

# Minimal Differentiation in the College Rankings Market

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## Abstract

College rankings have become an important source of information in the college admissions market. As the number of first-generation college students, the number of applications per student, and the geographic range of applications increased, rankings emerged as a means of navigating the complex college choice process. While previous literature studies the influence of college rankings on college and student decisions, it treats college rankings themselves as static and given. My thesis models product choice by college rankings firms. I extend the Hotelling model of horizontal product differentiation to allow for differences in consumers' willingness to pay and interactions in a two-sided market with advertisers and consumers. The model illustrates how a rise in the importance of advertising may help explain the observed decrease in the prices of and differentiation among rankings offered in the market. It also shows that increased advertising generates two opposing changes in total welfare. On one hand, increased advertising increases welfare by expanding access to rankings through lower prices. On the other hand, it may also decrease welfare by causing some consumers to lose access to their preferred product. Through these two opposing changes, advertising both aids and hinders the search for the "right" college.

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# 1 Introduction

*Every year, over two million high school graduates go to college. To make sure they end up at the right school, they spend several hundred million dollars on the admissions process...This book is your best bet.*

- 1992 Princeton Review Student Access Guide to the Best Colleges

From 1976 to 2010, the number of students enrolled in undergraduate institutions in the U.S. increased from 9.42 million to 18.09 million (“Total Fall Enrollment,” 2011). Many of these new applicants were minority and/or low-income students less informed about the admissions process and college market (Hossler et al., 2004). These applications also spanned a wider geographic range of colleges than in previous years (Hoxby, 1996). In 1989, only 16% of students applied to six or more colleges. In 2009, 33% applied to six or more institutions (Hoover, 2010). This widening in scope made determining to which colleges to apply more difficult. In response to this increase in demand for information, college rankings firms emerged as experts capable of providing comparisons of hundreds of colleges nationwide to students. Rankings supply information to students faced with the uncertain decision of which colleges to apply and ultimately attend.

The rise in demand for college ranking products has spurred an economics literature studying rankings. This literature focuses primarily on how rankings have altered the behaviors of students and colleges. In particular, it debates whether rankings facilitate efficiency or encourage wasteful behavior. However, this research treats college rankings themselves as static and given. While considering the strategies of students and colleges, they do not consider the strategies of the rankings firms themselves.<sup>1</sup>

My thesis endogenizes the rankings product. I argue that, with increased advertising demand, incumbent rankings firms offer similar products to maximize profit.

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<sup>1</sup>See, e.g. (Bastedow & Bowman, 2008, 2009, 2010a, 2010b; Ehrenberg & Monks, 1999; Griffith & Rask, 2005; Jin & Whalley, 2007; Meredith, 2004) and the papers cited there.

Advertising also encourages new entrants to enter with similar products. Consequently, with advertising, engaging in price-competition may be consistent with profit-maximization in an oligopoly, as offering consumers free rankings may allow for higher profits from advertisers. However, the market with advertising may still be inefficient from the perspective of a utilitarian social planner.

The structure of the rankings market makes these outcomes possible. Since rankings are information goods, product differentiation emerges as rankings firms' most viable escape from price competition, i.e. Bertrand's paradox. Escaping Bertrand's paradox is especially important to rankings firms because information goods typically have high fixed costs and low marginal costs. Charging price equal to marginal cost may be insufficient to recoup fixed costs (Arrow, 1962, p.614). Arrow (1962) points out that rankings also have a low marginal cost of redistribution. This makes consumers of information goods potential competitors. Consequently, product differentiation remains as one of the viable traditional escapes from price competition. The ability of consumers to reproduce and redistribute rankings renders capacity constraints difficult to maintain, as consumers can easily produce more rankings. Since the number of firms in the market potentially equals the number of consumers, collusion is also unsustainable. Thus, the remaining escape is for firms to produce differentiated goods.

In addition to product differentiation, two-sided markets may allow firms to earn positive profits. In a two-sided market, rankings firms sell rankings to consumers and advertising space in the rankings to advertisers. While it may be easy for consumers to enter the rankings side of the market, they encounter entry barriers to selling to advertisers. This occurs because advertisers value the number of viewers a rankings firm reaches.<sup>2</sup> Incumbents already have a large, established viewer base. Consumers, as new entrants, have few viewers. As such, their advertising space is not attractive

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<sup>2</sup>As I discuss later, these viewers could be a specific type of viewer.

to advertisers.

The tradeoff between product differentiation and advertising emerges when advertisers are interested in targeting a particular type of consumer. This makes it difficult for firms to benefit from product differentiation and two-sided markets at the same time. Product differentiation is effective when different consumers have different tastes for a given product. However, if advertisers are interested in only one type of consumer, then it is only beneficial for them to buy advertising space from the firm(s) whose product(s) reach that type of consumer. Thus, rankings firms must choose between commonly appealing to advertisers' target consumers or providing differentiated rankings appealing to different consumer types.

This approach to college rankings fits with a broader literature on what Gal-Or and Dukes (2003) call the *principle of minimal differentiation*. The *principle of minimal differentiation* states that firms may produce undifferentiated products in equilibrium. This literature explores how minimal differentiation occurs in the presence of advertising. However, it focuses on the impacts of advertising on the provision of public goods through radio and television.<sup>3</sup> Assuming media as free to consumers, the literature centers on how advertising prices are set.<sup>4</sup> My thesis extends this literature's insights on minimal differentiation to the provision of private goods in the college rankings market.

Demand for advertising in college rankings was likely small when U.S. News and World Report (USNWR) first released its rankings in 1983. USNWR sold advertising only to State Farm Insurance until 2001. In 2002, it sold advertising only to AIG.

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<sup>3</sup>See, e.g. (Steiner, 1952; Owen, 1977; Gabszewicz, 2001; Beebe, 1977; Gal-Or & Dukes, 2003; Coate, 2005), and the papers cited there.

<sup>4</sup>Coate (2005) briefly explores the consequences of allowing media firms to charge households positive prices. However, Coate models how the addition of positive consumer prices changes a market that originally profited only from advertising. I would like to look at the reverse, how the addition of advertising changes a market that had positive consumer prices. Coate finds that, given the option, media firms will charge households positive prices. My model demonstrates that the introduction of advertising may lead to zero household prices.

Within the rankings book, these advertisements filled roughly eight full, color pages. This suggests that USNWR had plenty of advertising capacity but few advertisers demanding space. Moreover, when the Princeton Review (PR) first released its rankings in 1992 it did not sell any advertising space.

When advertising demand was low, the firms in the market, USNWR, PR, and Money Magazine, produced rankings that served different needs and appealed to different types of consumers. USNWR appealed to parents with more general information about college life. PR targeted students by ranking based on student perspectives of colleges. Neither explicitly ranked using the “value” and “outputs” of a college education. Money Magazine uniquely catered to those interested in value, ranking “best buy” colleges.

After 2000, there is evidence that demand for advertising increased greatly. USNWR started selling advertising space to multiple firms. It also divided the many pages once allotted to a single advertiser among ten or so different advertisers. PR added a section at the end of its book dedicated to advertising. This suggests a rise in advertising demand, as USNWR could now sell smaller, less preferable advertising spaces to more advertisers, and PR could begin selling advertising space. Furthermore, this new demand was to reach financially-conscious consumers. The majority of advertisers were financial service firms looking to promote credit and loan services. Advertisements not explicitly for financial services also appear to target financially-conscious viewers; they include ads for community colleges, job search engines, and discount shopping.

This increase in advertising demand changed the rankings market in two ways. First, it decreased the degree of differentiation among rankings. Whereas they previously catered to those interested in college life, both USNWR and PR began ranking with a greater emphasis on value in terms of the cost of attending an institution and an institution’s impact on “output” or future earnings. Second, it increased the



number of entrants into the rankings market who provided value-oriented rankings. Many other firms such as Forbes, Wall Street Journal (WSJ), and Kiplinger have also begun releasing rankings that emphasize the “value” or outputs with which colleges provide their students (Kaminer, 2013).

Increased advertising demand decreased differentiation and increased entry because advertising acts as a subsidy to rankings firms. When advertisers prefer to reach one type of consumer, advertising revenue effectively subsidizes only one type of ranking. In turn, this encourages firms to create rankings serving that consumer type. Moreover, increased advertising demand increases profits from selling rankings, encouraging entry. While some of the availability of value-oriented rankings is due to consumer demand, as there would be no “eyeballs” for advertisers to reach without it, this advertising “subsidy” may lead more firms to produce value-oriented rankings *ceteris paribus*.

To demonstrate how advertising decreases differentiation in the college rankings market, my thesis models firm product choice in an oligopoly using a variant of the Hotelling model. There are two firms and two types of consumers defined by different preferences. Firms choose which rankings to sell and what price to charge for rankings. Demand functions for each firm are derived assuming consumer utility maximization. Household and advertising markets are separate. I treat the price of advertising space as exogenous. This is because rankings firms are price-setters in the rankings market but price-takers in the advertising market. A limited number of firms producing college rankings, but many firms sell advertising. Thus, rankings firms make some additional revenue from advertising for each ranking sold to the advertisers’ preferred consumer type. The amount of this additional revenue increases with advertising demand.

The model shows that two symmetric firms who act simultaneously will segment the market by choosing differentiated rankings and charging high consumer prices

when advertising demand is low. With the introduction of sufficient advertising, firms produce the same rankings and charge consumers zero prices. Furthermore, it demonstrates that advertising increases welfare by increasing consumer access to rankings but still leads to an inefficient provision of rankings.

Subsequently, I extend the model to illustrate how advertising affects entry into the college rankings market. I consider three firms that decide whether to enter the market and, if so, what rankings to offer sequentially. This extension demonstrates how increased advertising demand may induce more firms to enter and produce the same product. In terms of welfare, entry induced by advertising improves welfare as it expands consumer access to goods. However, at “high” levels of advertising, the provision of goods is also inefficient.

## 2 Rankings Industry Background

Since 1983 when USNWR started offering college rankings, the cost of attending college has been a concern for many families. However, during this time, there have been major changes in both the rankings available and the advertisements appearing in the college rankings product. Rankings content has become more similar, and more firms have begun offering rankings. Advertising demand has gone from playing a minor role to a prominent one. My thesis argues these changes are related.

### 2.1 Rankings 1983 to 2000

Prior to 2000, the rising cost of college was already a concern for the families of college-bound students. Across private (nonprofit) four-year, public four-year, and public two-year institutions, the average annual percentage increases in inflation-adjusted published tuition were 4.1%, 4.3%, and 5%, respectively, for the three types of institutions in the 1980s (“Average Rates,” 2014).

Before 2000, there were few firms in the rankings market, and these firms provided clearly differentiated rankings. (See Figure 1). These firms included USNWR, PR, and Money Magazine. USNWR emphasized objective data and college administration feedback that appealed to parents. It printed articles directed at parents such as “A Counselor’s Tips to Parents” in the “News You Can Use About College” section of USNWR’s 1989 edition, which “offer[ed] anxious parents advice on how to relieve rather than add to the stress involved in helping their children get into college” (39). PR emphasized student feedback that would appeal more to prospective students. In particular, PR titled their product “*Student Access Guide* to Best Colleges,” and addressed students directly with questions such as, “Do you qualify? Find out how competitive each school *really* is” (1992).

Contrary to USNWR and PR, Money Magazine uniquely focused on measuring the “affordability” and “bottom line” of a college education in its rankings. Absent from USNWR’s rankings criteria is an emphasis on “affordability” and “value.” USNWR initially based its rankings off of a survey sent to college presidents asking them to pick the five best undergraduate institutions similar to their own based on “quality of academic courses, professors, student bodies and general atmosphere of learning provided.” In 1988, this was changed to include, in order of importance, “the school’s selectivity; the strength of a school’s faculty and its instructional budget per student; the resources available for its educational programs; a college’s ability to see its entering students through graduation.” Similarly, of the approximately 63 separate lists of colleges PR provides detailing different aspects of student life, not one emphasized value or output.

