortunately, we will have to imagine such a scenario, because that is the fate that befell several of the children who were originally on the unstable fixed point (as well as Ruby's other shoe). On the bright side, we will not make Ruby go through this. Instead, we'll take a look at another child, on a sleepy Melvin, who had been taking a nap on the unstable fixed point when the first kid, our tall, lanky catalyst, had stretched and started the reaction that caused everyone to fall like a blown house of cards.

Poor Melvin had gotten so tired while standing on the unstable fixed point, he'd fallen asleep right on his feet. He'd been having a nice little dream too, a dream of a place where there were no fixed points, where children could stand on flat ground, could learn at a place called school and went home in the afternoon to play with toys and watch TV after doing their homework. In his dream, he had gotten in trouble for falling asleep in math class and his teacher had been loudly yelling at him when suddenly he woke up on the fixed point and realized that the yells were real and not coming from his fake math teacher at all. They were coming from the kids around him, and suddenly he was pushed and falling—no, he was flying!

Some force began to tug him up and away, his toes leaving the ground first slowly, but then the force worked faster and faster, increasing his speed, until he was zooming away and leaving the fixed point below. There were some kids flying up along with him too, but there were also others that were falling. He felt sorry for the falling ones: surely, flying up must be better than falling down, right?

The fixed point below him became so small and far that it disappeared. All that was around was him, the space, and those who were flying just like him. Melvin wrapped his arms around himself. Was it his imagination, or was his speed still increasing?

It wasn't Melvin's imagination: instead of eventually slowing down, like Ruby, he would only go faster and faster, moving so quickly that if he were to zoom by your window, you wouldn't even see him. Melvin tried looking up to see where he was headed, but everything around him was becoming a blur. Even the kids that were flying along with him were becoming hard to see. Melvin tried speaking to them, but he was already travelling faster than the speed of sound. Below him, sonic booms rung loud and clear. Even if anyone heard it, he couldn't tell if they'd replied. Unable to see anyone or communicate with them, Melvin felt that he was completely alone.

Incredibly, his speed was still increasing. Melvin was perplexed. Some part of him thought perhaps he was still dreaming, and that if he pinched himself hard enough he'd wake up still on the unstable fixed point. He pinched himself as hard as he could, but only succeeded in giving himself a bruise.

Now some of you might be remembering what Ruby and Niko had said: that anybody that fell off the wrong side of the unstable fixed point would be moving forever and ever. Well, they were right. Melvin did move for a rather long time, so long that if this writer were to write about it, our story would be much longer than it already is, and no one wants to read a huge book about that. Melvin had a lot of time to think while he was moving. He thought about how much he missed being able to stand still, and about the sound of other people's voices. He also slept a lot and tried to dream of things to entertain himself with. For a little bit, he even managed to return to the dream about the place

called school. He created classmates for himself, imagined homework that he'd forget to do, and pictured what it would be like to run through a hallway. But then he would wake up and be moving faster than before, and he would be so bored he'd fall right back asleep.

For Melvin, forever was not a very fun time.

At a certain point in time, however, Melvin felt something different.

He'd woken up, frightened by a nightmare in which he'd won a race, but instead of stopping after the finish line he'd kept running and running until his legs had fallen off. He was awake and still flying, but he noticed something peculiar about the speed that he was moving at. He used to know exactly what his speed was at all times, though eventually he stopped keeping track (there were so many digits in the numbers he lost track of them). But something was different about his speed this time. He felt lighter, looser, and the air around him was unusual, too. He felt troubled. He'd already lost the ability to see and hear, was he losing his sense of touch too? He started to poke at the bruise he'd left on his arm, to see if he could still feel pain, when he saw the first wave particle of light.

Melvin's eyes snapped towards it. A strange sensation tickled his stomach. What was this? Up until then, he'd only been able to see dark gray shadows around him, similar to what Ruby had described, but now, there was a little light wave particle flying alongside him, a sight so incredible and complex his small mind couldn't quite comprehend it. He stretched his arm out to it, when another one appeared beside it, and then another one, and another below him, until suddenly, all around, flying through



whatever pathway he was barreling through, were various little wave particles. They surrounded him like a school of fish, and he tried not to blink, to take everything in. He couldn't even describe their color and didn't even know how he knew they were wave particles. Like much about the mathematical universe, this was just another thing that he knew. He knew he wasn't dreaming, so he wondered if he were instead going crazy. If so, he decided being crazy wasn't so bad.

As suddenly as they appeared, however, Melvin soon began to leave the wave particles behind. His speed was still increasing, and it wasn't long before he was moving faster than them. Melvin despaired. He didn't want to be left alone again. He reached his arm out to grab hold of one of the light wave particles, but he missed and cried out.

"This isn't fair!" he screamed, unable to hear his words. "I want to go back! Take me back!"

This is where Melvin would get the biggest surprise of his life.

The instant he surpassed the light wave particles, Melvin was moving faster than the speed of light. Suddenly, the tunnel he was travelling through started shrinking, until he could feel its walls pressing in on him. He thought he was going to be suffocated into nonexistence. He tucked his knees up to his chest, squeezing his arms close to his sides, trying to hide himself from the shrinking walls, but they folded right over him. Melvin, still travelling faster than light, felt like he were being forced through a rubbed tube. His heart beat almost as fast as his speed, and he was extremely frightened. He closed his eyes, shutting them tight, feeling the compression around him increase and increase until...

*Pop!*

Wait! Reader, don't be alarmed! Melvin was not harmed!

The reality is that because Melvin was travelling so fast, he *travelled through time*. That *pop!* was the sound of his breaking the space time continuum, and because Melvin's wish was to go back, that's what he did. His eyes still squeezed shut, Melvin realized that he no longer felt like he were being squeezed. He also realized that he was no longer moving.

Gasping, he opened his eyes, only to see that he was standing back on the unstable fixed point, surrounded by the same children that had stood upon it with him. Beside him was the strong girl holding the tower of children, and he got so excited, so emotional, so thrown off by his change of situation, that he threw himself towards her and hugged her because he was no longer alone.

The girl, caught completely off guard by Melvin's hug, accidentally dropped the children sitting atop her, and our story takes another loop. The children again were thrown into disarray, falling off the unstable fixed point in various directions. Melvin, fearing that he'd be flying forever again, made an enormous effort, and twisted himself away from those flying up to join those falling down. He'd already gone off the wrong end of the unstable fixed point once. He didn't want to do it again. This time, he'd fall all the way to the stable fixed point, feel soft grass, and be able to stand for a long time without moving at all. And for a very long time, he was happy.

Back on the unstable fixed point, a little girl raised her head to see that all her friends were gone.

But we already know what happens to her.

**THE END**

No children were harmed in the making of this short story ☺