Initially, there was little advertising in the college rankings market. USNWR sold advertising only to State Farm Insurance, which “sponsored” the college rankings provided by USNWR in exchange for almost ten full pages of advertising space. USNWR’s parent-oriented content also aligned with the wishes of its State Farm

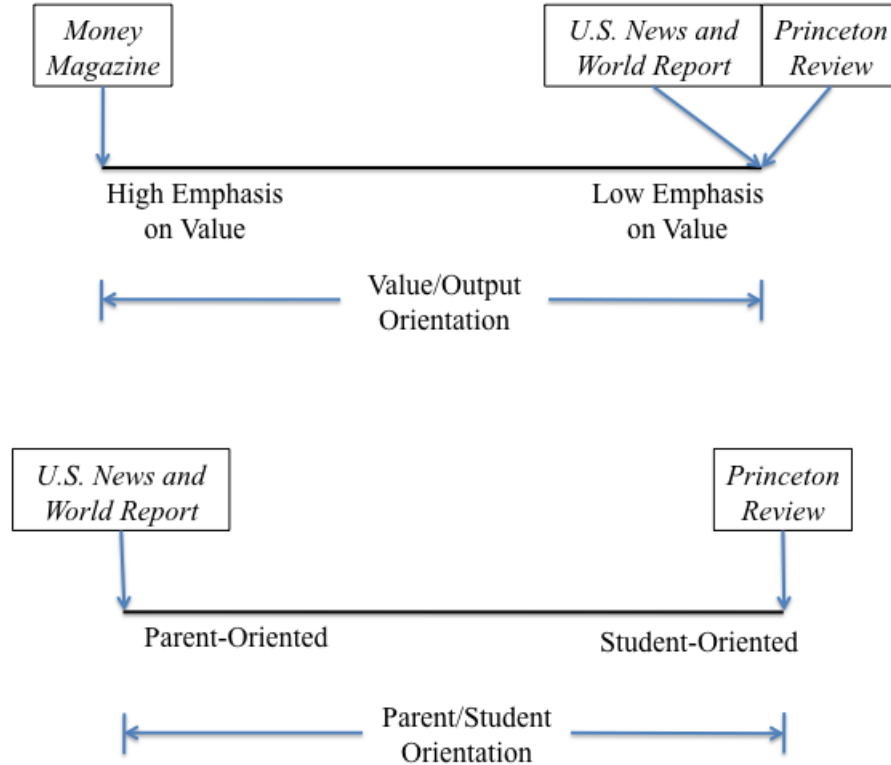


Figure 1: College Rankings Market 1983 to 2000

sponsor. Selling life insurance, State Farm used its advertising pages to appeal to parents to buy life insurance such that their children would be guaranteed “a quality education.” PR did not sell advertising space. Moreover, reflecting the small role of advertising, rankings were provided at a cost to households. Through the 1990s, USNWR charged about \$ 6.00. PR charged between \$ 16.00 and \$ 20.00 to households (Princeton Review, 1992, 1994, 2000, 2014).

## 2.2 2000 to Present

The rising cost of college continued to be a concern after the turn of millennium. Across private (nonprofit) four-year, public four-year, and public two-year institutions, the average annual percentage increases in inflation-adjusted published prices were 2.3%, 4.2%, and 3%, respectively in the 2000s (“Average Rates,” 2014). These

percentages are similar to those before 2000.

Furthermore, after 2000, the rankings offered became more similar, and many more firms entered the rankings market. (See Figure 2). New entrants included Forbes, Washington Monthly, the Wall Street Journal (WSJ), Kiplinger, and Payscale. Incumbents and entrants emphasized value and outputs as a criteria in their rankings (Kaminer, 2013). USNWR's rankings now emphasize "output measures" such as graduation rates and graduation rate performance. Whereas graduation statistics were at the bottom of USNWR's list of rankings criteria prior to 2000, they are now at the top, accounting for 30% of USNWR's rankings data ("Frequently Asked Questions," 2013). USNWR also now publishes its own "best value" list. Similarly, PR's collection of lists now includes a "Best Value" list longer and more prominently located than those about student life. Moreover, lists detailing elements of student social life such as "Students hang out in big groups" and "Aesthete schools" have been dropped in favor of lists such as "Best Career Services" and lists about "Best" facilities (1994, 2014).

Entrants similarly focus on the value and output of a college education. Echoing USNWR, Forbes promotes its focus on " 'output' over 'input,' " and emphasizes that rankings matter because college is a large investment (Howard, 2013). The Washington Monthly uses a "dollars and cents tabulation" to calculate its rankings. WSJ provides a list of colleges as preferred by job recruiters, Kiplinger a list of "best value" colleges, and Payscale a list of returns to college as an "investment". The Alumni Factor surveys college alumni and stresses the career preparation and financial returns of a college.

Concurrently, the amount of advertising in college rankings has also increased greatly. In 2003, USNWR ceased having exclusive advertising agreements with specific advertisers (State Farm and then AIG). It now sells advertising to a number of firms such as AXA Equitable, eBay, Wachovia, Dean College, American Express, Col-

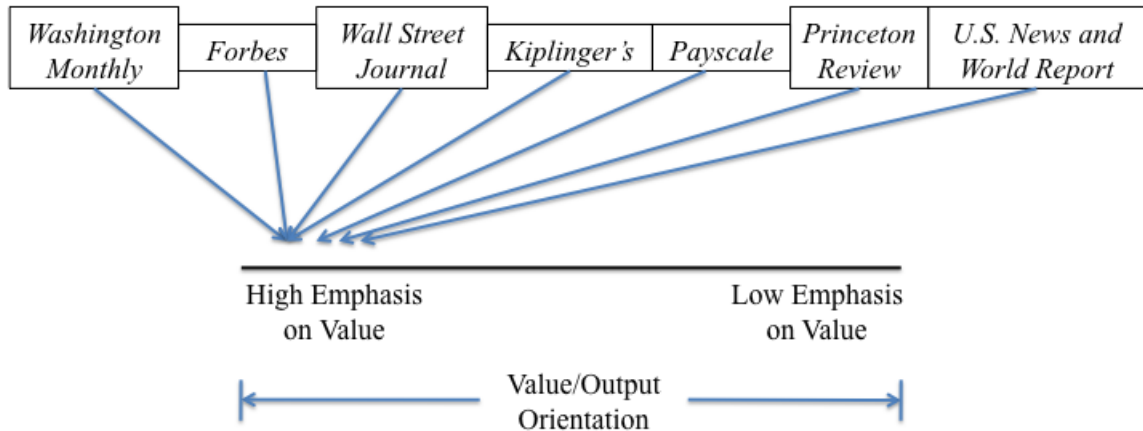


Figure 2: **College Rankings Market 2000 to Present**

lege Loan Corporation, and University of Richmond. Additionally, PR began selling advertisements to an assortment of colleges in a section called “School Says”. There are sixteen pages of advertising in its 2014 release. The majority of new entrants also coupled their rankings with advertising in the form of banners and pop-ups on their websites.

Notably, the type of advertisers buying ad space in college rankings target financially-conscious people. A large portion of advertisers are financial service providers promoting services such as banking, loans, insurance, credit cards, and investments. Advertisements that are not for financial services such as those for community colleges, employment services, and retail deals are also directed towards financially-conscious people. They seek to target individuals who are concerned about “dollars and cents tabulations.”

Another indication of the increased role of advertising is that many entrants provide their rankings to households for free or low prices. Moreover, these free or cheap rankings coincide with relatively high price tags for advertising space. A page of advertising in USNWR’s rankings costs tens of thousands of dollars (estimated \$30,000).

Online advertising for Forbes costs over a hundred dollars. USNWR highlights the popularity of its rankings in the section of its website for advertisers interested in purchasing space. This indicates that advertising is now an important revenue source for ranking firms.

That Money Magazine is no longer in the rankings market indicates that an increase in advertising supply by rankings firms did not increase the quantity of advertising. Money was an early producer of value-based rankings. However, it failed to generate enough revenue from advertisers and households to remain in the market. As such, it is likely that an increase in advertising demand led to the increase in rankings quantity, as advertisements in rankings also appear in other media reaching similar consumers (e.g. during various news hours on television).

### 3 Model

I model product choice by college rankings firms to further explore how changes in advertising influence the rankings offered and to study the effects of advertising on welfare in the college rankings market.

#### 3.1 Firms and Consumers

Consider a college rankings market with two firms that offer rankings products to two types of consumers. Firms 1 and 2 are symmetric and offer rankings  $(z_1, z_2)$  respectively. The firms' choices of rankings vary along a Hotelling line of length 1. The far left point of the line ( $z = 0$ ) represents a ranking with the maximum emphasis on college value and output. The far right point ( $z = 1$ ) represents a ranking with the maximum emphasis on non-value factors such as selectivity and college experience. Points inbetween represent some combination of value and other factors.

While Hotelling distributes consumers continuously along this line, I locate con-

sumers at the two endpoints to account for two discrete types of consumers: Type 1 and Type 2. All else equal, Type 1 consumers view products closer to  $z = 0$  or rankings that emphasize college value and output as having higher quality. All else equal, Type 2 consumers view products closer to  $z = 1$  or rankings that emphasize other factors as having higher quality. While consumer types are discrete, consumers' willingness to pay for higher quality rankings varies continuously within each consumer type. Thus, at each end point of the Hotelling line, there is a uniformly distributed continuum of consumers  $y_{st} \sim U[0, 1]$  where  $y_{st}$  reflects the willingness of Type  $t$  consumer  $s$  to pay for higher quality rankings. Higher values of  $y_{st}$  indicate a higher willingness to pay for quality. See Figure 3 for an illustration of this modified Hotelling model.

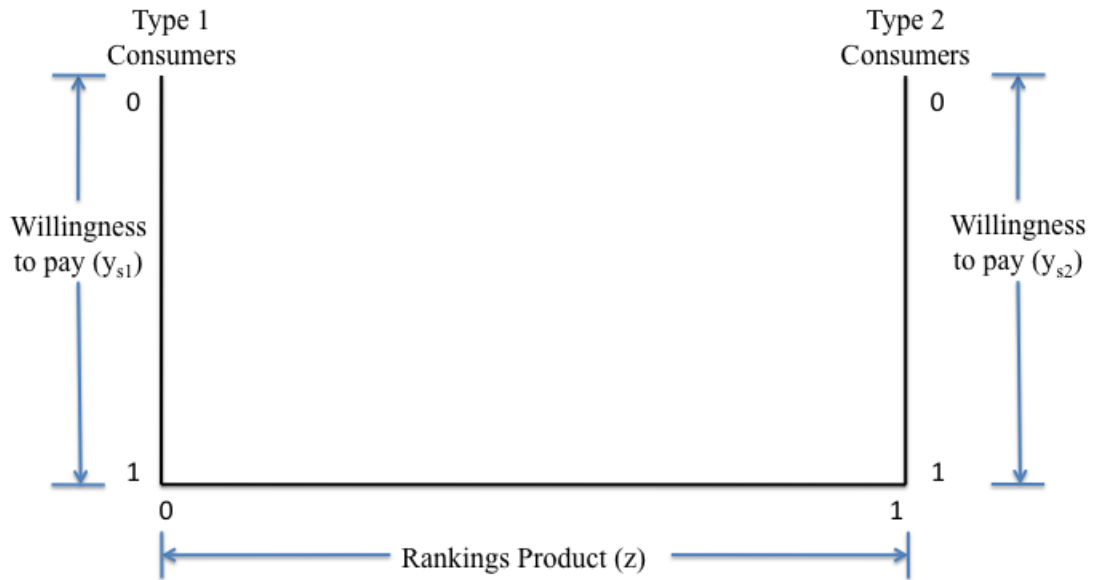


Figure 3: **Modified Hotelling Line**



Firm  $i$  selects some combination of rankings and price  $(z_i, p_{ci})$  in order to maximize profits. I assume that the marginal cost of producing rankings is zero, as the marginal cost of producing information goods has been understood to be very low (Arrow, 1962). I further assume that fixed costs are sunk in order to focus on the product choices of incumbent firms. Therefore, firm  $i$  faces the following profit function when it provides ranking  $z_i$  at  $p_{ci}$ , given rival firm  $j$  provides ranking  $z_j$  at  $p_{cj}$ :

$$\pi_i = p_{ci}(q_{1i}(z_i, p_{ci}; z_j, p_{cj}) + q_{2i}(z_i, p_{ci}; z_j, p_{cj}))$$

$q_{1i}$  and  $q_{2i}$  represent the quantity of rankings firm  $i$  sells to Type 1 and Type 2 consumers respectively for a given  $(z_i, p_{ci}; z_j, p_{cj})$ .

I model firms as choosing price and product type in a two-stage game. In the first stage, firms simultaneously choose a rankings product  $(z_1, z_2)$  to sell. In the second stage, firms observe  $(z_1, z_2)$  chosen in the first stage and simultaneously choose consumer prices  $(p_{c1}, p_{c2})$  given these values. Thus, firms maximize profits through sequential games of product choice then price choice.

Consumers choose to purchase  $z_1$ ,  $z_2$ , or no rankings product based on utility maximization. For simplicity and to emphasize horizontal differentiation such that different types of consumers have different preferences over rankings, I assume that each consumer only purchases at most one rankings product. The utility each consumer receives from purchasing no rankings product is normalized to zero, following the tradition in the discrete choice literature of consumer demand. The utility each consumer receives from purchasing  $z_1$  or  $z_2$  varies with consumer type and willingness to pay for rankings quality.

Each consumer  $s$  of Type 1 receives the following utilities from consuming firm 1

and firm 2's rankings, respectively:

$$\begin{aligned}
U_1(z_1) &= (\gamma_1 + \beta(1 - z_1))y_{s1} - \frac{1}{2}\alpha_1 p_{c1}^2 \\
U_1(z_2) &= (\gamma_1 + \beta(1 - z_2))y_{s1} - \frac{1}{2}\alpha_1 p_{c2}^2 \\
&\text{with } (\gamma_1, \beta, \alpha_1) > 0
\end{aligned}$$

Type 1 consumer  $s$  buys from firm 1 if  $U_1(z_1) > \max\{U_1(z_2), 0\}$ , from firm 2 if  $U_1(z_2) > \max\{U_1(z_1), 0\}$ , and from neither if  $0 > \max\{U_1(z_1), U_1(z_2)\}$ .

Each consumer  $s$  of Type 2 receives the following utilities from firm 1 and firm 2's rankings, respectively:

$$\begin{aligned}
U_2(z_1) &= (\gamma_1 + \beta z_1)y_{s2} - \frac{1}{2}\alpha_1 p_{c1}^2 \\
U_2(z_2) &= (\gamma_1 + \beta z_2)y_{s2} - \frac{1}{2}\alpha_1 p_{c2}^2 \\
&\text{with } (\gamma_1, \beta, \alpha_1) > 0
\end{aligned}$$

Type 2 consumer  $s$  buys from firm 1 if  $U_2(z_1) > \max\{U_2(z_2), 0\}$ , from firm 2 if  $U_2(z_2) > \max\{U_2(z_1), 0\}$ , and from neither if  $0 > \max\{U_2(z_1), U_2(z_2)\}$ .

Note,  $\gamma_1$  reflects consumers' willingness to pay for a ranking regardless of quality.  $\beta(1 - z_i)y_{s1}$  reflects how Type 1 consumers gain additional utility from purchasing  $z_i$  of higher quality (closer to  $z = 0$ ) and how this added utility increases with their willingness to pay for quality  $y_{s1}$ .  $\beta z_i y_{s2}$  reflects how Type 2 consumers gain additional utility from purchasing  $z_i$  of higher quality (closer to  $z = 1$ ) and how this added utility increases with their willingness to pay for quality  $y_{s2}$ .  $\frac{1}{2}\alpha_1 p_{ci}^2$  captures the disutility consumers experience from higher prices.

The quantity demanded  $q_{ti}$  by consumer Type  $t$  of firm  $i$ 's ranking is the length of the segment of the continuum of Type  $t$  consumers whose  $y_{st}$  is such that  $U_t(z_i) > \max\{U_t(z_j), 0\}$ . More specifically, I derive quantity demanded analytically for three main cases:

**Case 1:** There is no product differentiation ( $z_i = z_j = z^*$ ). In this case, one firm charges a higher price ( $p_{cj} > p_{ci}$ ),  $U(z_i) > U(z_j)$  for both types of consumers. Consequently, Type 1 consumers choose between buying from firm  $i$  or buying no rankings. Type 2 consumers similarly choose between buying from firm  $i$  or buying no rankings. No consumers purchase firm  $j$ 's ranking. The Type 1 consumer indifferent between purchasing firm  $i$ 's ranking and purchasing no rankings can be determined by solving for  $\tilde{y}_1$ , the value of  $y_{s1}$  such that  $U_1(z_i) = 0$ :

$$U_1(z_i) = (\gamma_1 + \beta(1 - z_i))y_{s1} - \frac{1}{2}\alpha_1 p_{ci}^2 = 0$$

$$\longrightarrow \quad \tilde{y}_1 = \frac{\alpha_1 \frac{1}{2} p_{ci}^2}{\gamma_1 + \beta(1 - z_i)}$$

If  $\tilde{y}_1 \in [0, 1]$ , then the consumer with  $y_{s1} = \tilde{y}_1$  is indifferent between purchasing  $z_i$  and purchasing no rankings, as she receives equal utility from both. Type 1 consumers whose  $y_{s1} \in (\tilde{y}_1, 1]$  receive higher utility from purchasing  $z_i$  compared to purchasing no rankings. As such, they will buy firm  $i$ 's product:  $q_{1i} = 1 - \tilde{y}_1$ . Oppositely, Type 1 consumers whose  $y_{s1} \in [0, \tilde{y}_1)$  gain higher utility from buying no rankings than buying firm  $i$ 's ranking. These consumers purchase no rankings. Consequently, if  $\tilde{y}_1 > 1$ , no Type 1 consumer buys firm  $i$ 's product:  $q_{1i} = 0$ .  $q_{2j}$  can be similarly derived.

However, charging a higher price is a dominated strategy when ( $z_i = z_j = z^*$ ) and  $p_{ci}, p_{cj} \geq 0$ .<sup>5</sup> As described above, the firm with the lower price sells to all of the consumers who purchase rankings and the firm with the higher price sells to no one.

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<sup>5</sup>I impose  $p_{ci}, p_{cj} \geq 0$ , because I do not observe any negative prices in the rankings market.

This means that firms have a strong incentive to undercut each other's prices until price is equal to marginal cost, as doing so allows the firm with the lower price to capture the whole market. Since I assumed a marginal cost of zero, firms will charge  $p_{ci} = p_{cj} = 0$  in the absence of product differentiation. When this occurs, I assume firm  $i$ 's expected demand for each consumer Type  $t$  is  $q_{ti} = \frac{1}{2}$ , as both firms offer identical rankings.

**Case 2:** Firms sell different rankings ( $z_i < z_j$ ) at the same price ( $p_{ci} = p_{cj} = p_c^*$ ). This implies  $U_1(z_i) > U_1(z_j)$  for Type 1 consumers and  $U_2(z_i) < U_2(z_j)$  for Type 2 consumers. Consequently, Type 1 consumers choose between buying from firm  $i$  or buying no rankings. Type 2 consumers choose between buying from firm  $j$  or buying no rankings. The Type 1 consumer indifferent between purchasing firm  $i$ 's rankings and purchasing no rankings can be determined by solving for  $\tilde{y}_1$ , the value of  $y_{s1}$  such that  $U_1(z_i) = 0$ .

$$U_1(z_i) = (\gamma_1 + \beta(1 - z_i))y_{s1} - \frac{1}{2}\alpha_1 p_{ci}^2 = 0$$

$$\rightarrow \tilde{y}_1 = \frac{\alpha_1 \frac{1}{2} p_{ci}^2}{\gamma_1 + \beta(1 - z_i)}$$

If  $\tilde{y}_1 \in [0, 1]$ , the consumer with  $y_{s1} = \tilde{y}_1$  is indifferent between purchasing firm  $i$ 's rankings and purchasing no rankings, as she receives equal utility from both options. Type 1 consumers whose  $y_{s1} \in (\tilde{y}_1, 1]$  receive greater utility from purchasing firm  $i$ 's ranking than purchasing no rankings. Thus, they will buy firm  $i$ 's product:  $q_{1i} = 1 - \tilde{y}_1$ . Oppositely, Type 1 consumers whose  $y_{s1} \in [0, \tilde{y}_1)$  receive higher utility from buying no rankings. It follows that if  $\tilde{y}_1 > 1$ , no Type 1 consumer buys firm  $i$ 's product:  $q_{1i} = 0$ .  $q_{2i}$  and  $q_{2j}$  can be similarly derived.

**Case 3:** Firms sell different rankings ( $z_i < z_j$ ) at different prices ( $p_{ci} > p_{cj}$ ).  $U_2(z_j) > U_2(z_i)$  for all Type 2 consumers, as their preferred ranking is cheaper. Thus,

Type 2 consumers choose between buying firm  $j$ 's ranking and buying no rankings to maximize utility, and solving for Type 2 consumer demand is the same as in Case 2. However, Type 1 consumers may purchase firm  $i$ 's ranking, firm  $j$ 's ranking, or no rankings. See Figure 4 below for an illustration.

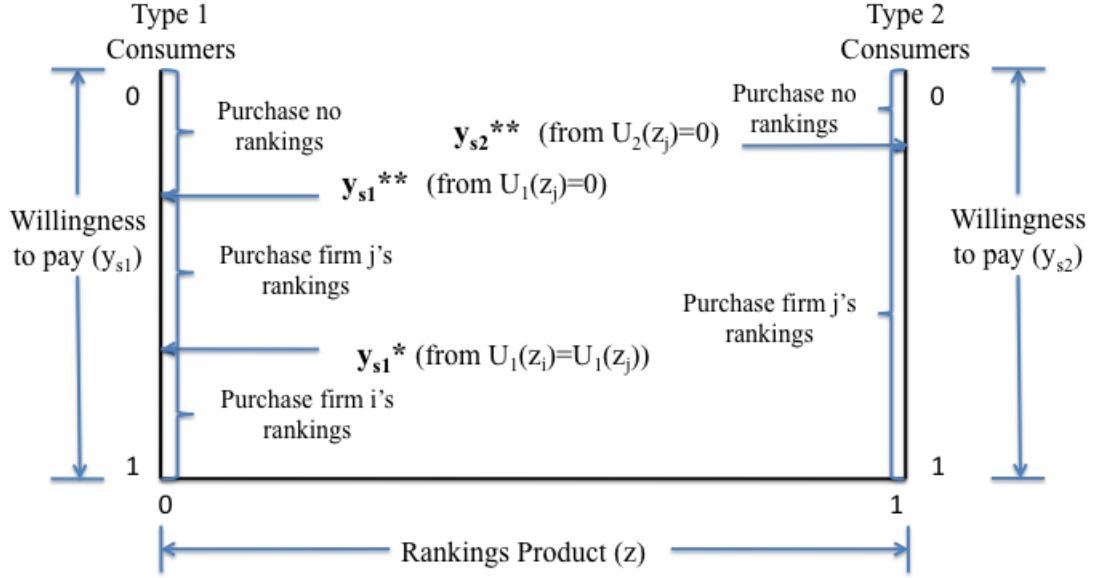


Figure 4: **Indifference Thresholds**  $z_i < z_j$ ,  $p_{ci} > p_{cj}$

The Type 1 consumer indifferent between purchasing  $z_i$  and  $z_j$  can be determined by solving for  $y_{s1}^*$ , the value of  $y_{s1}$  such that  $U_1(z_i) = U_1(z_j)$ . This consumer receives the same amount of utility from purchasing firm  $i$ 's ranking and purchasing firm  $j$ 's ranking.

$$\begin{aligned}
 U_1(z_i) &= U_1(z_j) \\
 (\gamma_1 + \beta(1 - z_i))y_{s1} - \frac{1}{2}\alpha_1 p_{ci}^2 &= (\gamma_1 + \beta(1 - z_j))y_{s1} - \frac{1}{2}\alpha_1 p_{cj}^2 \\
 \rightarrow y_{s1}^* &= \frac{\frac{1}{2}\alpha_1(p_{ci}^2 - p_{cj}^2)}{\beta(z_j - z_i)}
 \end{aligned}$$

The Type 1 consumer indifferent between purchasing  $z_j$  and no ranking can be

determined by solving for  $y_{s1}^{**}$ , the value of  $y_{s1}$  such that  $U_1(z_j) = 0$ . This consumer receives the same amount of utility from purchasing firm  $j$ 's ranking and from purchasing no rankings.

$$\begin{aligned}
 U_1(z_j) &= (\gamma_1 + \beta(1 - z_j))y_{s1} - \frac{1}{2}\alpha_1 p_{cj}^2 = 0 \\
 \longrightarrow \quad y_{s1}^{**} &= \frac{\frac{1}{2}\alpha_1 p_{cj}^2}{\gamma_1 + \beta(1 - z_j)}
 \end{aligned}$$

Consequently, if  $y_{s1}^* \in [0, 1]$ , Type 1 consumers whose  $y_{s1} \in (y_{s1}^*, 1]$  gain higher utility by purchasing firm  $i$ 's ranking in comparison to purchasing firm  $j$ 's ranking or purchasing no ranking. Thus, they will buy firm  $i$ 's product:  $q_{1i} = 1 - y_{s1}^*$ . Type 1 consumers with  $y_{s1} \in (y_{s1}^{**}, y_{s1}^*)$ , receive higher utility from purchasing firm  $j$ 's ranking in comparison to purchasing firm  $i$ 's ranking or purchasing no ranking. These consumers will buy firm  $j$ 's product:  $q_{1j} = y_{s1}^* - y_{s1}^{**}$ . Type 1 consumers with  $y_{s1} \in [0, y_{s1}^{**})$  receive the highest utility from purchasing no rankings.

Note,  $y_{s1}^* < y_{s1}^{**} < 1$  implies that the consumer indifferent between purchasing firm  $i$  and firm  $j$ 's rankings has a lower willingness to pay for quality than the consumer indifferent between purchasing firm  $j$ 's and no rankings. This contradicts that firm  $i$ 's ranking  $z_i < z_j$  is the higher quality good from the perspective of Type 1 consumers. It implies that consumers who receive the greatest utility from purchasing the lower quality, cheaper good have a higher willingness to pay for quality than consumers who receive the greatest utility from purchasing the higher quality, pricier good. Consequently,  $y_{s1}^* < y_{s1}^{**} < 1$  is a corner solution; no Type 1 consumers purchase rankings from firm  $j$  in this case, as all Type 1 consumers who buy rankings receive higher utility from  $z_i$ .

Let  $p_{ci} = p_{cj} + \epsilon$ .  $\epsilon$  is how much more firm  $i$  charges consumers for its ranking compared to firm  $j$ . Intuitively,  $\epsilon$  represents the degree to which firm  $i$  is more

aggressive than firm  $j$  in extracting Type 1 consumer surplus. The value of  $\epsilon$  at which the consumer indifferent between  $z_i$  and  $z_j$  and the consumer indifferent between  $z_j$  and no rankings are the same is:

$$\begin{aligned}
y_{s1}^* &= y_{s1}^{**} \\
\frac{\frac{1}{2}\alpha_1(2p_{cj} + \epsilon^2)}{\beta(z_j - z_i)} &= \frac{\alpha_1\frac{1}{2}p_{cj}^2}{\gamma_1 + \beta(1 - z_j)} \\
\longrightarrow \quad \epsilon_1 &= p_{cj}\left(\sqrt{1 + \frac{\beta(z_j - z_i)}{\gamma_1 + \beta(1 - z_j)}} - 1\right)
\end{aligned}$$

When  $\epsilon \leq \epsilon_1$ ,  $y_{s1}^* < y_{s1}^{**}$ , and no Type 1 consumers purchase rankings from firm  $j$ .  $\epsilon$  is small enough, such that the additional utility from purchasing the higher quality ranking outweighs the loss in utility from a higher price. Thus, Type 1 consumers choose between purchasing rankings from firm  $i$  or no rankings. The consumer indifferent between purchasing firm  $i$ 's good and purchasing no rankings has  $y_{s1}$  such that  $U_1(z_i) = 0$ :

$$\begin{aligned}
U_1(z_i) &= (\gamma_1 + \beta(1 - z_i))y_{s1} - \frac{1}{2}\alpha_1 p_{ci}^2 = 0 \\
\longrightarrow \quad y_{s1}^{***} &= \frac{\alpha_1\frac{1}{2}p_{ci}^2}{\gamma_1 + \beta(1 - z_i)}
\end{aligned}$$

The second corner solution occurs when  $1 < y_{s1}^* < y_{s1}^{**}$ . In this case,  $p_{ci}$  and  $p_{cj}$  are high enough such that purchasing no good is the utility maximizing choice for all Type 1 consumers. If  $p_{ci} = p_{cj} + \epsilon$ ,  $\epsilon_2$  is the value of  $\epsilon$  such that all of the consumers  $y_{s1} \in [0, 1]$  maximize utility by purchasing no rankings:

$$\begin{aligned}
& y_{s1}^{***} > 1 \\
& \frac{\alpha_1 \frac{1}{2} p_{ci}^2}{\gamma_1 + \beta(1 - z_i)} > 1 \\
\rightarrow \quad \epsilon_2 > -p_{cj} + \sqrt{\frac{2(\gamma_1 + \beta(1 - z_i))}{\alpha_1}}
\end{aligned}$$

When  $y_{s1}^{***} = 1$ , the consumer indifferent between purchasing firm  $i$ 's rankings and purchasing no rankings is the consumer with the highest willingness to pay for quality. Consequently, for consumers  $y_{s1} < 1$ , buying no ranking yields the highest utility. If  $y_{s1}^{***} > 1$ , even the consumer at the end of the distribution  $y_{s1} = 1$  receives higher utility from buying no rankings.

The third corner solution occurs when  $y_{s1}^* > 1$ . This indicates all Type 1 consumers gain higher utility from either purchasing no rankings or  $z_j$  than purchasing  $z_i$ ; there is no consumer  $y_{s1} \in [0, 1]$  indifferent between  $z_i$  and  $z_j$ . As such, firm  $i$  makes no sales to Type 1 consumers:  $q_{ci} = 0$ . The values of  $\epsilon$  for which this is true are:

$$\begin{aligned}
& y_{s1}^* > 1 \\
& \frac{\frac{1}{2}\alpha_1(2p_{cj} + \epsilon^2)}{\beta(z_j - z_i)} > 1 \\
\rightarrow \quad \epsilon_3 > -p_{cj} + \sqrt{p_{cj}^2 + \frac{2\beta(z_j - z_i)}{\alpha_1}}
\end{aligned}$$

Combining these three threshold values for  $\epsilon$ , when  $\epsilon \in (\epsilon_1, \min(\epsilon_2, \epsilon_3))$ , both firms  $i$  and  $j$  make positive sales to Type 1 consumers:  $q_{1i} = 1 - y_{s1}^*$ ,  $q_{1j} = y_{s1}^* - y_{s1}^{**}$ . When  $\epsilon \leq \epsilon_1$ , only firm  $i$  may sell to Type 1 consumers:  $q_{1i} = 1 - y_{s1}^{***}$ ,  $q_{1j} = 0$ . When  $\epsilon \geq \min(\epsilon_2, \epsilon_3)$ , only firm  $j$  may sell to Type 1 consumers:  $q_{1i} = 0$ ,  $q_{1j} = 1 - y_{s1}^{**}$  (assuming  $y_{s1}^{**} \leq 1$ ).



When firms sell different rankings ( $z_i < z_j$ ) at different prices ( $p_{ci} < p_{cj}$ ), deriving demand for Type 2 consumers is similar to deriving demand for Type 1 consumers when  $p_{ci} > p_{cj}$ .

## 3.2 Advertising

In order to consider the impact of advertisers who wish to target only Type 1 consumers interested in value and output oriented rankings, I alter the profit function to include the price to advertisers for consumer “eyeballs”  $p_a$ :

$$p_a = \frac{(rate)(space)}{(size)(viewers)}$$

where *rate* is the cost of a given advertisement, *size* is the amount of space occupied by that advertisement, *space* is the total amount of space available for advertising, and *viewers* is the number of eyeballs or consumers reading a given ranking.  $p_a$  reflects the price per viewer reached adjusted by the relative size of the ad. When the demand for advertising space increases,  $p_a$  also increases. Consequently,  $p_a$  captures changes in advertising demand, as shifts in demand *ceteris paribus* lead to corresponding price changes in the same direction.

Notably,  $p_a$  acts as an exogenous subsidy rather than a strategic variable because, while rankings firms set prices in the rankings market, they are price takers in the market for advertising. There are a limited number of firms that produce college rankings. However, there are many firms in addition to college rankings firms who sell space to advertisers. On the internet, almost every single website (e.g. Facebook, Yahoo) sells space on their web-pages to advertisers. In terms of print publications, publications that do not publish college rankings such as *The Economist* sell advertising space.

It follows that the profit function for firm  $i$  with possible advertising revenue is:

$$\pi_i = (p_{ci} + p_a)(q_{1i}(z_i, p_{ci}; z_j, p_{cj})) + p_{ci}(q_{2i}(z_i, p_{ci}; z_j, p_{cj}))$$

This reflects how advertisers wish to target only Type 1 consumers. For each additional unit of quantity  $q_{1i}$  sold, firms receive revenue of  $p_a$  in addition to price charged.

### 3.3 Solving for Equilibrium

I discretize firms' choices of rankings and prices to simplify solving for the Nash equilibria.<sup>6</sup> Firm  $i$  chooses rankings from  $z_i \in \{0, 0.5, 1\}$ ; it may produce consumer Type 1's ideal ranking, consumer Type 2's ideal ranking, or a ranking halfway between the two ideals. Firm  $i$  chooses price from  $p_{ci} \in \{0, p_L, p_H\}$ . It may charge price 0 such as in the case of price competition when firms produce the same rankings, some low price  $p_L$ , or some high price  $p_H$ . Based on the differences among  $\{0, p_L, p_H\}$ , we can determine under which of the three cases mentioned earlier demand falls.

Finding the equilibrium involves solving two-stages of 3x3 normal form games. Solving backwards, the second stage determines the price each firm charges for a given choice of  $(z_1, z_2)$ , as firms choose prices that maximize profit given their competitor's choice. In the first stage, firms select the rankings that maximize over the profits determined in the second stage. As John Nash (1950) showed, given this overall game can be expressed as a finite strategic form game, the existence of a Nash equilibrium is guaranteed.

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<sup>6</sup>Solving the general problem is computationally difficult. I leave it for future research.

### 3.3.1 Parameters

Consider the model under the following parameter values:  $\alpha_1 = 10$ ,  $\beta = 1$ , and  $\gamma_1 = 10$ .  $\alpha_1$  is set high to reflect how consumers are very price responsive. When consumers are very price responsive, price competition is a greater concern for firms, as each unit decrease in price leads to a much larger increase in quantity demanded. Thus, setting  $\alpha_1 = 10$  allows me to demonstrate in the base case how, in the absence of advertising, firms offer highly differentiated rankings to avoid price competition.

$\beta$  and  $\gamma_1$  are chosen such that there are cases in which both firms make positive sales to both types of consumers.  $\beta$  is smaller than  $\gamma_1$  to reflect how having access to some information is more important to consumers than having information that perfectly match their preferences.

For these parameters, I explore discrete prices  $p_L = 0.15$ , and  $p_H = 0.3$ , as they yield interior (non-corner) solutions (as described in Case 3) for the demand functions for all non-equal price combinations. With these prices, the firm with the lower price makes sales to both Type 1 and Type 2 consumers. The firm with the higher price sells only to the type of consumer that considers its product to be of higher quality. This is shown in greater detail in Table 1 and Table 2.

In Table 1,  $\epsilon$  is how much more firm 1 charges for its rankings when  $p_{c1} > p_{c2}$ .  $\epsilon_1$  shows how large  $\epsilon$  must be such that there are Type 1 consumers who gain higher utility by purchasing  $z_2$ .  $\min(\epsilon_2, \epsilon_3)$  shows at what value  $\epsilon$  becomes so large that no consumers purchase  $z_1$ .

Table 1 also demonstrates how  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  vary with values of  $p_{c2}$ ,  $z_1$ , and  $z_2$ . When  $p_{c2} > 0$ , there are positive values of  $\epsilon$  at which firm 1 charges a higher price than firm 2 and all Type 1 consumers who buy rankings buy from firm 1. The closer  $z_1$  and  $z_2$  are to each other, the smaller the range of positive  $\epsilon$  at which firm 1 can still capture all of Type 1 consumer demand. As  $z_2$  closer to  $z_1$  implies that  $z_2$  is

a more favorable substitute for Type 1 consumers, this also limits how much higher firm 1 can charge before it makes no sales.  $\delta$  is how much more firm 2 charges for its rankings when  $p_{c2} > p_{c1}$ . The explanations of the thresholds of  $\delta$  in Table 2 are similar to those for  $\epsilon$ .

Table 1: Threshold Values for  $\epsilon$

$p_{c2}$	$(z_1 < z_2)$	$\epsilon_1$	$\min(\epsilon_2, \epsilon_3)$
0	(0,1)	0	0.4472
0	(0,0.5)	0	0.3162
0	(0.5,1)	0	0.3162
0.15	(0,1)	0.0074	0.3216
0.15	(0,0.5)	0.0036	0.2
0.15	(0.5,1)	0.0038	0.2

Table 2: Threshold Values for  $\delta$

$p_{c1}$	$(z_1, z_2)$	$\delta_1$	$\min(\delta_2, \delta_3)$
0	(0,1)	0	0.4472
0	(0,0.5)	0	0.3162
0	(0.5,1)	0	0.3162
0.15	(0,1)	0.0074	0.3216
0.15	(0,0.5)	0.0038	0.2
0.15	(0.5,1)	0.0036	0.2

### 3.3.2 Base Case

Consider when there is no advertising revenue:  $p_a = 0$ . As illustrated in Figure 5, there are two pure-strategy equilibria in the first stage of the game:  $(z_1, z_2) = (0, 1)$  and  $(z_1, z_2) = (1, 0)$  where firms charge  $p_{c1} = p_{c2} = 0.3$  and earn  $\pi_1 = \pi_2 = 0.2877$ . This base case demonstrates how, without advertising, both firms segment the market by producing goods with the maximum possible differentiation in the presence of highly price-sensitive consumers.<sup>7</sup>

<sup>7</sup>See the Appendix for an example of how the payoffs in the first stage are derived from each second stage game.

		Firm 2		
		Z=0	Z=0.5	Z=1
Firm 1	$P_a=0$ Z=0	$\Pi_1=0, \Pi_2=0$	$\Pi_1=0.2877,$ $\Pi_2=0.2871$	$\Pi_1=0.2877,$ $\Pi_2=0.2877$
	Z=0.5	$\Pi_1=0.2871,$ $\Pi_2=0.2877$	$\Pi_1=0, \Pi_2=0$	$\Pi_1=0.2871,$ $\Pi_2=0.2877$
	Z=1	$\Pi_1=0.2877,$ $\Pi_2=0.2877$	$\Pi_1=0.2877,$ $\Pi_2=0.2871$	$\Pi_1=0, \Pi_2=0$

\*\*\*For (0,0.5), (0.5,0), (0.5,1), and (1,0.5), there are actually two pure strategy Nash in the second stage pricing game. I include only the higher payoffs for both firms to emphasize that these are not at intersection of the firms' best responses in the first stage.

Figure 5: **First Stage Game**  $p_a = 0$

### 3.3.3 Shifts in $p_a$

Consider when there is “low” advertising demand:  $p_a = 0.1$ . As shown in Figure 6, for this “low” level of advertising, there are two pure-strategy equilibria:  $(z_1, z_2) = (0, 1)$  and  $(z_1, z_2) = (1, 0)$  where both firms charge  $p_{c1} = p_{c2} = 0.3$  and earn  $\pi_1 = 0.3836$ ,  $\pi_2 = 0.2877$ . Thus, when the price of advertising is low, it offers insufficient incentives for firms to risk price competition by producing similar goods. In particular,  $p_a$  provides insufficient incentives for the firm producing  $z = 1$  to appeal to Type 1 consumers by shifting its product offering towards  $z = 0$  or charging lower prices.

As  $p_a$  becomes larger, there is first a change in the (0, 1) and (1, 0) stage-two equilibrium prices. For  $0.2769 < p_a < 0.659$ , advertising is now substantial enough such that, when the firm offering  $z = 0$  charges a high price of 0.3, the firm offering  $z = 1$  prefers to offer its rankings at a lower price 0.15 in order to appeal to Type 1 consumers and gain advertising revenue (instead of just profiting from Type 2 consumers and charging 0.3, which occurs when the  $z = 0$  firm charges 0.15).<sup>8,9</sup>

<sup>8</sup>The upper bound 0.659 is the threshold value of  $p_a$  at which best responses in the second-stage pricing games change.

<sup>9</sup>I solve for these threshold values by calculating values of  $p_a$  that give firm equal profits for

		Firm 2		
		Z=0	Z=0.5	Z=1
Firm 1	Z=0	$\Pi_1=0.05,$ $\Pi_2=0.05$	$\Pi_1=0.2475,$ $\Pi_2=0.1484$	$\Pi_1=0.3836,$ $\Pi_2=0.2877$
	Z=0.5	$\Pi_1=0.1484,$ $\Pi_2=0.2475$	$\Pi_1=0.05,$ $\Pi_2=0.05$	$\Pi_1=0.2473,$ $\Pi_2=0.1485$
	Z=1	$\Pi_1=0.2877,$ $\Pi_2=0.3836$	$\Pi_1=0.1485,$ $\Pi_2=0.2473$	$\Pi_1=0.05,$ $\Pi_2=0.05$

Figure 6: **First Stage Game**  $p_a = 0.1$

This results in there only being a mixed strategy equilibrium where firms randomize between charging 0.15 and 0.3, as the firm offering  $z = 0$  prefers to charge the same price as the firm offering  $z = 1$  to capture all of Type 1 consumer demand and advertising revenue, and the firm offering  $z = 1$  prefers to charge the lower price or the higher price.

However,  $p_a$  does not induce firms to offer the same types of rankings until  $p_a \approx 0.4753$ . Once  $p_a > 0.4753$ , advertising is substantial enough such that in equilibrium both firms offer  $z = 0$  and engage in price competition (charge  $p_{ci} = p_{cj} = 0$ ) in order to compete for a larger share of Type 1 consumer demand and consequently a larger different pricing decisions. The best response strategies of firms vary according to these thresholds. For example, for a given  $(z_i, z_j)$  the following yields the threshold  $p_a$  when, given firm  $i$  charges  $p_{ci} = p_L$ , firm  $j$  makes equal profits when charging  $p_L$  and 0:

$$\begin{aligned} \pi_j(p_{ci} = p_L, p_{cj} = 0) &= \pi_j(p_{ci} = p_L, p_{cj} = p_L) \\ p_a \left( \frac{\frac{1}{2}\alpha_1 p_L^2}{\beta(z_j - z_i)} \right) &= p_L \left( 1 - \frac{\frac{1}{2}\alpha_1 p_L^2}{\gamma_1 + \beta z_j} \right) \\ p_a &= p_L \left( 1 - \frac{\frac{1}{2}\alpha_1 p_L^2}{\gamma_1 + \beta z_j} \right) \left( \frac{\beta(z_j - z_i)}{\frac{1}{2}\alpha_1 p_L^2} \right) \end{aligned}$$

share of advertising revenue.

For example, consider a “high” level of advertising:  $p_a = 0.6$ . As shown in Figure 7, this value of advertising is large enough to induce both firms in equilibrium to produce the same ranking  $z = 0$  and offer it to consumers at zero price. In this case, firms make all of their profits  $\pi_1 = \pi_2 = 0.3$  from advertisers. Notably, this is reminiscent of the many online rankings in which rankings are offered to consumers for free but advertisers must pay for advertising space.

		Firm 2		
		$Z=0$	$Z=0.5$	$Z=1$
Firm 1	$Z=0$	<b><math>\Pi_1=0.3, \Pi_2=0.3</math></b>	<b><math>\Pi_1=0.7423,</math> <math>\Pi_2=0.1484</math></b>	<b><math>\Pi_1=0.7751,</math> <math>\Pi_2=0.2877</math></b>
	$Z=0.5$	$\Pi_1=0.1484,$ <b><math>\Pi_2=0.7423</math></b>	$\Pi_1=0.3, \Pi_2=0.3$	$\Pi_1=0.742,$ $\Pi_2=0.1485$
	$Z=1$	$\Pi_1=0.2877,$ <b><math>\Pi_2=0.7751</math></b>	$\Pi_1=0.1485,$ $\Pi_2=0.742$	$\Pi_1=0.3, \Pi_2=0.3$

Figure 7: **First Stage Game**  $p_a = 0.6$

### 3.3.4 Shifts in $\beta$

$\beta$  reflects the strength of a consumer’s willingness to pay for quality. Consequently, it also reflects the degree to which firms can earn higher profits from product differentiation. The more consumers are willing to pay for what they believe to be quality, the more firms profit from serving them. As such, when  $\beta$  increases, profits from positive prices increase.

For example, consider a deviation in the base case such that  $\beta = 2$  instead of

$\beta = 1$ , all else equal. While there are still two Nash equilibria of  $(z_1, z_2) = (0, 1)$  and  $(z_1, z_2) = (1, 0)$  where  $(p_{c1}, p_{c2}) = (0.3, 0.3)$ , profits are higher. Firms now earn  $\pi_1 = \pi_2 = 0.2888$  instead of just 0.2877.

Since  $\beta$  makes product differentiation more profitable to firms, increasing  $\beta$  increases the  $p_a$  that induces a mixed strategy equilibrium for prices when  $(z_i, z_j) = (0, 1)$  and the  $p_a$  that induces both firms to supply  $z = 0$ .

When  $\beta = 2$ , there is only a mixed strategy for prices in the  $(0, 1)$  and  $(1, 0)$  second stage games when  $0.2796 < p_a < 0.7348$ . Firms randomize between charging 0.15 and 0.3. This threshold is greater than when  $\beta = 1$ :  $0.2796 > 0.2769$ . As product differentiation is more profitable under  $\beta = 2$ , Firm  $j$ , the firm offering the ranking  $z = 1$  not preferred by advertisers, requires a higher incentive to sacrifice profits from Type 2 consumers and compete for a share of Type 1 consumer demand and advertising profits by charging a lower price of 0.15 when firm  $i$  charges 0.3.

Similarly, the threshold at which firms both choose to produce  $z = 0$  instead of differentiating products increases from approximately 0.4753 to approximately 0.5755 when  $\beta$  increases from 1 to 2, as differentiation is now more profitable.

### 3.3.5 Shifts in $\alpha_1$

$\alpha_1$  describes how sensitive consumers are to price and thus how much of a concern price competition is to firms. The higher  $\alpha_1$  *ceteris paribus*, the lower a firm's profits, as a higher  $\alpha_1$  renders purchasing no rankings the utility maximizing option for more consumers. A higher  $\alpha_1$  also indicates price competition is more of a concern, because higher consumer sensitivity to price increases the additional profits from undercutting the other firm.

For example, consider a deviation in base case such that  $\alpha_1 = 12$  instead of  $\alpha_1 = 10$  *ceteris paribus*. This yields two Nash equilibria of  $(z_1, z_2) = (0, 1)$  and



$(z_1, z_2) = (1, 0)$ , where  $(p_{c1}, p_{c2}) = (0.3, 0.3)$ , and  $\pi_1 = \pi_2 = 0.2853$ . Though firms choose the same rankings and prices, equilibrium profits are lower than in the base case,  $0.2853 < 0.2877$ , as a result of fewer consumers purchasing rankings due to greater disutility from price.

Notably, increasing  $\alpha_1$  increases the marginal revenue of charging  $z = 0$  or  $z = 1$  instead of  $z = 0.5$ . This increase renders price competition more of a danger and consequently increases incentives to differentiate products. If  $\alpha_1 = 12$  *ceteris paribus*, when the other firm charges  $z = 0$ , the marginal gain to charging  $z = 1$  instead of  $z = 0.5$  is 0.0007 (considering the pure strategy equilibrium of  $(0.3, 0.3)$  in the  $(0, 0.5)$  second stage game). This is greater than the marginal gain of 0.0006 when  $\alpha_1 = 10$ .

Changing  $\alpha_1 = 12$  also alters the second-stage price games corresponding to rankings combinations  $(0, 0.5)$  and  $(0.5, 1)$ ,  $\epsilon = p_H - p_L = 0.3$  and  $\delta = p_H - p_L = 0.3$  now exceed  $\min(\epsilon_1, \epsilon_3) = 0.2886$  and  $\min(\delta_1, \delta_3) = 0.2886$ . As a result of increasing  $\alpha_1$  and thus the price-sensitivity of consumers, charging 0.3 when the other firm charges 0 now becomes “too much higher” of a price, and no consumers gain higher utility from purchasing the higher-price ranking.

It also changes the  $(0, 1)$  and  $(1, 0)$  second stage games. When  $\alpha = 12$ , there is only a mixed strategy equilibrium between prices 0.15 and 0.3 when  $0.2003 < p_a < 0.5484$ . This is a lower threshold than when  $\alpha_1 = 10$  *ceteris paribus*:  $0.2003 < 0.2769$ . As consumers are more price sensitive, the firm offering  $z = 1$  makes lower profits from selling at a high price to only Type 2 consumers and thus profits more from offering a cheaper good that also attracts more advertiser “subsidized” Type 1 consumers.

An increase in the price-sensitivity  $\alpha_1$  also decreases the amount of advertising needed to induce both firms to locate at  $z = 0$  because increased price sensitivity decreases profits from consumers in general. In this case, when  $p_a \approx 0.4155$ , firms make equal profits when splitting the market and producing the same good  $z = 0$ .

If  $p_a > 0.4155$ , they maximize profits by offering the same good. When consumer price-sensitivity  $\alpha_1$  increases by 2, the amount of advertising  $p_a$  that leads firms to prefer minimum differentiation decreases from 0.4753 to 0.4155.

### 3.3.6 Shifts in $\gamma_1$

$\gamma_1$  reflects consumers' indiscriminate willingness to purchase rankings. It is the amount of utility that a consumer earns from purchasing any given ranking, regardless of how close that ranking is to her preferences. Consequently, *ceteris paribus*, increases in  $\gamma_1$  increase profits that firms make from selling any rankings to consumers by increasing the number of consumers for whom buying some rankings maximizes utility.

Consider increasing consumers' indiscriminate willingness to buy rankings  $\gamma_1$  from 10 to 12 *ceteris paribus*. In this case, there are two Nash equilibria of  $(z_1, z_2) = (0, 1)$  and  $(z_1, z_2) = (1, 0)$  where firms charge  $(p_1, p_2) = (0.3, 0.3)$  and earn profits  $\pi_1 = \pi_2 = 0.2896$ . These new profits under  $\gamma_1 = 12$  reflect how increasing the willingness of consumers to buy any rankings increases profits made by firms as they are greater than profits ( $\pi_i = \pi_j = 0.2877$ ) when  $\gamma_1 = 10$ .

By increasing firm profits independent of ranking offered, increasing  $\gamma_1$  also increases the  $p_a$  needed for there to exist only a mixed strategy price equilibrium in the second stage games of  $(0, 1)$  and  $(1, 0)$ . If  $(z_i, z_j) = (0, 1)$ , there is only a mixed-strategy equilibrium where firms switch between charging 0.15 and 0.3 when  $0.2795 < p_a < 0.6606$ . As firm  $j$  can now make higher profits from the consumers who prefer  $z_j$ , it needs a greater incentive to compete with firm  $i$  for Type 1 demand and advertising profits by lowering its prices.

Similarly, increasing  $\gamma_1$  increases the  $p_a$  that makes firms indifferent between segmenting the market and producing different rankings. When firms produce the same rankings, they engage in price competition and make zero profits from consumers.

As such, being able to earn higher profits from consumers when they segment the market due to increases in  $\gamma_1$  implies that the advertising revenue required to make producing  $(z_i, z_j) = (0, 0)$  more profitable must also increase. Thus, the  $p_a$  required to induce firms to offer the same rankings  $z = 0$  increases from 0.4753 to 0.4768 when  $\gamma_1$  increases by 2.

### 3.4 Welfare

This two-firm game reveals that the introduction of significant advertising leads to two opposing changes in total surplus. On one hand, advertising increases total surplus by increasing consumers' access to rankings overall because it leads firms to charge lower prices. On the other hand, advertising leads to a misallocation of trade because it decreases product differentiation. With decreased differentiation, some consumers lose access to their preferred rankings. In particular, these consumers are the ones who would gain higher utility from purchasing their preferred rankings even at a higher price.

The intuition for these two changes in welfare comes from the analogy of  $p_a$  to a subsidy. One traditional effect of subsidies is that they lead firms to produce more of the subsidized good and offer it to consumers at a lower price in order to sell additional units. In the case of rankings,  $p_a$  has the similar effect of leading firms to supply more rankings at lower price of zero in the market. Without advertising, there is not enough trade as firms exercise market power by withholding quantity to charge higher prices. Consequently, the increase in consumer welfare from significant advertising reflects how subsidies expand access to a given good. Notably, whereas subsidy expenditures not transferred to consumers or producers are usually considered dead-weight loss when provided by the government, this does not apply to advertisers; by providing  $p_a$ , advertisers receive the benefit of reaching consumers.

Looking at how subsidies may distort trade provides intuition for why advertising

creates inefficiency from a misallocation of trade when advertisers wish to reach only one type of consumer. Subsidies distort trade by altering the marginal revenue faced by firms. In particular, they increase the marginal revenue of the subsidized good (A) relative to that of some unsubsidized good (B). Consequently, firms that in the absence of the subsidy would offer B, instead offer A, as offering A has become relatively more profitable. By effectively subsidizing only  $z = 0$ , advertisers cause firms to switch from producing  $z = 1$  to  $z = 0$ .

With regards to consumer surplus, I calculate maximum willingness to pay (marginal value) of a Type 2 consumer for some ranking  $z_i$  when the alternative is buying no rankings. This enables me to later compare pairwise changes in consumer surplus under the provision of different rankings. As such, the maximum willingness to pay of a Type 2 consumer  $y_{s2}$  for ranking  $z_i$  is:

$$\begin{aligned}
 U_2(z_i) &= 0 \\
 (\gamma_1 + \beta z_i)y_{s2} - \frac{1}{2}\alpha_1 p_{ci}^2 &= 0 \\
 p_{ci}^{max2} &= \sqrt{\frac{2(\gamma_1 + \beta z_i)y_{s2}}{\alpha_1}}
 \end{aligned}$$

If  $p_{ci} > p_{ci}^{max2}$ , the consumer with  $y_{s2}$  would receive higher utility from purchasing no rankings. The maximum willingness to pay of Type 1 consumers  $p_{ci}^{max1}$  can be similarly derived from  $U_1(z_i) = 0$ .

Consider the same parameters from the previous section. For these parameters:

$$p_{ci}^{max2} = \sqrt{\frac{(10 + z_i)y_{s2}}{5}}$$

Using  $p_{ci}^{max}$ , we can calculate the difference in total surplus without advertising  $p_a = 0$  and with “high” advertising  $p_a = 0.6$ . Total surplus is the sum of firm profits and consumer surplus. Type 2 consumer surplus is:

$$U_{2total}(z_i, p_{ci}; z_j, p_{cj}) = \int_{y_{s2}^{**}}^1 (p_{ci}^{max2} - p_{ci}) dy_{s2}$$

where  $y_{s2}^{**}$  is the value of  $y_{s2}$  such that  $U_2(z_i) = 0$ . Type 1 consumer surplus can be similarly derived from  $p_{cj}^{max1}$ ,  $y_{s1}^{**}$ .

Recall, when  $p_a = 0$ , one firm offers  $z = 0$ , the other firm offers  $z = 1$ , and both firms charge a price of 0.3. It follows that Type 1 consumer surplus is 0.6929, and Type 2 consumer surplus is likewise 0.6929. Notably, both types of consumers receive access to their preferred rankings. Each firm makes a profit of 0.2877. Total surplus is approximately 1.9612.

When  $p_a = 0.6$ , both firms offer  $z = 0$  at zero price. In this case, Type 1 consumer surplus increases to 0.9888, Type 2 consumer surplus increases to 0.9428, and total profits across firms increases to 0.6. Total surplus is thus 2.5316. Notably, total surplus is higher in this case because  $p_a$  is large enough to provide firms with higher profits and induce lower consumer prices. Lower consumer prices increase welfare through increasing the number of consumers who can purchase rankings.

Though Type 2 consumers gain utility from lower prices, they lose utility because

they no longer have access to their preferred rankings  $z = 1$ . That they can only purchase less-preferred rankings for which they have a lower marginal value ( $p_{ci}^{max2}$ ) indicates that they can earn higher utility by purchasing their preferred rankings at some higher price. Define  $y_{s2}^*$  as the value of  $y_{s2}$  for which  $U_2(z_i) = U_2(z_j)$ . For some positive price  $p_{ci} > p_{cj}$ , the total utility of Type 2 consumers with  $y_{s2} \in \{y_{s2}^*, 1\}$  who purchase  $z_i > z_j$  is:

$$U_{2total}(z_i, p_{ci}; z_j, p_{cj}) = \int_{y_{s2}^*}^1 (p_{ci}^{max2} - p_{ci}) dy_{s2}$$

When  $p_{cj} = 0$ ,  $z_i = 1$ , and  $z_j = 0$ :

$$U_{2total}(z_i = 1, p_{ci}; z_j = 0, p_{cj} = 0) = \frac{2}{3} \sqrt{\frac{11}{5}} - p_{ci} - \left(\frac{10}{3} \sqrt{11} - 5\right) p_{ci}^3$$

Thus, some Type 2 consumers may earn greater utility from purchasing  $z_i = 1$  at some  $p_{ci} > 0$  compared to purchasing  $z_j = 0$  at  $p_{cj} = 0$ . I calculate the highest price  $p_{ci}$  such that firm  $i$  offering  $z = 1$  and firm  $j$  offering  $z = 0$  at  $p_{cj} = 0$  leads to greater surplus for Type 2 consumers:

$$U_{2total}(z_i = 1, p_{ci}; z_j = 0, p_{cj} = 0) > U_{2total}(z_i = 0, p_{ci} = 0; z_j = 0, p_{cj} = 0)$$

$$\frac{2}{3} \sqrt{\frac{11}{5}} - p_{ci} - \left(\frac{10}{3} \sqrt{11} - 5\right) p_{ci}^3 - \frac{2}{3} \sqrt{2} > 0$$

$$p_{ci} < 0.0454$$

Thus, when the price of  $z = 1$  is lower than 0.0454, the total surplus of Type 2 consumers increases from the provision of  $z = 1$ . Note, consumers  $y_{s2} \in [0, y_{s2}^*)$  still

purchase  $z_j$ . Their utility remains the same regardless of what product firm  $i$  offers because  $p_{cj} = 0$ .

To illustrate how, when  $p_a = 0.6$ , a segmented market that offers  $z = 1$  at some positive price increases total surplus in comparison to one with minimal differentiation, I consider the game when  $p_a = 0.6$  and  $p_L = 0.01$  instead of  $p_L = 0.15$  such that  $p_{ci} \in \{0, 0.01, 0.3\}$ .  $p_L = 0.01$  allows for a pricing option that increases total surplus. With this change, all of the threshold requirements still hold such that demand corresponds to the interior solution in Case 3 when firms do not charge the same price.

Consequently, in equilibrium, both firms offer  $z = 0$  at 0 price, and earn  $\pi_i = \pi_j = 0.3$ ; Type 2 consumer utility from purchasing  $z = 0$  at  $p_{ci} = 0$  is  $\frac{2}{3}\sqrt{2} \approx 0.9428$ .

If one firm were to deviate from its best response and offer  $z = 1$  at 0.01, total surplus in terms of both consumer surplus and profits increase. Under this deviation, Type 1 consumer surplus remains the same, as Type 1 consumers still receive their ideal ranking  $z = 0$  for free. However, Type 2 consumer surplus increases by roughly 0.036. This increase in Type 2 consumer surplus is not from increased access to rankings; all consumers buy rankings in equilibrium and in the deviation because both include a ranking offered for free. As such, the increase in Type 2 consumer welfare when one firm does not play its best response is due to the provision of a better product  $z = 1$  even at a higher price 0.01. Notably, total profits across all firms also increase by approximately 0.01, as two firms are no longer competing over a set amount of advertising revenue. Thus, total surplus increases.

## 4 Extension: Entry

The model can also be adjusted to explore how increases in advertising  $p_a$  affect entry into the rankings market. Instead of considering only two firms, 1 and 2, I now

consider three firms, 1, 2, and 3. I also expand the strategy space to include the option of not entering the market by assuming positive fixed costs that are not sunk.

The game has the same two stages. In the first stage, firms choose which, if any, rankings to produce. In the second stage, firms choose prices. However, instead of choosing rankings simultaneously, I now assume that firms choose rankings sequentially. For simplicity, firm 1 chooses first, firm 2 chooses second, and firm 3 chooses third. Prices are still chosen simultaneously.

In order to solve for equilibrium, I first calculate the equilibria and corresponding profits that result from each second stage pricing game. Subsequently, I use these profits to derive the best responses and equilibrium (equilibria) in the first stage rankings choice game.

With the addition of a third firm, there is an additional threshold value (in addition to those discussed in Case 3) to consider when all three firms potentially make sales to one type of consumer. For  $z_i < z_j < z_k$ , all three firms may make positive sales to Type 1 consumers when  $p_{ci} > p_{cj} > p_{ck}$ .<sup>10</sup> Consequently, there may now be two (instead of one) Type 1 consumers who will be indifferent between two different pairs of rankings. One consumer with  $y_{s1} = y_s^a$  receives equal utility from  $z_j$  and  $z_k$ , and another consumer with  $y_{s1} = y_s^b$  receives equal utility from  $z_j$  and  $z_i$ :

$$\begin{aligned}
 & U_1(z_j) = U_1(z_k) \\
 \longrightarrow & y_s^a = \frac{\alpha_1}{2\beta(z_k - z_j)}(p_{cj}^2 - p_{ck}^2) \\
 & U_1(z_j) = U_1(z_i) \\
 \longrightarrow & y_s^b = \frac{\alpha_1}{2\beta(z_j - z_i)}(p_{ci}^2 - p_{cj}^2)
 \end{aligned}$$

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<sup>10</sup>As firms are symmetric, all three firms may make positive sales to Type 2 consumers when  $p_{ck} > p_{cj} > p_{ci}$ .



Thus, in order for firm  $j$  to make positive sales to Type 1 consumers,  $y_s^a < y_s^b$  must hold true, as consumers  $y_{s1} \in (y_s^a, y_s^b)$  receive the highest utility from purchasing the “middle” ranking  $z_j$ . When  $y_s^a > y_s^b$ , there are no such consumers. If  $p_{ci} = p_{cj} + \mu$ ,  $\mu$  must be high enough such that some consumers will gain higher utility from a moderately priced ranking of “middle” quality than from a high price, high quality rankings:

$$\begin{aligned}
& y_s^a < y_s^b \\
& \frac{\alpha_1}{2\beta(z_k - z_j)}(p_{cj}^2 - p_{ck}^2) < \frac{\alpha_1}{2\beta(z_j - z_i)}(p_{ci}^2 - p_{cj}^2) \\
& \frac{\alpha_1}{2\beta(z_k - z_j)}(p_{cj}^2 - p_{ck}^2) < \frac{\alpha_1}{2\beta(z_j - z_i)}(2p_{cj}\mu + \mu^2) \\
& \mu > -p_{cj} + \sqrt{p_{cj}^2 + \frac{z_j - z_i}{z_k - z_j}(p_{cj}^2 - p_{ck}^2)}
\end{aligned}$$

The previous thresholds (from Case 3) capture the relationship between the consumer with  $U_1(0) = U_1(z_k)$  and the consumer with  $U_1(z_k) = U_1(z_j)$  as well as the relationship between the consumer with  $U_1(z_k) = U_1(z_j)$  and the end of the distribution  $y_{s1} = 1$ .

#### 4.1 Parameters

I use the same parameter values as in the Base Case to illustrate an entry game. To reiterate,  $\alpha_1 = 10$ ,  $\beta = 1$ , and  $\gamma_1 = 10$ . Firm  $i$  chooses  $z_i$  from  $\{0, 0.5, 1\}$ . The choices of prices are also the same where firm  $i$  chooses  $p_{ci}$  from amongst  $\{0, 0.15, 0.3\}$ . With the new assumption that FC is not already sunk, I consider  $FC = 0.1$ .

The additional threshold requires us to examine if  $p_H$  is substantially high enough such that firm  $j$ , charging a middle price  $p_L$  and offering a middle-preferred product  $z_j = 0.5$ , can make positive sales when  $z_i < z_j < z_k$  and  $p_{ci} > p_{cj} > p_{ck}$ . Using the parameters, prices, and rankings choices from the previous sections, this condition is

satisfied as:

$$p_H - p_L = 0.15 > -p_{cj} + \sqrt{p_{cj}^2 + \frac{z_j - z_i}{z_k - z_j}(p_{cj}^2 - p_{ck}^2)} = 0.0622$$

## 4.2 Changes in $p_a$

First, consider “low” levels of advertising  $p_a = 0.1$ . In this case, firm 1 will produce ranking  $z_1 = 0$  at  $p_{c1} = 0.3$ , firm 2 will produce ranking  $z_2 = 1$  at  $p_{c2} = 0.3$ , and firm 3 will not enter the rankings market. Given a fixed cost of 0.1, advertising does not increase profits sufficiently to induce firm 3 to enter. This reflects the period with limited entry before 2000 in the rankings market.

Next, consider an increase in advertising to a “moderate” level  $p_a = 0.3$ . Now, all three firms enter. Firm 1 chooses  $z_1 = 1$  and  $p_{c1} = 0.3$ . Firms 2 and 3 both choose  $z_2 = z_3 = 0$ , and, locating in the same position engage in price competition,  $p_{c2} = p_{c3} = 0$ . Due to increased advertising, profits are high enough for firm 3 to enter profitably. However, advertising does not “subsidize”  $z = 0$  sufficiently to induce firm 1 to compete with later entrants over Type 1 consumer demand and advertising revenue.

Lastly, consider a further increase in advertising to a “high” level  $p_a = 0.51$ . All three firms still enter. However, all three firms now choose rankings  $z_1 = z_2 = z_3 = 0$  and charge consumers  $p_{c1} = p_{c2} = p_{c3} = 0$ . Profits from advertising are high enough that all three firms maximize profits by engaging in price competition with respect to consumers in order to make more revenue from advertisers. The change in firm 1’s rankings product when  $p_a$  increases from 0.3 to 0.51 potentially reflects why USNWR, for instance, may have been slower to market itself as a ranking emphasizing “output” measures compared to newer entrants who entered the market with such rankings.

### 4.3 Welfare

This version of the model shows that advertising may improve welfare through encouraging entry. Advertising may increase total surplus relative to no entry by expanding access to rankings through lower prices. However, it also shows that “too much” advertising may lead to an inefficient equilibrium. Increases in  $p_a$  allow new firms to enter and produce the same ranking as one of the incumbents, but, at some levels, it may also discourage the provision of rankings preferable to some consumers. I calculate total surplus the same way as in Section 3.4 to facilitate pairwise comparisons.

First, consider “low”  $p_a = 0.1$  compared with “moderate”  $p_a = 0.3$  to see how advertising increases welfare. When  $p_a=0.1$ , firm 3 does not enter the market, and total profits across firms are  $0.6213 - 0.2 = 0.4213$ . Type 1 consumers receive total surplus of 0.6929, and Type 2 consumers also receive total surplus of 0.6929. As such, total surplus is 1.8071.

By comparison, when  $p_a = 0.3$ , profits from advertising are high enough that firm 3 enters and total profits across all three firms are 0.165, as firm 3 enters and produces the same ranking  $z = 0$  as firm 2, leading to price competition. However, this entry increases consumer surplus because it leads to lower prices that increase consumer access to rankings. Due to this increased access, consumer surplus for Type 1 consumers increases by 0.2959, and consumer surplus for Type 2 consumers increases by 0.1170. Thus, when  $p_a$  increases to 0.3, total surplus also increases to 1.9637 as a result of increased access to goods induced by entry.

Now, consider  $p_a = 0.6$  to see how “high” levels of advertising lead to inefficient outcomes by reducing the types of rankings available. In equilibrium,  $p_a = 0.6$  induces firms to offer and charge ( $z_i = 0, p_{ci} = 0; z_j = 0, p_{cj} = 0$ ). In the same way that  $p_a = 0.3$  increases total surplus,  $p_a = 0.6$  increases total surplus to 2.2316. However, this outcome is inefficient because total surplus can be further increased if one firm

(firm D) deviates from its best response and offers  $z = 1$  at a price of 0.3. While firm D's profits decrease from 0.1 to 0.065, total profits across firms increase from 0.3 to 0.465. As Type 2 consumers now purchase their preferred rankings, the increase in profits outweighs the decrease of 0.1329 in Type 2 consumer surplus from higher prices. Consequently, total surplus increases by 0.0321.

## 5 Conclusion

College rankings have become an important source of information in the college admissions market. As the number of first-generation college students, the number of applications per student, and the geographic range of applications increased, rankings emerged as a means of navigating the complex college choice process. However, even though prospective students and families may rely on rankings for information about colleges, the rankings available do not necessarily match their preferences. This occurs because, in determining what rankings product to offer, rankings firms consider revenue from advertising in addition to considering revenue from consumers.

In particular, even if consumer preferences remain relatively similar, the presence of advertising may change the rankings offered on the market. To study how advertising affects the provision of rankings goods, I modeled advertising as a two-sided market in which firms profit from both advertisers and consumers. My model demonstrated how, when advertisers target a specific subgroup of consumers, increases in advertising induce minimal differentiation between rankings because advertising effectively subsidizes one out of many possible rankings products. The welfare implications of this are two-fold. On one hand, advertising increases consumer access to goods by lowering prices. On the other hand, the equilibrium is still inefficient, as a subset of consumers loses access to its preferred product under minimal differentiation.

Considered together, these implications are troubling because of what may not

be captured in the welfare calculations. The model reflects how, with the aid of advertising, college rankings have become the widely available source of information on colleges, unlike private college counseling services and more generic guidebooks. Unlike these other resources, they are largely available free of charge to anyone who has access to the internet. As reflected by the popular obsession with college rankings, rankings are positioned to have wide-spread influence on our valuation of colleges and consequently on college education itself.

Thus, minimal differentiation may have negative effects in ways not captured by welfare calculations because it proliferates a limited vision of what a college education ought to be. Reducing considerations down to a narrow set of goals, rankings have the potential power to move college education itself towards a heavier focus on “value” and “output.” What are broader implications of viewing college education chiefly in terms of its net outputs?

Moreover, as the widely available source of information on colleges, rankings also widely influence the matches formed between students and colleges. However, looking at the measurable final payoffs to college education may not reflect much about how those end results are achieved. Different colleges may achieve similar graduation rates and measures of affordability through different methods. As students rely on rankings that elide these variations to make decisions, they may end up at institutions that do not best serve their needs.

While advertising helps make rankings available to more people, it may also influence college education and admissions in more profound, negative ways by encouraging a one-sided approach to assessing education.

## 6 Appendix: Normal Form Games

These are the first and second stage normal form games. Table 3 is the first stage game. Table 4 is the second stage game for restricted values of  $\epsilon$  and  $\delta$ , as outlined in Case 3 of Section 3.1.

Table 3: Stage 1 Rankings Game  
Firm  $i$  vertical, Firm  $j$  horizontal

	0	0.5	1
0	$\pi_i = \frac{1}{2}p_a, \pi_j = \frac{1}{2}p_a$	$\pi_i(z_i = 0, z_j = 0.5), \pi_j(z_i = 0, z_j = 0.5)$	$\pi_i(z_i = 0, z_j = 1), \pi_j(z_i = 0, z_j = 1)$
0.5	$\pi_i(z_i = 0.5, z_j = 0), \pi_j(z_i = 0.5, z_j = 0)$	$\pi_i = \frac{1}{2}p_a, \pi_j = \frac{1}{2}p_a$	$\pi_i(z_i = 0.5, z_j = 1), \pi_j(z_i = 0.5, z_j = 1)$
1	$\pi_i(z_i = 1, z_j = 0), \pi_j(z_i = 1, z_j = 0)$	$\pi_i(z_i = 1, z_j = 0.5), \pi_j(z_i = 1, z_j = 0.5)$	$\pi_i = \frac{1}{2}p_a, \pi_j = \frac{1}{2}p_a$

Table 4: Stage 2 Price Game for  $z_i < z_j$   
 $\epsilon \in (\epsilon_2, \min(\epsilon_1, \epsilon_3)), \delta \in (\delta_2, \min(\delta_1, \delta_3))$

Firm  $i$  vertical, Firm  $j$  horizontal

$j/i$	0	$p_L$	$p_H$
0	$\pi_i = p_a$ $\pi_j = 0$	$\pi_i = p_a$ $\pi_j = p_L(1 - \frac{\frac{1}{2}\alpha_1 p_L^2}{\beta(z_j - z_i)})$	$\pi_i = p_a$ $\pi_j = p_H(1 - \frac{\frac{1}{2}\alpha_1 p_H^2}{\beta(z_j - z_i)})$
$p_L$	$\pi_i = (p_L + p_a)(1 - \frac{\frac{1}{2}\alpha_1(p_L^2)}{\beta(z_j - z_i)})$ $\pi_2 = p_a(\frac{\frac{1}{2}\alpha_1(p_L^2)}{\beta(z_j - z_i)})$	$\pi_i = (p_L + p_a)(1 - \frac{\frac{1}{2}\alpha_1(p_L^2)}{\gamma_1 + \beta(1 - z_i)})$ $\pi_j = p_L(1 - \frac{\frac{1}{2}\alpha_1(p_L^2)}{\gamma_1 + \beta z_j})$	$\pi_i = (p_L + p_a)(1 - \frac{\frac{1}{2}\alpha_1(p_L^2)}{\gamma_1 + \beta(1 - z_i)})$ $+ p_L(\frac{\frac{1}{2}\alpha_1(p_H^2 - p_L^2)}{\beta(z_j - z_i)} - \frac{\frac{1}{2}\alpha_1 p_L^2}{\gamma_1 + \beta z_j})$ $\pi_j = p_H(1 - \frac{\frac{1}{2}\alpha_1(p_H^2 - p_L^2)}{\beta(z_j - z_i)})$
$p_H$	$\pi_i = (p_H + p_a)(1 - \frac{\frac{1}{2}\alpha_1 p_H^2}{\beta(z_j - z_i)})$ $\pi_j = p_a(\frac{\frac{1}{2}\alpha_1 p_H^2}{\beta(z_j - z_i)})$	$\pi_i = (p_H + p_a)(1 - \frac{\frac{1}{2}\alpha_1(p_H^2 - p_L^2)}{\beta(z_j - z_i)})$ $\pi_j = (p_L + p_a)(\frac{\frac{1}{2}\alpha_1(p_H^2 - p_L^2)}{\beta(z_j - z_i)} - \frac{\frac{1}{2}\alpha_1 p_L^2}{\gamma_1 + \beta(1 - z_j)})$ $+ p_L(1 - \frac{\frac{1}{2}\alpha_1 p_L^2}{\gamma_1 + \beta z_j})$	$\pi_i = (p_H + p_a)(1 - \frac{\frac{1}{2}\alpha_1 p_H^2}{\gamma_1 + \beta(1 - z_i)})$ $\pi_j = p_H(1 - \frac{\frac{1}{2}\alpha_1 p_H^2}{\gamma_1 + \beta z_j})$

## References

- Anderson, S., & Coate, S. (2005). Market Provision of Broadcasting: A Welfare Analysis. *Review of Economics Studies*, 72, 947–972.
- Arrow, K. (1962). Economic Welfare and the Allocation of Resources for Invention. *National Bureau of Economic Research*, (The Rate and Direction of Inventive Activity: Economic and Social Factors), 609–626.
- Average Rates of Growth of Published Charges by Decade. (2014). *College Board*. Retrieved from <http://trends.collegeboard.org/college-pricing/figures-tables/average-rates-growth-tuition-and-fees-over-time>
- Bastedo, M., & Bowman, N. (2009). *College Rankings as an Interorganizational Dependency: Establishing the Foundation for Strategic and Institutional Accounts*.
- Beebe, J. J. (1977). Institutional Structure and Program Choices in Television Markets. *The Quarterly Journal of Economics*, 15–37.
- Bowman, N., & Bastedo, M. (2008). *Getting on the Front Page: Organizational Reputation, Status Signals, and the Impact of U.S. News and World Report on Student Decisions*.
- Bowman, N., & Bastedo, M. (2010a). Anchoring Effects in World University Rankings: Exploring Biases in Reputation Scores.
- Bowman, N., & Bastedo, M. (2010b). U.S. News & World Report College Rankings: Modeling Institutional Effects on Organizational Reputation. *American Journal of Education*.
- Frequently Asked Questions: 2014 Best Colleges Rankings - US News. (2013, September 9). *US News & World Report*. Retrieved from <http://www.usnews.com/education/best-colleges/articles/2013/09/09/frequently-asked-questions-2014-best-colleges-rankings>
- Gabszewicz, J. J., Laussel, D., & Sonnac, N. (2000). TV-broadcasting competition and advertising (CORE Discussion Paper No. 2000006). *Université catholique de*



- Louvain, Center for Operations Research and Econometrics (CORE). Retrieved from <http://ideas.repec.org/p/cor/louvco/2000006.html>
- Gal-Or, E., & Dukes, A. (2003). Minimum Differentiation in Commercial Media Markets. *Journal of Economics & Management Strategy*, 12(3), 291–325.
- Griffith, A., & Rask, K. (2005). The Influence of the U.S. News and World Report Collegiate Rankings on the Matriculation Decision of High-Ability Students: 1995-2004. *Cornell Higher Education Research Institute (CHERI)*. Retrieved from <http://digitalcommons.ilr.cornell.edu/cheri/29>
- Hoover, E. (2010, November 5). College Applications Continue to Increase. When Is Enough Enough? *The New York Times*. Retrieved from <http://www.nytimes.com/2010/11/07/education/edlife/07HOOVER-t.html>
- Hossler, D., Kinzie, J., Palmer, M., Hayek, J., & Jacob, S. (2004). Fifty Years of College Choice: Social, Political and Institutional Influences on the Decision-making Process. *Lumina Foundation for Education*. Retrieved from <http://www.luminafoundation.org/publications/Hossler.pdf>
- Howard, C. (2013, July 14). America's Top Colleges 2013. *Forbes*. Retrieved from <http://www.forbes.com/sites/carolinehoward/2013/07/24/americas-top-colleges-2013/>
- Jin, G., & Whalley, A. (2007). The Power of Information: How Do U.S. News Rankings Affect the Financial Resources of Public Colleges?
- Kaminer, A. (2013, October 27). Lists That Rank Colleges' Value Are on the Rise. *The New York Times*. Retrieved from <http://www.nytimes.com/2013/10/28/education/lists-that-rank-colleges-value-are-on-the-rise.html>
- Meredith, M. (2004). Why Do Universities Compete in the Ratings Game? An Empirical Analysis of the Effects of the “U.S. News and World Report” College Rankings. *Research in Higher Education*, 45(5), 443–461.
- Nicholson, W., & Snyder, C. (2012). *Microeconomic Theory: Basic Principles and Extensions* (11th ed.). Mason: South-Western Cengage Learning.

Princeton Review. Various Issues 1992-2014. *The Best Colleges*. New York: Random House, Inc.

Spence, M., & Owen, B. (1977). Television Programming, Monopolistic Competition, and Welfare. *The Quarterly Journal of Economics*, 91(1), 103–126.  
doi:10.2307/1883140

Steiner, P. (1952). Program Patterns and Preferences, and the Workability of Competition in Radio Broadcasting. *The Quarterly Journal of Economics*, 194–223.

Total fall enrollment in degree-granting institutions, by level of student, sex, attendance status, and race/ethnicity: Selected years, 1976 through 2010. (2011, November). *Digest of Education Statistics*. Retrieved from [http://nces.ed.gov/programs/digest/d11/tables/dt11\\_237.asp](http://nces.ed.gov/programs/digest/d11/tables/dt11_237.asp)

U.S. News and World Report. Various Issues 1983-2010. *America's Best Colleges*.